Synergetic Approach for the Selection of a Furnace Fuel in Production of Ferrous Castings in an Eco-friendly Environment

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Abstract

The problem of selection of fuel for the production of castings in most economical way and in an eco-friendly environment is of great relevance in the present time. Traditionally, such problems were addressed using conventional techniques of Multi Criteria Decision Making, such as, Analytical Hierarchy Process (AHP). This paper proposes a methodology for solving common problem of fuel selection by modifying conventional AHP by incorporating 'fuzzy Linguistic variables' in place of numbers. The methodology will create a fuzzy interface for conversion of input and output variables into suitable linguistic variables. Further, employing the fuzzification process by assigning the linguistic variables to numerical values of the membership functions and formulating suitable decision rules, the procedure culminates into the defuzzification process for converting fuzzy output into crisp value and obtaining the result in the form of fuzzy scores. The model is explained using a numerical example and also presents a validation of the proposed methodology.

Keywords: Fuel Selection, Fuzzy, AHP.

1. INTRODUCTION

Agra produces all grades of castings ranging from normal grey iron castings to graded and ductile castings. There are approximately 340 small and medium scale cast iron foundries in Agra. They manufacture general castings along with the graded and quality castings as required by large and renowned private and public sector undertakings like Escorts, Kirloskar, Maruti, ABB, Punjab Tractors, etc. The quality of castings produced is excellent and because of it there is a huge turnover. Beside this, several units are export oriented exporting C.I. pipes and fittings to Middle East and African countries. A few units are also exporting C.I. castings to Great Britain. A number of small scale foundries produce general grey cast iron castings e.g. Cylinder block, flywheel, gearbox body etc. which are used in Diesel engines, diesel pumps and generator industry. The diesel engines produced by these units are not only indigenously used in agricultural and domestic markets but are exported World wide also.

Thousands of small machining workshops/ industries have sprung up in Agra whose primary business is to procure castings from the foundries and after finishing and machining, to supply them to leading diesel engines manufacturers.

Further, there exist hundreds of small-scale foundries using crucible furnaces as melting technique. Unfortunately the gases emitted by these foundries are harmful pollutants. The major pollutants that are generated are suspended particulate matter (SPM), carbon monoxide CO, carbon dioxide CO_2 and sulphur dioxide SO_2. A recent investigation carried out in foundries has shown that the emission levels are exceedingly high. In large number of foundries the average SPM level in the exhaust gasses is about 1500 mg/Nm3, which are ten times the permissible limits. The gasses emitted by these foundries are pollutants that violate the Clean Air Legislation Act.

Almost all the foundries used coke-fired cupolas for the melting. However, melting by coke fired cupola does not obey the environmental regulations. Therefore, all the 310 foundries lying in Taj Trapezium Zone have been closed down by the order of The Honourable Supreme Court of India. The emission of the gases, suspended particles, dust and grit has not been found much beyond the acceptance limits of the pollution control limits.

This has affected approximately 3 lakh people related to this industry in Agra directly or indirectly. Because of the closure of foundries, the Diesel Engines and Generator industry is also on the verge of closing. Thus, there is a need for a melting technique with suitable fuel which is technically feasible, economically viable and environmental friendly. Rotary furnace came out to be an alternative for production of quality castings in an eco friendly environment. Lot of experiments have been performed on self designed and fabricated Rotary Furnace by the Department of Mechanical Engineering, Faculty of Engineering, Dayalbagh Educational Institute with LDO (Light Diesel Oil) as fuel. The results are quite encouraging but still the pollutants are not within the prescribed limits of CPCB (Central Pollution Control Board). Considering the grave situation, it is imperative to find a feasible, affordable and adequate fuel by using which not only Taj Mahal but human life is also protected.

In this paper an attempt has been made to find such an alternative fuel which enables to rehabilitate the foundries without affecting the grandeur of Taj Mahal. So there is a need for a better fuel which will give the casting in an eco-friendly environment to the full satisfaction of CPCB. This problem of Fuel selection is, one of the complicated ones in which each alternative is to be judged on a multidimensional scale for which no industry wide standard exist.

Several researchers have addressed themselves to the problem of Robot selection over the past few years. An overview of the important techniques is given below:
(a) Multi Criteria Decision Making (MCDM) Techniques: Techniques like AHP by Saaty (1988) and Multi Attribute Utility Theory (MAUT) by Keeny and Raiffa(1993) have been proposed in literature for the selection of discrete set of technological alternatives as per their relative importance. However, these techniques are known to become increasingly difficult for the decision makers with increase in the number of criteria of evaluation. Goh (1997) has addressed the problem of Robot selection using AHP and Nnaji and Yannacoupoulou (1988) has addressed it using MAUT

Computer Assisted Models: Models proposed by Offodile et.al. (1987). Agrawal et. al. (1991) etc., use an expert system to evaluate a large number of Robots attributes and criteria for concluding a feasible list of Robots. Such a process is usually based on theoretical assumptions and provides an imperfect fitment.

Techniques Employing Soft Computing Tools: Recent tools of fuzzy reasoning, Genetic algorithm etc. are being used for such problems with fair success. Khouja and Booth (1995) and Liao(1996) have used clustering technique and 'Multi Criteria Decision Making Heuristic' employing Fuzzy logic and given a new direction to this problem. Other techniques inspired by various tools of Management and Financial analysis such as Economic Feasibility Evaluation, Strength – Weakness – Opportunity – Threat Analysis (SWOT) and Cost-Benefit Analysis etc., have been used with limited success.

1.1 Analytic Hierarchy Process

One very popular technique of MCDM variety is the Analytic Hierarchy Procedure (AHP) suggested by *Saaty (1988)*. It involves breaking down of the decision problem into a hierarchy of interrelated 'decision elements'. The first level of the 'hierarchy of objectives' is made up of one element, the subsequent levels of the hierarchy contain attributes which contribute to the nature of the overall objective and the elements of the lowest level are the alternatives (element to be evaluated). Once the hierarchy of objectives has been constructed, the process requires that each pair of elements (A_i, A_j) be taken and the expert(s) asked to respond to the pair wise comparison of "Which of A_i and A_j is more important and by how much (how many times)?" with a ratio. Saaty suggests that a 1-9 scale be used to quantify the 'decision makers' strength of feeling between any two attributes with respect to any given criterion. This involves introducing individual judgements into the analysis by equating them to arbitrary numerical values. Such judgements are supposed to represent the articulation of “the tradeoffs among the conflicting criteria” and are often highly subjective in nature. Therefore the process makes use of a suitable process to estimate relative weights of the decision elements and culminates into their aggregation in order to arrive at the outcome.

Verbal judgements	Equally Preferred	Moderately Preferred	Strongly Preferred	Very Strongly Preferred	Extremely Preferred
Numerical	1	3	5	7	9
Linguistic	Very low	Low	Medium	High	Very High

TABLE 1: Comparison of Saaty’s numerical ratings with authors’ linguistic ratings.

Triantaphyllou and Mann (1994) have pointed out another drawback. They demonstrate for an AHP, that as the alternative increases, even a small change in the numerical rating or the weighting factor increases the probability that the model will drastically fail.

1.2. Fuzzy Logic Theory

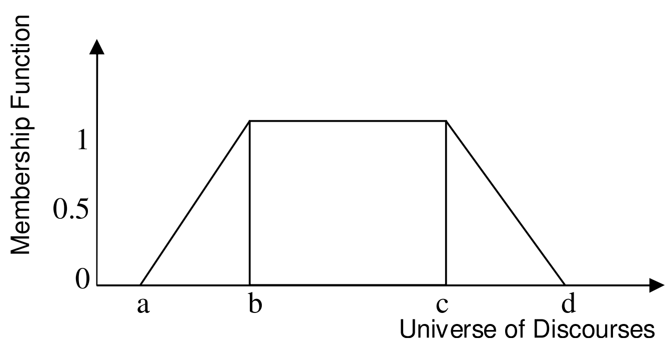
It was introduced by Zadeh (1965) and has since gained much importance in practical applications such as process control, Flexible manufacturing, Flexible automation, MCDM (Multi Criteria Decision Making), etc. Fuzzy Logic aims at providing a body of concepts and techniques for dealing with modes of reasoning which are “approximate rather than exact”.

1.3. Fuzzy Numbers

It is a recent endeavour of mathematicians to express numbers in an approximate manner and to carry out computations on them using fuzzy arithmetic. By a Fuzzy Number is meant a number that is ‘a special fuzzy subset of real numbers’.
The membership function (uA) of a fuzzy number is a continuous mapping from R to a closed interval [0,1].In this paper Trapezoidal Fuzzy numbers have been used which are of the form [a,b,c,d] and have straight line segments for µA(x) in [a,b] and [c,d].

Mathematically, a Fuzzy number A in R is defined as a Trapezoidal Fuzzy number (a, b, c, d) if its membership function µA: R → [0, 1] is given by eq. (1) and graphically represented in figure1.

$$\mu_A(x) = \begin{cases} (x-a)/\,(b-a) & \text{for } a \le x \le b \\ 1 & \text{for } b \le x \le c \\ (x-d)/(c-d) & \text{for } c \le x \le d \\ 0 & \text{otherwise} \end{cases} \quad \ldots (1)$$



Value between b & c belong to fuzzy Number completely
Value before a or after d do not belong to it at all
Value between a & b or c & d belong partially

FIGURE 1: Membership function diagram of a ‘Trapezoidal Fuzzy Number’

1.4. Fuzzy Linguistic Variables

By a ‘Fuzzy Linguistic Variable’ we mean “an expression in natural or artificial language” that represents a variety of values .They are commonly depicted as being spread over the ‘Universe of Discourse’ by means of overlapping triangles, trapezoidal or any other suitable geometric representations. In this paper Linguistic Values from the term set {Very Low, Low, Medium, High, Very High} are employed to explicate the fuzzy linguistic variables 'preference' and Linguistic values from the term set {Very Poor, Poor, Fair, Good, Very Good} are employed to explicate 'suitability'.

As against the use of non-fuzzy numerical values for quantification of subjective opinions, the concept of "Fuzzy Linguistic Variables' provides us with a convenient means of making subjective judgements about complex or ill-defined situations.

2. THE PROBLEM

The problem of selection of fuel is a complex exercise that poses impediments of the following kind:

- (i) The options available for selection offer a wide spectrum of choice.
- (ii) Fuel needs to be evaluated over both subjective and objective parameters.
- (iii) Parameters for selection are usually context dependent and most of the criteria have an element of fuzziness.
- (iv) All the parameters may not be of equal importance for any activity. There are certain parameters of the 'must' type while others are merely of desirable type.(Groover et.al.(1986)) There is lack of industrial standards and benchmarks for the performance evaluation of the fuels to decide on the suitability of any alternative.

Hence, the problem of Fuel selection for melting cast iron is still an open problem and, it appears that, we need an approach that offers a means of modelling situations which are known sketchily or approximately and possess an inherent vagueness.

2.1 Selection Parameters

Groover (Gro 1986) suggested to divided the list of technical features in to two categories: 'must' and 'desirable'. The must features are those, which should essentially be satisfied by any fuel to perform satisfactorily. The desirable features are those which are not necessarily required to accomplish any task but would be highly beneficial during installation and / or operation. To select parameters influencing the melting process, a survey has been carried out by taking the views of the experts of the Foundry. A questionnaire has been developed to decide about the parameters and their effectiveness that have to be taken for consideration while selecting the melting fuel. The expert identified *Performance, Quality* and *Cost* as the three main criteria for evaluation. They found that all the three vaguely known and hence realized that they may more appropriately be described fuzzily. Further, some parameters have been identified by them on which these three main criteria depend.

(a) Attributes Influencing Fuel Performance

Though performance of a fuels is a term which cannot ordinarily be broken up into smaller elements, however its dependence may be investigated over (i) Thermal efficiency, (ii) Specific fuel consumption, (iii) Emission, (iv) Melting Rate. A study of various fuels from the fuel suppliers show that following three are most common and sensitive parameters taken to specify a fuel. The performance of the fuel is mainly dependent on the above four sub criteria in the proposed methodology.

(b) Attributes Influencing Quality

The quality of any product has been defined as "the totality of features and characteristics that bear on its ability to satisfy stated or implied needs." Despite the fact that various fuels users may have different needs and the exercise could be very context dependent, the experts suggest sensitive attributes viz., Cetane number, Viscosity, Flash point and Fire point for the purpose of evaluating the fuel quality.

(c) Attributes Influencing Cost

The attribute cost needs little explaining since it's among the most common criteria for any selection. Also, there is always an element of fuzziness attached to it when it is said that a fuel costs between Rs. 30/liter to Rs. 70/liter. The cost criteria can be further be subdivided into three more sub-criteria viz., Percentage yield, Availability and Fuel Refining cost.

3. CONCEPT, METHODOLOGY AND ANALYSIS

In this paper the authors propose a methodology that links Fuzzy Logic with conventional AHP and use it to solve common problems of fuel Selection. In order to incorporate fuzziness in the conventional AHP, the authors propose to measure the values of the input variables and output variables, used in the process of selection, by converting them into phrases or words from a natural language. This is also known as creation of the Fuzzy interface for any algorithm and is carried out by using trapezoidal distribution over the normalized 'Universe of Discourse'.

The generation of the membership function values for the fuzzy linguistic variables used in this analysis is an outcome of a field study involving 'direct' allocation of domain space for several linguistic values by a large number of people. The term set {Very low, low, medium, high, Very high} has been taken to represent the verbal attitudes of a group of decision makers for evaluating the preference of various alternatives and {Very Poor, Poor, fair, Good, Very Good} has been taken to represent the Term set for evaluating the suitability of various alternatives. The approach adopted here is hierarchical in nature. For any 'p' number of fuel alternative to be evaluated by any 'q' number of decision makers over any 'i' number of criteria.

3.1. Methodology of Fuzzy AHP

Listed below is the step by step methodology of Fuzzy AHP based fuel selection procedure developed by the authors:

1. Identify the fuel selection problem that requires Multi criteria Decision Making.
2. Choose the criteria on the basis of which of selection needs to be carried out. The choice may be of parameters that can be described qualitatively as well as those that are described quantitatively. Take the number of criteria identified for analysis to be i.
3. From the possible fuels available for selection, carry out preliminary screening and identify the number of alternatives that needs rigorous evaluation. Take the number of alternatives available for this rigorous exercise as *p* and the number of decision makers carrying out the evaluation to be *q.*
4. Decide Term Sets of Linguistic values to be used like {Very low, low, medium, high, Very high} for 'preference' and {Very Poor, Poor, fair, Good, Very Good} for evaluating the 'suitability' of various alternatives against the identified criteria; and choose appropriate values for their Membership Functions.
5. Ask each of the *q* decision makers to assign relative weights to each criterion using Linguistic Values from the Term set for '*preference*'. Thus, if the weight assigned for any j^{th} criterion (such that j = 1, 2,…, i), by the r^{th} decision maker (such that r = 1, 2,…, q) is as given W_{jr} ,then this step would provide us with a total of i×q weights, each of the form : $W_{jr} = (a_{jr}, b_{jr}, c_{jr}, d_{jr})$, j = 1, 2,…,i; r =1,2,…, q.
6. From the assigned values, calculate aggregate preference weights reflecting the collective opinion of all decision makers by using the mean Operator. Any other operator may also be used if the designer so desires. If we denote the mean aggregated preference weight for any criterion C_j by W_j then it may be computed as given in Eq. (2).
   $W_j = [\sum W_{jr}]/q$ where r = 1 to q. …(2)
   Where $W_j = (a_j, b_j, c_j, d_j)$ and $a_j = [\sum a_{jr}] /q$, $b_j = [\sum b_{jr}] /q$, $c_j = [\sum c_{jr}] /q$,
   $d_j = [\sum d_{jr}] /q$.
7. If any of the i criteria can be further subdivided and evaluated using related sub criteria then the overall influence of these sub criteria with respect to the criterion in question may also be calculated using the above aggregation formula. For example, if any jth criterion has say k sub criteria, then each sth sub criterion (such that s = 1,2,…,k) would have an observed weight of the form $W_{jsr} = (a_{jsr}, b_{jsr}, c_{jsr}, d_{jsr})$, j = 1,2,…,i; r = 1,2,...,q; m = 1,2,...,p. An aggregated weight of the form $W_{js} = (a_{js}, b_{js}, c_{js}, d_{js})$, may similarly be obtained using Eq. (2). This state is an optional one and may be skipped if such detailing is not desired.
8. Ask each of the q decision makers to evaluate the suitability of each p alternative against each i criterion using linguistic variables from the Term Set for 'Suitability'. Thus, if the suitability assigned to any mth alternative (such that m = 1,2,…,p) when evaluated against any jth criterion (such that j =1,2,…,i),by the r^{th} decision maker (such that r=1,2,…,q) is as given by S_{jmr} ,then this step would provide the evaluator with a total of i×p×q observations (I tables of p×q entries), each of the form: $S_{jmr} = (a_{jmr}, b_{jmr}, c_{jmr}, d_{jmr})$. However, if any criterion demonstrates dependence over say, k sub criteria, then it would

provide k tables each of p×q size for that criterion alone. Each observation in such a situation may be denoted by S_{mr} , s=1,2,…,k; m=1,2,…,p; r=1,2,…,q. If this is the case, aggregation of 'suitability' for such sub criteria may be obtained using Steps 9 & 10. Otherwise, if none of the criteria shows dependence over sub criteria then go directly to step 12.

9. Use the formula for weighted mean given in Eq. (2) to get the aggregation of suitability for each of the s^{th} sub criterion (such that s=1,2,…,k) with respect to that jth main criterion (which has dependence over k sub criteria) and report it in all p×q segments of the resulting table. If we denote the weighted aggregation of suitability for such a criterion by S_{jmr} then its value may be computed as given in Eq. (3).

   $S_{jmr} = \sum[S_{smr} \times W_{js}]/ \sum W_{js}$ where s=1 to k ....(3)

   While S_{jmr} obtained thus is the form of $(a_{jmr}, b_{jmr}, c_{jmr}, d_{jmr})$, it is not a trapezoidal Fuzzy variable. It merely represents a weighted approximation for observed suitability of jth criterion, as evaluated by the r_{th} decision maker for mth alternative, aggregated over k sub criteria.
10. Check if the values obtained in the step 9 satisfy the condition ($0\le a_{jmr} \le 1$, $0\le b_{jmr} \le 1$, $0\le c_{jmr} \le 1$, $0\le d_{jmr} \le 1$) for every S_{jmr} .If they do, proceed ahead. If not, use the process of normalization to make them conform to this condition.
11. Check if any of the remaining criteria can be further subdivided and evaluated using related sub-criteria. If yes, then the overall influence of those sub criteria with respect to the criterion in question may also be calculated using the aggregation formula in Eq. (3). If no, then proceed to Step 12.
12. Calculate the suitability index for each mth alternative against every j_{th} criterion using Eq. (4). Since m=1,2,…,p; j=1,2,….,i; this stage would give us a total of p×i entries.

   For any C_j ; $S_{jm} = [\sum S_{jmr}]/q$ ....(4)

   Where; $S_{jm} = (a_{jm}, b_{jm}, c_{jm}, d_{jm})$, and $a_{jm} = [\sum a_{jmr}] /q$, $b_{jm} = [\sum b_{jmr}] /q$, $c_{jm} = [\sum c_{jmr}] /q$, $d_{jm} = [\sum d_{jmr}] /q$.
13. Calculate the fuzzy score for each m_{th} alternative using Eq. (5).

   $\Gamma_m = [\sum(S_{jm} \times W_j)]/i$ for j=1,2,…,i; m=1,2,….,p ....(5)
14. Convert each Fuzzy Score into its corresponding crisp value using the process of defuzzification by Weighted Average Method as given in Eq. (6). This method weighs each x_{th} value of the base variable by its respective maximum membership value $\mu_m (x)$ for any 'm' Fuzzy Number (or variable).

   $\Gamma_m (mean) = [\sum\mu_m (x) \times x] / \sum \mu_m (x)$ ....(6)

   Since the fuzzy score obtained from equation Eq. (5) is not a trapezoidal Fuzzy number, it is suggested that only four landmark values of the base variable for each Fuzzy number (or variable) of the form (a, b, c, d) viz., [{a+(b-a)/2}, b, c and {c+(d-c)/2}] be taken up for this computation.
15. The alternative giving highest value for Γ is the best choice from the available Fuels alternatives.

3.2. Analysis of a Hypothetical Problem

Step 1:- Consider the fuels which are to be selected for production of castings in a most economical way and in an eco-friendly environment. As the experts have ascertained that MCDM is required; it is decided to solve the problem using the proposed Fuzzy AHP.

Step 2:- Assuming that the experts identify Performance, Quality and Cost as the three main criteria for evaluation. They find that all the three are vaguely known and hence realize that they may more appropriately be described fuzzily.

Steps 3:- After a preliminary screening the experts identify the number of alternatives for rigorous evaluation as 3. They are:

(i) Jatropha oil
(ii) Karanja oil
(iii) LDO.

Let $D_1,D_2,…,D_r,…,D_q$ be the notations used for 'q' numbers of decision makers and A_1, $A_2,…,A_r,…,A_p$ for 'p' alternatives being evaluated over $C_1,C_2,…,C_r,…,C_j$ criteria. For this example p=3, q=4 and j=3.

Step 4:- Experts use {Very low, low, medium, high, Very high} for evaluating the preferences and {very poor, poor, fair, good, very good} for evaluating the suitability of various alternatives against the identified criteria.

Let the Membership functions of the five linguistic variables for 'preference' from the term set {Very low, low, medium, high, Very high}, herein abbreviated as {VL, L, M, H, VH}, be trapezoidal in shape and be numerically expressed as : VL={0.00,0.00,0.05,0.25}; L={0.05,0.20,0.25,0.40}; M={0.25,0.40,0.45,0.60}; H ={0.50,0.65,0.70,0.90}; VH={0.75,0.90,1.00,1.00}.

Let the membership functions of the linguistic variables for suitability from the set {Very poor, Poor, Fair, Good, Very good} abbreviated as {VP, P, F, G, and VG}. These are numerically expressed as

VP={0.00,0.00,0.05,0.25}; P={0.05,0.20,0.25,0.40}; F={0.25,0.40,0.45,0.60};
G={0.50,0.65,0.70,0.85}; VG={0.75,0.90,1.00,1.00}

Step 5:- Let the four decision makers weigh their preference for the three criteria as given in table 2.

Alternatives	D_1	D_2	D_3	D_4
Jatropha	VH	VH	H	M
Karanja	M	M	H	VH
LDO	L	L	M	H

TABLE 2: Relative importance of the three criteria for the purpose of determining their weightage.

Step 6:- After evaluation let the preference weights expressed by each expert be aggregated using Eq. (2) as demonstrated below. For the first criterion C_1,

$W_1 = [W_{11} + W_{12} + W_{13} + W_{14}]/ 4$

$W_1 = (H+VH+H+M)/4$

$= [(0.75+0.75+0.5+0.25)/4, (0.9+0.9+0.65+0.4)/4, (1+1+0.7+0.45)/4, (1+1+0.9+0.6)/4]$

$W_1 = (0.56, 0.71, 0.78, 0.87)$

Let the corresponding weight $W_2 = (0.43, 0.58, 0.65, 0.77)$ for the second criteria C_2, and $W_3 = (0.20, 0.31, 0.36, 0.53)$ for the third criterion C_3 be calculated in a similar manner.

Step 7:- Let criterion C_1(Performance) be taken to depend on four sub criteria viz. ***Thermal efficiency*** (C_{11}), ***Specific fuel consumption*** (C_{12}), ***Emission*** (C_{13}) and ***Melting Rate*** (C_{14}). Let the procedure laid out in previous stages be repeated for evaluation of preference of C_{11}, C_{12}, C_{13} and C_{14} with respect to C_1 using linguistic variables from the term set {VL,L,M,H,VH} and let be reported in table 3.

Alternatives	D1	D2	D3	D4
Thermal efficiency (C11)	H	VH	H	VH
Specific fuel consumption (C12)	M	VH	M	M
Emission (C13)	M	M	M	L
Melting Rate (C14)	L	VL	VH	VL

TABLE 3: Relative importance of sub-criteria for the purpose of determining their weightage

Let the relative weights for different sub criteria once again be evaluated using the mean operator explained in Eq. (2) and let the values obtained be represented as:

W_{11}={0.62,0.77,0.85,0.95}; W_{12}={0.37,0.52,0.58,0.7}; W_{13}={0.2,0.35,0.4,0.55};
W_{14} ={0.2,0.27,0.33,0.47}

Step 8:- Let the four experts evaluate suitability of each 3 alternatives against each criterion C_i using Linguistic Variables from the term set {VP, P, F, G, VG}. Since the first criterion C_1 has four sub criteria, let the observed suitability of each alternative with respect to C_{11}, C_{12}, C_{13} and C_{14} be given in the table 4.

Alternatives	**D_1**	**D_2**	**D_3**	**D_4**
a. *Observed suitability of each alternative with respect to (C_{11})*				
Jatropha	VG	G	F	VG
Karanja	F	F	P	F
LDO	P	P	VP	G
b. *Observed suitability of each alternative with respect to (C_{12})*				
Jatropha	VG	G	G	F
Karanja	G	VG	F	P
LDO	F	F	P	VP
c. *Observed suitability of each alternative with respect to (C_{13})*				
Jatropha	VG	F	G	G
Karanja	G	G	F	VG
LDO	F	VG	P	G
d. *Observed suitability of each alternative with respect to (C_{14})*				
Jatropha	F	G	VG	G
Karanja	G	F	P	VP
LDO	G	P	F	P

TABLE 4: Suitability of each alternative with respect to each attribute influencing Performance

Step 9:- Let the suitability of each alternative for C_{11}, C_{12}, C_{13} and C_{14} with respect to C_1 be aggregated using the weighted mean method. Let the resulting aggregated values be obtained according to Eq. (3) and reported in all p×q segments. This gives the weighted aggregation of suitability for j^{th} criterion (here j =1 to 3), as evaluated by r^{th} decision maker (here r =1 to 3) for each m^{th} alternative (here m = 1 to 3), aggregated over k sub criteria (here k = 4).

Alternatives	D_1	D_2	D_3	D_4
Jatropha	S_{111}	S_{112}	S_{113}	S_{114}
Karanja	S_{121}	S_{122}	S_{123}	S_{124}
LDO	S_{131}	S_{132}	S_{133}	S_{134}

TABLE 5: Suitability of each alternative with respect to C1 (Performance)

Then m=1 and r = 1, in the case of the first criterion (j=1) the aggregation would be:
S_{111}= [{VG×W_{11}} + {VG×W_{12}} + {VG×W_{13}} + {F×W_{14}}] / [W_{11}+ W_{12}+ W_{13}+ W_{14}]
S_{111} = [0.67, 0.82, 0.91, 0.92]

Alternatives	D_1	D_2	D_3	D_4
Jatropha	0.67,0.82,0.91,0.92	0.46,0.60,0.65,0.79	0.42,0.58,0.64,0.78	0.54,0.68,0.74,0.83
Karanja	0.38,0.54,0.60,0.76	0.41,0.58,0.64,0.75	0.13,0.29,0.34,0.49	0.23,0.37,0.43,0.56
LDO	0.19,0.35,0.41,0.57	0.20,0.38,0.44,0.57	0.05,0.14,0.20,0.38	0.30,0.40,0.45,0.61

TABLE 6: The aggregated values of suitability of each alternative with respect to C_1

Step 10:- Let it so happen that the values of Table 6 are not contained within the range of 0 to 1. Let this be rectified using the process of normalization. Let each entry of the Table 6 of the form (a, b, c, d) be divided by the largest of all the entries from the entire table which is of the form {a_{max}, b_{max}, c_{max}, d_{max}}. Let this step provide values that are between 0 and 1, each of the form (a/a_{max}, b/b_{max}, c/c_{max}, d/d_{max}). Expectedly, the slot having {a_{max}, b_{max}, c_{max}, d_{max}} would become unity of the form {1.0, 1.0, 1.0, 1.0} after normalization. Let the resulting values (after normalization) be given in Table 7.

Alternatives	D_1	D_2	D_3	D_4
Jatropha	1.0,1.0,1.0,1.0	0.68,0.72,0.71,0.85	0.62,0.70,0.70,0.84	0.80,0.82,0.81,0.90
Karanja	0.57,0.66,0.65,0.81	0.61,0.70,0.70,0.81	0.19,0.35,0.37,0.53	0.34,0.45,0.47,0.61
LDO	0.28,0.42,0.44,0.61	0.3,0.46,0.48,0.61	0.08,0.17,0.22,0.41	0.44,0.49,0.49,0.65

TABLE 7: The final values of suitability of each alternative after normalization

Step 11:- Let criterion C_2 (Quality) be also taken to depend on three sub criteria viz. ***Cetane Number*** (C_{21}), ***Viscosity*** (C_{22}) and ***Flash point & Fire point*** (C_{23}). Let this procedure laid out in previous stages be repeated for evaluation of preference of C_{21}, C_{22} and C_{23} with respect to C_2 using Linguistic Variables from the Term Set {VL, L, M, H, VH} and let it be reported in Table 8.

Alternatives	D_1	D_2	D_3	D_4
Cetane Number (C_{21})	VH	H	H	VH
Viscosity (C_{22})	H	M	M	M
Flash point & Fire point (C_{23})	M	VH	VH	H

TABLE 8: Three sub criteria with respect to C2 (Quality) for the purpose of determining their weightage

Let the relative weights for the three sub criteria be again evaluated using Eq. (2) and let the aggregated values be given as:

W_{21} =(0.625,0.775,0.85,0.95);W_{22} =(0.313, 0.463, 0.513,0.675);W_{23}=(0.563, 0.713,0.788,0.88)

The decision makers evaluate suitability of each alternative against C_{21}, C_{22} and C_{23} and let it be reported in the Table 9. Proceeding as above, let the final values of suitability of each alternative with respect to C_2 be obtained and reported in table 11. Let the resulting values (after normalization) be given in table 12.

Alternatives	D_1	D_2	D_3	D_4
a. *Observed suitability of each alternative with respect to C_{21}* (Cetane Number)				
Jatropha	VG	G	VG	F
Karanja	VG	G	F	F
LDO	F	P	P	P
b. *Observed suitability of each alternative with respect to C_{22}* (Viscosity.)				
Jatropha	F	VG	G	VG
Karanja	G	G	F	P
LDO	VG	F	G	F
c. *Observed suitability of each alternative with respect to C_{23} (Flash point & Fire point)*				
Jatropha	G	F	P	F
Karanja	VG	G	G	G
LDO	F	P	F	P

TABLE 9: Suitability of each alternative with respect to each attribute influencing Quality

Alternatives	D_1	D_2	D_3	D_4
Jatropha	S_{211}	S_{212}	S_{213}	S_{214}
Karanja	S_{221}	S_{222}	S_{223}	S_{224}
LDO	S_{231}	S_{232}	S_{233}	S_{234}

TABLE 10: Suitability of each alternative with respect to C_2 (Quality)

Alternatives	D_1	D_2	D_3	D_4
Jatropha	0.61,0.69,0.75,0.83	0.45,0.61,0.67,0.80	0.43,0.58,0.65,0.74	0.35,0.51,0.58,0.70
Karanja	0.69,0.84,0.92,0.95	0.5,0.65,0.7,0.85	0.34,0.49,0.54,0.68	0.30,0.44,0.49,0.63
LDO	0.35,0.51,0.58,0.70	0.09,0.24,0.29,0.45	0.21,0.37,0.43,0.59	0.09,0.24,0.29,0.45

TABLE 11: The aggregated values of suitability of each alternative with respect to C_2

Alternatives	D_1	D_2	D_3	D_4
Jatropha	0.88,0.82,0.81,0.87	0.65,0.73,0.73,0.83	0.62,0.69,0.70,0.78	0.50,0.61,0.62,0.73
Karanja	1.0,1.0,1.0,1.0	0.71,0.77,0.75,0.88	0.49,0.58,0.58,0.71	0.43,0.52,0.53,0.66
LDO	0.50,0.61,0.62,0.73	0.13,0.29,0.32,0.47	0.31,0.45,0.46,0.61	0.13,0.29,0.32,0.47

TABLE 12: The final values of suitability of each alternative after normalization

Step 12:- Let criterion C_3 (Cost) be also taken to depend on three sub criteria viz. ***Percentage yield*** (C_{31}), ***Availability*** (C_{32}) and ***Fuel refining cost*** (C_{33}). Let this procedure laid out in previous stages be repeated for evaluation of preference of C_{31}, C_{32} and C_{33} with respect to C_3 using Linguistic Variables from the Term Set {VL, L, M, H, VH} and let it be reported in Table 13.

Alternatives	D_1	D_2	D_3	D_4
Percentage yield (C_{31})	VH	H	H	H
Availability (C_{32})	H	M	VH	M
Fuel refining cost (C_{33})	M	L	M	L

TABLE 13: Three sub criteria with respect to C_3 (Cost) for the purpose of determining their weightage.

Let the relative weights for the three sub criteria be again evaluated using Eq. (2) and let the aggregated values be given as:
W_{31}=(0.563, 0.713, 0.775, 0.93); W_{32} = (0.438, 0.588, 0.65, 0.775); W_{33} = (0.15, 0.3, 0.35, 0.5)
The decision makers evaluate suitability of each alternative against C_{31}, C_{32} and C_{33} and let it be reported in the table 14. Proceeding as above, let the final values of suitability of each alternative with respect to C_3 be obtained and reported in table 16. Let the resulting values (after normalization) be given in table 17.

Alternatives	D_1	D_2	D_3	D_4
a. *Observed suitability of each alternative with respect to C_{31} (*Percentage yield*)*				
Jatropha	G	F	VG	G
Karanja	P	VP	P	P
LDO	VG	G	G	F
b. *Observed suitability of each alternative with respect to C_{32} (*Availability*)*				
Jatropha	VG	P	VG	G
Karanja	G	G	G	VG
LDO	P	VP	F	F
c. *Observed suitability of each alternative with respect to C_{33} (*Fuel refining cost*)*				
Jatropha	F	G	VG	VG
Karanja	VP	P	F	P
LDO	G	F	G	G

TABLE14: Suitability of each alternative with respect to each attributes influencing cost.

Alternatives	D_1	D_2	D_3	D_4
Jatropha	S_{311}	S_{312}	S_{313}	S_{314}
Karanja	S_{321}	S_{322}	S_{323}	S_{324}
LDO	S_{331}	S_{332}	S_{333}	S_{334}

TABLE 15: Suitability of each alternative with respect to C_3 (Cost)

Alternatives	D_1	D_2	D_3	D_4
Jatropha	0.56,0.69,0.76,0.84	0.20,0.37,0.42,0.58	0.75,0.9,1.0,1.0	0.53,0.69,0.75,0.88
Karanja	0.21,0.32,0.37,0.52	0.19,0.27,0.32,0.49	0.24,0.40,0.45,0.60	0.31,0.45,0.52,0.61
LDO	0.45,0.59,0.66,0.75	0.27,0.36,0.41,0.58	0.40,0.55,0.60,0.76	0.28,0.44,0.49,0.65

TABLE 16: The aggregated values of suitability of each alternative with respect to C_3

Alternatives	D_1	D_2	D_3	D_4
Jatropha	0.75,0.77,0.76,0.84	0.27,0.42,0.43,0.59	1.0,1.0,1.0,1.0	0.71,0.77,0.76,0.89
Karanja	0.28,0.36,0.38,0.52	0.26,0.30,0.32,0.49	0.33,0.45,0.46,0.60	0.42,0.51,0.53,0.61
LDO	0.60,0.66,0.67,0.76	0.37,0.40,0.41,0.59	0.54,0.62,0.61,0.76	0.38,0.49,0.49,0.66

TABLE 17: The final values of suitability of each alternative after normalization

Step 13:- Let the aggregated Suitability Index (S_{jm}) be calculated for each criterion using Eq. (4) as demonstrated below:

$S_{jm} = [\sum S_{jmr}] / q$

Let this calculation for suitability index (S_{jm}) provide a weighted approximation for the observed suitability of j_{th} criterion (here j=1 to j=3), as evaluated by r_{th} decision maker (here r =1 to 4) for m_{th} alternative (here m = 1 to 3), aggregated over k sub criteria. For the first criterion C_1,

$S_{11} = [S_{111} + S_{112} + S_{113} + S_{114}]/4$

S_{11}= (1+0.68+0.62+0.80)/4,(1+0.72+0.70+0.82)/4,(1+0.71+0.71+0.81)/4,(1+0.86+0.84+.90)/4]

S_{11} = (0.777, 0.814, 0.811, 0.902)

Similarly other values for S_{jm} are:

S_{12} = (0.431, 0.543, 0.552, 0.693); S_{13} = (0.279, 0.390, 0.412, 0.577)

S_{21} = (0.667, 0.717, 0.719, 0.807); S_{22} = (0.660, 0.721, 0.717, 0.815)

S_{23} = (0.270, 0.414, 0.432, 0.575); S_{31} = (0.683, 0.740, 0.736, 0.829)

S_{32} = (0.325, 0.406, 0.420, 0.558); S_{33} = (0.471, 0.546, 0.546, 0.688)

Step 14.Finally for the purpose of final ranking, let Eq. (5) be used to obtain a simple fuzzy score (Γ) for each alternative.

$\Gamma_m = [\sum (S_{jm} \times W_j)]/ I$; For j = 1 to i & m = 1 to p; Here p=3

$\Gamma_m = (1/i) [(S_{1m} \times W_1) + (S_{2m} \times W_2) + \ldots + (S_{jm} \times W_i)]$

For the first alternative Jatropha oil; $\Gamma_1 = 1/3[(S_{11} \times W_1) + (S_{21} \times W_2) + (S_{31} \times W_3)]$

Γ_1=1/3[(0.777×0.56), (0.814×0.71), (0.811×0.78), (0.902×0.87) + (0.667×0.43), (0.717×0.58), (0.719×0.65), (0.807×0.77) + (0.683×0.20), (0.740×0.31), (0.736×0.36), (0.829×0.53)]

Γ_1=1/3[(0.437,0.580,0.638, 0.789)+(0.291,0.421,0.467,0.625)+(0.136,0.231,0.266,0.445)]

Γ_1 = [0.288, 0.411, 0.457, 0.620]

For Karanja oil; Γ_2 = [0.198, 0.312, 0.351, 0.513]

For LDO; Γ_3 = [0.123, 0.230, 0.268, 0.440]

Step 15.Let the fuzzy score of each alternative be converted into its corresponding crisp value using the process of Defuzzification as given in Eq. (6)

Let the defuzzified score for each oil be obtained as follows:

For Jatropha oil

Γ_1(mean)=[(0.349×0.5) + (0.411×1.0) + (0.457×1.0) + (0.539×0.5)] / (0.5+1.0+1.0+0.5)

= **0.437**

For Karanja oil;

Γ_2(mean)=[(0.255×0.5) + (0.312×1.0) + (0.351×1.0) + (0.432×0.5)] / (0.5+1.0+1.0+0.5)
= **0.336**

For LDO;

Γ_3(mean)={(0.177×0.5) + (0.230×1.0) + (0.268×1.0) + (0.177×0.5)] / (0.5+1.0+1.0+0.5)
= **0.254**

Step 16.Let the alternatives giving highest value for Γ be the best choice from the available Fuel alternatives.

As alternative 1 i.e. Jatropha has the highest value of Γ, it is declared as the best choice of the available 3 oils alternatives.

4. CONCLUSION

For the methodology of Fuzzy AHP explained above a program was written in C language and was tested on the example presented in the paper and was found giving satisfactory results.

The final scores came out to be Γ1 (mean) > Γ_2(mean) > Γ_3(mean). This implies that oil 1 is most suitable choice followed by oil 2 and 3.

For the purpose of validation of the methodology used in this paper, the authors compare the solutions rendered by existing models of the two kinds. One, Khouza and Booth (1995) using computerized fuzzy clustering procedure for selection from twenty seven alternatives over four criterion and the other using non-fuzzy or crisp selection process of Goh(1997) which uses the conventional AHP for the selection from four alternatives over three criterion viz., performance, quality and cost. The results reported by Goh are reproduced in table 18.

For the purpose of validation both the problems were suitably coded and solved by the methodology presented in this paper. As can be seen from the table 18 the obtained results are in total agreement with those obtained by other researcher. It establishes the validity of the proposed methodology.

Alternative	**Authors' Results**		**Goh's Results**	
	Fuzzy Score	Oil Ranking	Fuzzy Score	Ranking
1	0.437	1	0.42	1
2	0.336	2	0.30	2
3	0.254	3	0.21	3

TABLE 18: Comparison of solution to the problem given by conventional AHP

However the methodology proposed in this paper is an improvement over the conventional technique as it can even be used for solving problems based on purely subjective parameters which allows freedom to the evaluators to express their views. Further, in this methodology there is no serious limitation in increasing the evaluators or alternatives but the methodology uses the unique property of approximating the information offered by fuzzy logic rather than carrying out the precise analysis. The methodology is equally effective for crisp values and non-fuzzy situations.

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