On the Analysis of the Laminar to Turbulent Flow Patterns in the Treatment of a Patient Receiving Oxygen

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Abstract

For a fluid, the transition from laminar to turbulent flow is a function of the fluid's speed, direction, applied pressure, pipe length, pipe radius, fluid viscosity, and fluid density. For human breathing, all of these parameters are generally beyond control, except for the fluid's density and viscosity. If the human has trouble breathing, laminar flow is preferred since the person does less work for each breath. In our analysis, the pipe is the airway (or breathing tube) from lips to bifurcation; the throat/pipe radius is known or can be determined; the differential pressure is the excess pressure above or below atmospheric pressure; fluid flow rate is the person's tidal lung volume divided by the breathing rate. We analyze 13 widely different humans (with differing values for throat length, radius, etc.) to see the effect of breathing two different fluids: air (20% oxygen, 80% nitrogen) and HeOx (20% oxygen, 80% helium). The onset of turbulent flow occurs for the critical radius, and this is calculated for each patient. For 12 patients, the critical radius is much smaller than the throat/tube radius, if HeOx is used--the flow is laminar. For all patients breathing air, the critical radius is larger than the throat/tube radius--the flow is turbulent. Thus, HeOx is shown to be superior in treating patients with breathing problems.

Keywords: Laminar, Turbulent, Viscosity, HeOx, Endotracheal.

1. INTRODUCTION

The focus of this paper is to derive the critical radius r_c, where r_c is the radius of the pipe (throat) such that laminar flow and turbulent flow are of equal intensity. To put this another way, r_c defines the boundary between turbulent and laminar flow. When radius < r_c, the flow is turbulent. If the radius exceeds r_c, the flow becomes laminar.

The governing equation for fluid flow in a pipe is [1 - 5]

$$\Delta P = \frac{8\eta L F}{\pi r^4} + \frac{16 L \rho F^2}{10^4 R_e \, \pi^2 r^5} \tag{1}$$

Where the first term describes laminar fluid flow and the second term describes turbulent flow. Equation (1) is a form of Rohrer's equation [3], with values of radius r set at the point where the laminar and turbulent flows are equal. If we ignore the second term, then equation (1) becomes Ohm's law, and all fluid flow is laminar. In Ohm's Law, an electromotive force (voltage) causes a flow of current through a resistor; the length and area of the resistor factor in to the value of the resistance. In (1) a mechanical force (the change in pressure above or below the atmospheric value (expressed in Pascals)) produces a current or flow F (cm^3/sec) multiplied by the resistance. The resistance is proportional to η the viscosity (Pascal.second), the pipe length L (cm), and inversely proportional to the pipe radius r (cm). The viscosity has a value of 18.3 (air), 20.3 (oxygen), 19.4 (helium), and 19.6 (HeOx), in units of micropascal.seconds. L is approximately 20 to 23 cm in length for an adult. Whether the patient is intubated or not, this value remains the same. However, the value of r is either the radius of the ETT (endotracheal tube) used to intubate or the average radius of the airway. ETT's are 7 to 8.5 mm, inner diameter. Patient's airways are larger, and they vary according to the age, sex, weight, height, and health of the patient. See references [6, 7] for assorted data on various types of patients. The data that we quote in this paper comes from specific patient medical records. NOTE: all medical records quoted here are purged of any specific patient identification.

Equation 1 is true for any radius of pipe, if we ignore the second (i.e. turbulent) term. Equation 1 is true for both terms, if the radius is equal to the critical radius (r_c) or close to the value of the critical radius, i.e. the radius where term # 1 equals term # 2. The reason for this limited range of applicability is due to the fact that the nonlinear regime for fluid flow is complex and requires a power series expansion in the variable F (fluid flow) to fully quantify all effects that can occur. Our concept of turbulent flow (as limited to its use in this paper) is to describe one stage of complexity greater than simple laminar flow. Turbulent flow is governed by F raised to the second power; the Reynolds number is fixed at 2000. As the flow increase still further, the turbulence must be described by F raised to the third power, and later the forth, etc. Since it is not our purpose to dwell on all of the mathematics governing the flow process, we refer to Figures 1 and 2. Normal stream lines represent laminar flow. Erratic streamlines represent turbulence. Turbulent flow increases with increasing velocity and with bends and twists in the pipe.

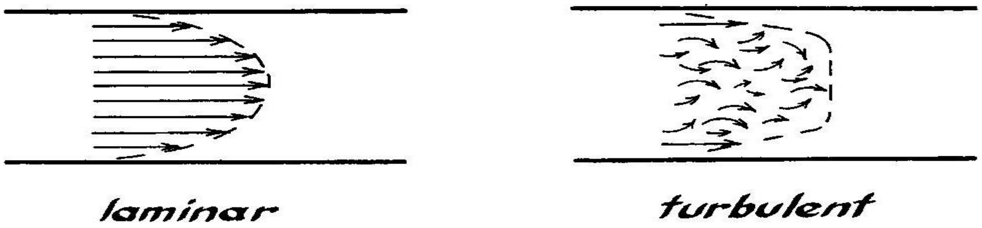

FIGURE 1: Velocity profiles for laminar and turbulent flow in circular pipes [1].

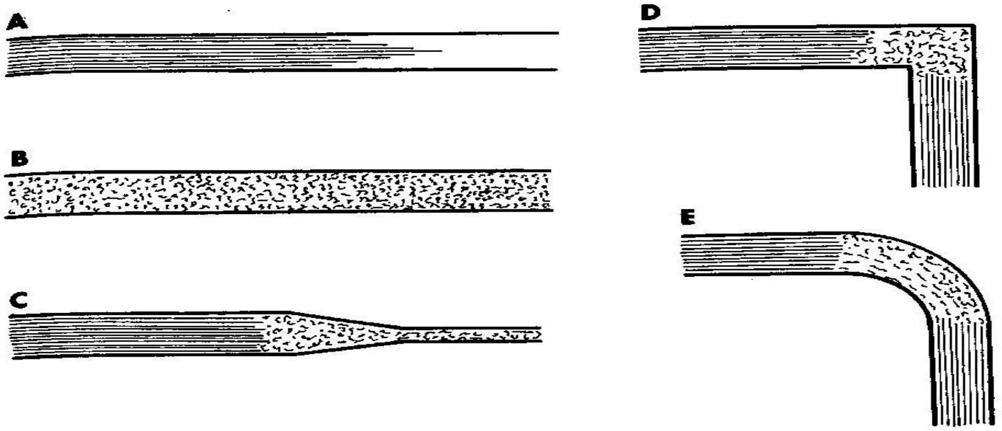

FIGURE 2: Laminar flow (A) versus turbulent flow (B) is shown by normal straight streamlines versus short erratic lines. (C) As a pipe's radius decreases the fluid's velocity increases, and the fluid changes from laminar to turbulent. For sharp bends (D) and even for gradual bends (E) in the pipe, the laminar flow becomes turbulent, even if it is only a local effect. This figure is adapted from Reference 8.

The pipe length is the distance from lips to bifurcation. Please note: in some hospital settings, an endotracheal tube is inserted into the patient's mouth. The distance from lips to bifurcation is generally 20 to 24 cm (depending on the patient and assuming the patient is an adult). However, the tube can be inserted to a greater length (at least 4 cm longer) if it diverts to only one lung. This practice is not recommended, however, since it is inefficient for a person with 2 functioning lungs. It is only used for patients in whom one lung is missing or defective to the extent that the oxygen best serves the patient's other lung.

It is well known in electricity that Ohm's Law breaks down if we approach the saturation current of the conductor. This is never seen for normal conductors like copper. The reason is that the saturation current for a normal sized copper wire is over a billion amps [9]. It takes only several hundred amps to vaporize a copper wire. Therefore, it is impossible to obtain the saturation current. By contrast, consider a semiconductor. Since the conductivity is orders of magnitude lower than copper or metal, the saturation current is much lower. Consider the conduction channel in a JFET (junction field effect transistor). With no gate voltage applied, the channel current (called the source current) will saturate if the drain-source voltage is made too large. The value of this current is of the order of milliamps.

Just as a large increase in applied voltage pushes current to saturate, a large pressure difference causes fluid flow to saturate. The region between linear (laminar) and saturation flow is a transition state, which we know as turbulent flow. Complete turbulence leads to chaos and the fluid flow is a fixed value, no matter how high the pressure. Less turbulence leads to a relationship which is quantified as the second term in equation (1).

Term # 2 in (1) shows that the pressure difference varies as the square of the flow rate. Re is the Reynolds number, and for the transition value between the laminar/turbulent flow in a pipe, Re = 2000 [5]. Density ρ is in units of (kg/m^3), with the pressure still in Pascals. If we set the first and second terms in (1) equal, we can come up with the critical radius (r_C), i.e. the radius where 50% of the flow is Laminar and 50% turbulent.

Here is the approach we will take with the sick human being, i.e. patient. Ambient pressure is the atmospheric pressure (1.01×10^5 Pascals). During normal human breathing, a human will exert a pressure difference to inhale (inspiration) or exhale (expiration) air. The value of this extra pressure can be positive or negative and it typically varies from a value of zero to 30 Pascals. TV is the tidal volume (in cm^3), i.e. the extra volume of the lungs as they expand to take in the fresh air. The time for inspiration and expiration is not the same; generally it takes twice as long to expire a breath than it did to inspire it [10]. As an example, if a person breathes at the rate of 20 breaths per minute, it takes 3 seconds to complete a breath, with one second spent on inspiration and 2 spent on expiration. Since the flow rate equals the tidal volume divided by time, the flow is larger for inspiration. To put this another way, there is more likelihood of a patient having breathing trouble during inspiration, since the flow rate is twice as large. In this paper, we will use the time of inspiration to focus on laminar vs. turbulent flow with the idea that whatever our results for inspiration, our results for expiration will be better, i.e. more likely laminar, since the flow rate is cut to one half. One other thing to be noted is patients with COPD (chronic obstructive pulmonary disease). Their expiration time is longer than normal. Hence, the inspiration time once again becomes the more sensitive parameter in determining breathing problems.

Our focus is to show that the HeOx solution is easier to breathe than regular air [11, 12]. The change in pressure is fixed. If the patient is on a ventilator or breathing on his own, then the change in pressure above below atmospheric pressure is fixed for a given person. The length and radius of the patient are also fixed and depend on the patient's airway or ETT.

Our analysis proceeds in this fashion: set term # 1 and term #2 equal to each other. This assumes that laminar and turbulent flows are equal. The length and all other parameters are fixed, and we compute the radius, i.e. the critical radius (r_c). We actually compute 2 values for the critical radius, one for HeOx and one for air. Then compare this to the radius of the patient's throat or ETT.

Table I lists the critical radius (r_c) for air and HeOx as well as the radius of the person's throat or ETT. This data comes from the personal medical files in a hospital with the patient's ID removed. In all cases, HeOx is better than air, i.e. in all cases the air flow remains more like a laminar than a turbulent flow.

Patient description	#1 Radius parameters for a patient's throat or ETT (cm)	#2 Critical radius (r_c) using air (cm)	#3 Critical radius (r_c) using HeOx (cm)	#4 Flow as tidal volume divided by inspiration time (cm^3/seconds)
18 year-old female, 110 pounds, Caucasian 4'9", ETT-L= 21 cm, no past medical history	0.375	0.94	0.31	500/1
39 year-old male, 205 pounds, Caucasian, 6'2", no ETT, L = 22 cm, healthy	0.5 to 0.6	0.83	0.27	800/2
52 year-old male, 176 pounds, Italian, 5'8", Tachycardia (rapid heart rate) and breathless, no ETT, L = 30 cm	0.40	0.72	0.22	560/1.7
60 year-old female, 154 pounds, Hispanic, after Coronary Artery graft bypass, L=22 cm, no ETT	0.37 to 0.45	1.47	0.45	700/1
20 year-old female, 132 pounds, Black, ETT-L = 22 cm, following minor surgery	0.375	0.63	0.33	600/2

56 year-old female, 132 pounds, Hispanic, ETT-L = 22 cm, smoker and COPD	0.36	0.63	0.19	600/2
25 year-old female, 125 pounds, Caucasian, Pregnant, ETT-L = 21 cm	0.36	0.77	0.24	550/1.5
45 year-old male, 180 pounds, Caucasian, in remission for cancer – underwent right lung lobectomy to remove 30% of right lung, emphysema, takes shallow breaths at rapid rate on ventilator and requires higher lung volumes as bullae from on eroding alveoli, ETT-L = 22 cm	0.40	0.96	0.30	550/1.5
61 year-old male, 154 pounds, Caucasian,following coronary artery bypass, ETT-L = 22 cm	0.40	0.47	0.14	600/1.5
27 year-old female, 132 pounds, Hispanic, undergoes lymph node breast biopsy, no past medical history and no illness, ETT-L = 23 cm	0.375	0.48	0.15	450/2
30 year-old male, 154 pounds, Causian, 5’9”, following an asthma attack but not 0.57 intubated, L = 12.7 cm; Note: normal flow is 500/1, but this is reduced to 300 to 1 to include effects of asthma	0.54 to 0.57	0.60	0.20	300/1
68 year-old male, 154 pounds, 5’6”, mixed Asian, chronic lung disease/emphysema, decreased lung capacity, ETT-L = 22 cm	0.40	0.53	0.16	250/1

TABLE 1: Measured throat radius (or ETT radius), critical radius, and flow rate are cited for 13 patients.

2. CONCLUSION

There are several conclusions that we can obtain from this work. First and most important, the mixture of oxygen and helium produces a substitute for air that is laminar, even under the most adverse conditions. Normal, healthy people can breathe air in a fashion that is turbulent. Every sharp twist and turn between the lips, throat, and bifurcation can cause simple, laminar flow to go turbulent. Rapid breathing also increases the probability of turbulent flow. All of these conditions are of no significance in a healthy person. But for a sick person with breath difficulty or even for a healthy person who has undergone surgery and is intubated, the process of breathing laminar is very important. The ETT itself is generally free from sharp bends and kinks which promote turbulent flow. In addition, the HeOx mixture lowers the critical radius (r_c) for the onset of turbulent flow by over 300 % (or a factor of more than 3). See Table I.

A second thing to be noted from Table I is that sometimes the radius of the airway is a variable, due to the lack of simple smoothness of the throat. Even if the person is intubated, the ETT may not have the value of radius for which it is listed. For example, an ETT with inner diameter of 8 mm has a radius of 0.4 cm. However, often the sick patient produces secretions which compromise this value so that the actual value of the tube's radius is lower by up to 20% [13, 14, 15]. But even in this case, the HeOx mixture proves up to the task of maintaining laminar flow.

A third thing to be noted from our data is that there is nothing significant about the race or sex or age of the patient. All persons studied were adults. If children and infants were included, there would, of course, be a profound effect. But neither race nor sex nor age played a role in our overall analysis. Rather, the patient's size and medical history were the determining factors in our analysis. Granted, an older patient who smokes is more likely to have a long medical history than a younger person who smokes. In that sense, age is a strong factor. But the old and the young patient can have the same results if their medical history and size are the same.

It should be noted that our mathematical analysis of the data in Table I shows HeOx to be better than air in 12 of 13 cases, but in practice, all 13 patients improved their comfort with HeOx, i.e. they breathed easier.

3. ACKNOWLEDGEMENT

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