Propagation of Love Waves Through an Irregular Surface Layer in the Presence of a Finite Rigid Barrier

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Abstract

The effect of irregularities present in the surface layer has been discussed in the present paper. The irregularity is in the form of a finite rigid barrier in the surface layer. The surface layer has been assumed to be homogeneous, isotropic and slightly dissipative. The reflected, transmitted and scattered waves have been obtained by Fourier transform and Wiener-Hopf technique. The numerical computation has been done by taking the barriers of different sizes. The amplitude of the scattered and the reflected waves has been plotted against the wave number. The scattered waves behave as decaying cylindrical waves at distant points. The amplitude of the scattered waves falls off rapidly as the wave number increases slowly. The amplitude of the reflected Love wave decreases rapidly with the wave number and ultimately becomes saturated which shows that the reflected Love wave takes a very long time to dissipate making these the most destructive waves during the earthquake.

Keywords: Cylindrical Waves, Fourier Transforms, Scattered Waves, Surface Layer, Wiener-Hopf Technique.

1. INTRODUCTION

Love waves are surface seismic waves that cause horizontal shifting of earth during the earthquake. The particle motion of Love waves forms a horizontal line perpendicular to direction of propagation. The theory of elastic waves finds numerous applications in seismology and geophysics. Seismic signals are applied to investigate the internal structure of earth. During earthquake seismic waves such as Love waves are generated from interior of earth. Love waves are transversely propagated surface waves which we feel directly during earthquake. They are reflected and transmitted due to the presence of irregularities like rocks, mountains, ditches, trenches etc. in the crustal layer of earth. The scattering of Love waves due to the surface defects' results in large amplification of the waves during earthquake, making these the most destructive surface waves. The propagation of love waves in the presence of rigid barrier in the layer of thickness H superimposed on a solid half space $z \geq 0$ has been discussed herewith using Wiener-Hopf technique [6] and Fourier transform [7].

This paper is based on a paper by Sato [8] who studied the problem of reflection and transmission of Love wave at a vertical discontinuity in a surface layer. The author found the approximate solution of the problem and showed the relationship between reflection and transmission coefficients by graphs, by assuming the layer of small thickness as compared with the wavelength. Ashgar and Zaman [1] have solved the problem of diffraction of Love waves normally incident on two parallel perfectly weak half planes lying in surface layer. Tomar and Kaur [9] have studied the problem of reflection and transmission of a plane SH-wave at a corrugated interface between a dry sandy half space and an anisotropic elastic half space. They used the Rayleigh's method of approximation for studying the effect of sandiness, the anisotropy, the frequency and the angle of incidence on the reflection and transmission coefficients. Chattopadhyay et al. [2] have studied the similar type of problem by taking shear waves in viscoelastic medium at parabolic irregularity. They found that amplitude of reflected wave decreases with increasing length of notch and increases with increasing depth of irregularity. Kaur et al. [5] have studied the reflection and refraction of SH-waves at a corrugated interface between two laterally and vertically heterogeneous viscoelastic solid half-spaces. The propagation of seismic waves has also been studied by Zaman [10], and Zhang and Chan [11]. In all the earlier studies, comparative discussion is missing. Here we discuss the propagation of Love waves through irregularity in form of rigid barrier and for comparison purpose, the numerical computations have been discussed by taking the different sizes of rigid barrier.

2. PERLIMINARIES

The scattering of incident Love waves at the rigid barrier in the surface layer has been discussed in the present paper. The problem is two dimensional and is being analyzed in zx plane. The z axis has been taken vertically downwards and x axis along the interface. The geometry of the problem is shown in figure 1.

The incident Love wave is given by

$$v_{0,1} = A\cos\theta_{2N} H e^{-(\theta_{1N} z + i k_{1N} x)}, \quad z \geq 0, \tag{1}$$

$$v_{0,2} = A\cos\theta_{2N}(z+H) e^{-i k_{1N} x}, \quad -H \leq z \leq 0 \tag{2}$$

where,

$$\theta_{2N} = \sqrt{k_2^2 - k_{1N}^2}, \quad \theta_{1N} = \sqrt{k_{1N}^2 - k_1^2}, \ |k_1| < |k_2| \tag{3}$$

and k_{1N} is a root of the equation

$$\tan\theta_{2N} = \gamma \frac{\theta_{1N}}{\theta_{2N}}, \ \gamma = \frac{\mu_1}{\mu_2}, \tag{4}$$

$\mu_1 \, and \, \mu_2$ being the rigidities of shear waves in the half space and in the crustal layer respectively.

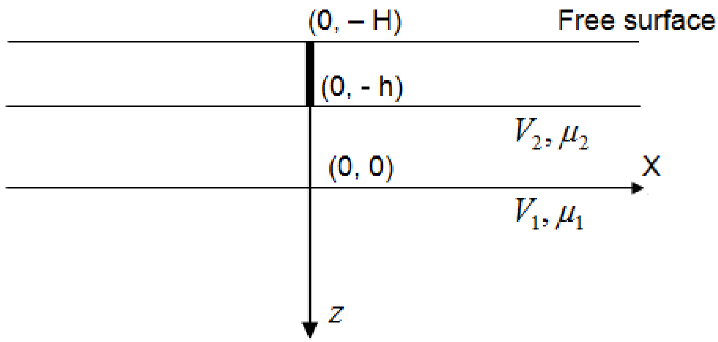

FIGURE1: Geometry of the problem

The wave equation in two dimensions is given as

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}+\frac{\varepsilon}{c^2}\frac{\partial u}{\partial t}, \tag{5}$$

where, $\varepsilon > 0$ is the damping constant and c is the velocity of propagation. If the displacement be harmonic in time, then

$$u(x,z,t) = v(x,z)e^{-i\omega t} \tag{6}$$

and equation (5) reduces to

$$\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial z^2}+k^2 v=0. \tag{7}$$

The above wave equation in the present study can be written as

$$(\nabla^2 + k_j^2)v_j = 0, \quad j=1,2 \tag{8}$$

where,

$$k_j = \sqrt{\frac{\omega^2 + i\varepsilon\omega}{V_j^2}} = k_j^{'} + ik_j^{''}. \tag{9}$$

V_1 and V_2 are respectively the velocities of shear waves in the half space $z \geq 0$ and in the layer $-H \leq z \leq 0$.

Let the total displacement be given by

$$v = v_{0,1} + v_1, \qquad z \geq 0 \quad , \quad -\infty < x < \infty, \tag{10}$$

$$= v_{0,2} + v_2, \quad -H \leq z \leq 0 \quad , \quad -\infty < x < \infty. \tag{11}$$

The boundary conditions are

$$\nu_{0,2} + \nu_2 = 0, \qquad -H \le z \le -h,\ x = 0\,, \tag{12}$$

$$\frac{\partial}{\partial z}(\nu_{0,2} + \nu_2) = 0, \qquad x \ge 0, \quad x \le 0, \quad z = -H, \tag{13}$$

$$\nu_1 = \nu_2\,,\ \mu_1 \frac{\partial \nu_1}{\partial z} = \mu_2 \frac{\partial \nu_2}{\partial z}, \qquad z = 0,\ -\infty < x < \infty. \tag{14}$$

The condition (12) implies that the barrier is rigid and there is no displacement across it. Condition (14) specifies the continuity of displacement at the interface. From equations (2) and (12), we get

$$\nu_2 = -A\cos\theta_{2N}(z+H)e^{-ik_{1N}x}, \quad -H \le z \le -h, \quad x = 0. \tag{15}$$

Taking Fourier transform of equation (8), we obtain

$$\frac{\partial^2 \bar{\nu}_j(p,z)}{dz^2} - \theta_j^2 \bar{\nu}_j\,(p,z) = 0, \tag{16}$$

where $\theta_j = \pm\sqrt{p^2 - k_j^2}$ and $\bar{\nu}_j\,(p,z)$ represents Fourier transform of $\nu_j\,(p,z)$ which can be defined as

$$\bar{\nu}_j\,(p,z) = \int_{-\infty}^{\infty} \nu_j(x,z)e^{ipx}dx, \qquad p = \alpha + i\beta$$

$$= \int_{-\infty}^{0} \nu_j(x,z)e^{ipx}dx + \int_{0}^{\infty} \nu_j(x,z)e^{ipx}dx$$

$$= \bar{\nu}_{j-}(p,z) + \bar{\nu}_{j+}(p,z). \tag{17}$$

If for a given z, as $|\,x\,| \to \infty$ and $M, \tau > 0,\ |\,\nu_j(x,z)\,| \sim Me^{-\tau|x|}$, then $\bar{\nu}_{j+}(p,z)$ is analytic in $\beta > -\tau$ and $\bar{\nu}_{j-}(p,z)$ is analytic in $\beta < \tau\,(= lm(k_j))$. By analytic continuation, $\bar{\nu}_j\,(p,z)$ and its derivatives are analytic in the strip $-\tau < \beta < \tau$ in the complex p - plane [3].

Solving equation (16) and choosing the sign of θ_j such that its real part is always positive, we obtain

$$\bar{\nu}_1\,(p,z) = A(p)e^{-\theta_1 z}, \quad z \ge 0 \tag{18}$$

and

$$\bar{\nu}_2\,(p,z) = B(p)e^{-\theta_2 z} + C(p)e^{\theta_2 z}, \quad -H \le z \le 0. \tag{19}$$

Solving equation (19) by using boundary condition (14), we get

$$\bar{v}_2(p,z) = A(p)\frac{[\theta_2 \cosh\theta_2 z - \gamma\theta_1 \sinh\theta_2 z]}{\theta_2}. \tag{20}$$

Differentiating equation (20) with respect to z and putting $z=-h$ and denoting $\bar{v}_j(p,-h)$ by $\bar{v}_j(p)$ etc., eliminating $A(p)$, we obtain

$$\bar{v}_2(p) = [\bar{v}_{2+}(p) + \bar{v}_{2-}(p)] = -\frac{\theta_2 \cosh\theta_2 h + \gamma\theta_1 \sinh\theta_2 h}{\theta_2(\theta_2 \sinh\theta_2 h + \gamma\theta_1 \cosh\theta_2 h)} \times [\bar{v}'_{2+}(p) + \bar{v}'_{2-}(p)]. \tag{21}$$

Now multiplying equation (8) by e^{ipx} and integrating from 0 to ∞ $(j=2)$

$$\frac{d^2}{dz^2}[\bar{v}_{2+}(p,z)] - \theta_2^2\,\bar{v}_{2+}(p,z) = \left(\frac{\partial v_2}{\partial x}\right)_{x=0} - ip(v_2)_{x=0}. \tag{22}$$

Changing p to - p in equation (22) and subtracting the resulting equation from it, we get

$$\frac{d^2}{dz^2}[\bar{v}_{2+}(p,z) - \bar{v}_{2+}(-p,z)] - \theta_2^2[\bar{v}_{2+}(p,z) - \bar{v}_{2+}(-p,z)] = -2ip(v_2)_{x=0}. \tag{23}$$

Using equation (15) in equation (23), we obtain

$$\frac{d^2}{dz^2}[\bar{v}_{2+}(p,z) - \bar{v}_{2+}(-p,z)] - \theta_2^2[\bar{v}_{2+}(p,z) - \bar{v}_{2+}(-p,z)] = 2ipA\cos\theta_{2N}(z+H). \tag{24}$$

The solution of equation (24) is given as

$$\bar{v}_{2+}(p,z) - \bar{v}_{2+}(-p,z) = D_1(p)e^{-\theta_2 z} + D_2(p)e^{\theta_2 z} - \frac{2ipA\cos\theta_{2N}(z+H)}{p^2 - k_{1N}^2}. \tag{25}$$

Using boundary condition (13) in equation (25), we find

$$\bar{v}_{2+}(p,z) - \bar{v}_{2+}(-p,z) = D(p)\cosh\theta_2(z+H) - \frac{2ipA\cos\theta_{2N}(z+H)}{p^2 - k_{1N}^2}. \tag{26}$$

Differentiating equation (26) with respect to z, putting $z=-h$ in resulting equation and in (26) and then eliminating $D(p)$, we have

$$\bar{v}_{2+}(p) - \bar{v}_{2+}(-p) = \frac{\coth\theta_2\delta}{\theta_2}\left[\bar{v}'_{2+}(p) + \bar{v}'_{2+}(-p) - \frac{2ipA\,\theta_{2N}\sin\theta_{2N}\delta}{p^2 - k_{1N}^2}\right] - \frac{2ipA\cos\theta_{2N}\delta}{p^2 - k_{1N}^2} \tag{27}$$

where $\delta = H - h$.

Eliminating $\bar{v}_{2+}(p)$ from equations (21) and (27), we get

$$\frac{\coth\theta_2\delta iA\theta_{2N}\sin\theta_{2N}\delta}{\theta_2(p+k_{1N})}+\frac{iA\cos\theta_{2N}\delta}{p+k_{1N}}-\frac{f_1(p)\bar{v}_{2+}'(p)}{\theta_2 f_2(p)\sinh\theta_2\delta}=\bar{v}_{2-}(p)-\bar{v}_{2+}(-p)$$
$$+\frac{f_3(p)\bar{v}_{2-}'(p)}{\theta_2 f_2(p)}-\frac{iA\cos\theta_{2N}\delta}{p-k_{1N}}-\frac{\coth\theta_2\delta iA\theta_{2N}\sin\theta_{2N}\delta}{\theta_2(p-k_{1N})}-\frac{\coth\theta_2\delta\bar{v}_{2+}'(-p)}{\theta_2} \tag{28}$$

where,

$$f_1(p)=\theta_2\sinh\theta_2 H+\gamma\theta_1\cosh\theta_2 H, \tag{29}$$

$$f_2(p)=\theta_2\sinh\theta_2 h+\gamma\theta_1\cosh\theta_2 h, \tag{30}$$

$$f_3(p)=\theta_2\cosh\theta_2 h+\gamma\theta_1\sinh\theta_2 h. \tag{31}$$

The equation (28) is the Wiener-Hopf type differential equation whose solution will give $\bar{v}_{2+}(p)$.

2.1 Solution of the Wiener-Hopf Equation

For solution of equation (28), we factorize $\left(\frac{\theta_2\delta}{\sinh\theta_2\delta}\right)\frac{f_1(p)}{f_2(p)}$ [8] as

$$\left(\frac{\theta_2\delta}{\sinh\theta_2\delta}\right)\frac{f_1(p)}{f_2(p)}=K_+(p)K_-(p), \tag{32}$$

where,

$$K_+(p)=K_-(-p)=\frac{L_+(p)}{H_+(p)}\prod_{n=1}^{\infty}\frac{(p+p_{1n})}{(p+p_{2n})}, \tag{33}$$

$p=\pm p_{1n}$ and $p=\pm p_{2n}$ being the zeros of $f_1(p)$ and $f_2(p)$ respectively.

We now decompose $\frac{\coth\theta_2\delta}{\theta_2\delta}$ as

$$\frac{\coth\theta_2\delta}{\theta_2\delta}=F_+(p)+F_-(p), \tag{34}$$

where,

$$F_+(p)=F_-(-p)=-\frac{1}{2k_2\delta(p+k_2)}+\sum_{n=1}^{\infty}\frac{1}{p_n\delta(p+ip_n)}. \tag{35}$$

Also we write,

$$\frac{f_3(p)}{\theta_2 f_2(p)}=R(p)=R_+(p)R_-(p), \tag{36}$$

and $\frac{f_3(p)}{f_2(p)}$ tends to 1 as $|p| \to \infty$.

Now $\frac{f_3(p)}{f_2(p)}$ can be factorized by infinite product theorem as

$$\frac{f_3(p)}{f_2(p)} = \prod_{n=1}^{\infty} \frac{p^2 - p_{3n}^2}{p^2 - p_{2n}^2} G_-(p) G_+(p), \tag{37}$$

where,

$$\log G_+(p) = \frac{1}{\pi}\int_0^{\infty} \frac{N_1 - N_2}{t - ip} dt - \frac{1}{\pi}\int_0^{k_1} \frac{M_1 - M_2}{t + p} dt - \frac{1}{\pi}\int_{k_1}^{k_2} \frac{1}{t + p} dt, \tag{38}$$

and

$$\tan N_1 = \frac{\alpha' \cos \alpha' h}{\gamma (t^2 + k_1^2)^{1/2} \sin \alpha' h},$$

$$\tan N_2 = \frac{\gamma (t^2 + k_1^2)^{1/2} \cos \alpha' h}{\alpha' \sin \alpha' h},$$

$$\tan M_1 = \frac{\alpha'' \cos \alpha'' h}{\gamma (k_1^2 - t^2)^{1/2} \sin \alpha'' h},$$

$$\tan M_2 = \frac{\gamma (k_1^2 - t^2)^{1/2} \cos \alpha'' h}{\alpha'' \sin \alpha'' h},$$

$\alpha' = \sqrt{t^2 + k_2^2}$ and $\alpha'' = \sqrt{k_2^2 - t^2}$.

Hence we can write,

$$R(p) = \frac{G_-(p) G_+(p)}{\sqrt{p^2 - k_2^2}} \prod_{n=1}^{\infty} \frac{(p \pm p_{3n})}{(p \pm p_{2n})} = R_+(p) R_-(p) \tag{39}$$

$\pm p_{3n}$ being the zeros of $f_3(p)$. Now using equations (32), (35) and (39) in equation (28) and simplifying, we obtain

$$\frac{iA\cos\theta_{2N}\delta}{(p+k_{1N})K_-(-k_{1N})}+\frac{iA\theta_{2N}\sin\theta_{2N}\delta F\_(-k_{1N})}{(p+k_{1N})K\_(-k_{1N})}-\frac{1}{(p-k_2)\delta}\left[\frac{\bar{v}_{2+}^{;}(p)K_+(p)}{p+k_2}-\frac{\bar{v}_{2+}^{;}(k_2)K_+(k_2)}{2k_2}\right]$$

$$-\frac{\bar{v}_{2-}^{;}(-p_{2m})R_+(p)R\_(-p_{2m})}{K\_(-p_{2m})}+\sum_{n=1}^{\infty}\frac{i}{p_n\delta(p+ip_n)K\_(-ip_n)}\bar{v}_{2+}^{;}(ip_n)-\frac{\bar{v}_{2+}^{;}(k_2)}{2k_2\delta(p+k_2)K\_(-k_2)}$$

$$+\frac{iA\theta_{2N}\sin\theta_{2N}\delta}{2k_2\delta(p+k_2)K\_(-k_2)(k_2+k_{1N})}+A\theta_{2N}\sin\theta_{2N}\delta\sum_{n=1}^{\infty}\frac{1}{p_n\delta(p+ip_n)K\_(-ip_n)(k_{1N}+ip_n)}$$

$$+\frac{iA\theta_{2N}\sin\theta_{2N}\delta}{2k_2\delta(p+k_2)K\_(-k_2)(k_2-k_{1N})}-\frac{iA\theta_{2N}\sin\theta_{2N}\delta}{2k_2\delta(p+k_{1N})K\_(-k_{1N})(k_2-k_{1N})}$$

$$-A\theta_{2N}\sin\theta_{2N}\delta\sum_{n=1}^{\infty}\frac{1}{p_n\delta(p+ip_n)K\_(-ip_n)(k_{1N}-ip_n)}+\frac{A\theta_{2N}\sin\theta_{2N}\delta}{(p+k_{1N})K\_(-k_{1N})}\sum_{n=1}^{\infty}\frac{1}{p_n\delta(k_{1N}-ip_n)}$$

$$=Q\_(p), \tag{40}$$

where $p_{2m}=k_2$ *and* $Q\_(p)$ includes the terms which are analytic in $\beta<\tau$ and left hand member of equation (40) is analytic in the region $\beta>-\tau$. Therefore, by analytic continuation each member tends to zero in its region of analyticity as $|p|\to\infty$. Hence by Liouville's theorem, the entire function is identically zero. So equating to zero the left hand side of equation (40), we obtain

$$\bar{v}_{2+}^{;}(p)=\left[\frac{T(p^2-k_2^2)}{p+k_{1N}}+\frac{\bar{v}_{2+}^{;}(k_2)K_+(k_2)(p+k_2)}{2k_2}+\frac{i\bar{v}_{2-}^{;}(-p_{2m})R_+(p_{2m})(p^2-k_2^2)\delta R_+(p)}{K_+(p_{2m})}\right.$$

$$-2iA\theta_{2N}\sin\theta_{2N}\delta.(p^2-k_2^2)\sum_{n=1}^{\infty}\frac{1}{(k_{1N}^2+p_n^2)(p+ip_n)K_+(ip_n)}-\frac{\bar{v}_{2+}^{;}(k_2)(p-k_2)}{2k_2K_+(k_2)}$$

$$\left.+\sum_{n=1}^{\infty}\frac{i\bar{v}_{2+}^{;}(ip_n)(p^2-k_2^2)}{p_n(p+ip_n)K_+(ip_n)}+\frac{iA\sin\theta_{2N}\delta(p-k_2)}{\theta_{2N}K_+(k_2)}\right]\frac{1}{K_+(p)}, \tag{41}$$

where,

$$T=\left[A\theta_{2N}\sin\theta_{2N}\delta.\sum_{n=1}^{\infty}\frac{1}{p_n(k_{1N}-ip_n)}+iA\theta_{2N}\sin\theta_{2N}\delta F_+(k_{1N})-\frac{iA\theta_{2N}\sin\theta_{2N}\delta}{2k_2(k_2-k_{1N})}\right.$$

$$\left.+iA\delta\cos\theta_{2N}\delta\right]\frac{1}{K_+(k_{1N})}. \tag{42}$$

Similarly, we find

$$\bar{v}_{2-}^{;}(p)=\left[\frac{N(p^2-k_2^2)}{p-k_{1N}}-\frac{\bar{v}_{2-}^{;}(-k_2)K_+(k_2)(p-k_2)}{2k_2}-\frac{\bar{v}_{2+}^{;}(p_{2m})R_+(p_{2m})(p^2-k_2^2)\delta R\_(p)}{K_+(p_{2m})}\right.$$

$$+2iA\theta_{2N}\sin\theta_{2N}\delta.(p^2-k_2^2)\sum_{n=1}^{\infty}\frac{1}{(k_{1N}^2+p_n^2)(p-ip_n)K_+(ip_n)}+\frac{\bar{v}_{2-}^{;}(-k_2)(p+k_2)}{2k_2K_+(k_2)}$$

$$\left.-\sum_{n=1}^{\infty}\frac{i\bar{v}_{2-}(-ip_n)(p^2-k_2^2)}{p_n(p-ip_n)K_+(ip_n)}-\frac{iA\sin\theta_{2N}\delta(p+k_2)}{\theta_{2N}K_+(k_2)}\right]\frac{1}{K_-(p)}, \qquad (43)$$

where,

$$N=\left[-A\theta_{2N}\sin\theta_{2N}\delta.\sum_{n=1}^{\infty}\frac{1}{p_n(k_{1N}-ip_n)}-iA\theta_{2N}\sin\theta_{2N}\delta F_+(k_{1N})+\frac{iA\theta_{2N}\sin\theta_{2N}\delta}{2k_2(k_2-k_{1N})}\right.$$

$$\left.-iA\delta\cos\theta_{2N}\delta\right]\frac{1}{K_+(k_{1N})}. \qquad (44)$$

The displacement $v_2(x,z)$ is obtained by inversion of Fourier transform, as

$$v_2(x,z)=\frac{1}{2\pi}\int_{-\infty+i\beta}^{\infty+i\beta}\bar{v}_2(p,z)e^{-ipx}dp$$

$$=\frac{1}{2\pi}\int_{-\infty+i\beta}^{\infty+i\beta}\frac{-1}{\theta_2}\left[\frac{\theta_2\cosh\theta_2 z-\gamma\theta_1\sinh\theta_2 z}{\theta_2\sinh\theta_2 h+\gamma\theta_1\cosh\theta_2 h}\right][\bar{v}_{2+}^{;}(p)+\bar{v}_{2-}^{;}(p)]e^{-ipx}dp \qquad (45)$$

where $\bar{v}_{2+}^{;}(p)$ and $\bar{v}_{2-}^{;}(p)$ are as given in equations (41) and (43) respectively .

2.2 Scattered Waves

The incident Love waves are scattered when these waves encounter with surface irregularities like rigid barrier in the crustal layer of earth. For finding the scattered component of the incident Love waves, we evaluate the integral in equation (45). There is a branch point $p=-k_1$ in the lower half plane. We put $p=-k_1-it$; t being small. The branch cut is obtained by taking $\mathrm{Re}(\theta_1)=0$. Now $\theta_1^2=p^2-k_1^2$, gives $\theta_1=\pm i\bar{\theta}_1$ and $\theta_2=\bar{\theta}_2$ .The contour of integration is shown in figure 2. The imaginary part of θ_1 has different signs on two sides of the branch cut.

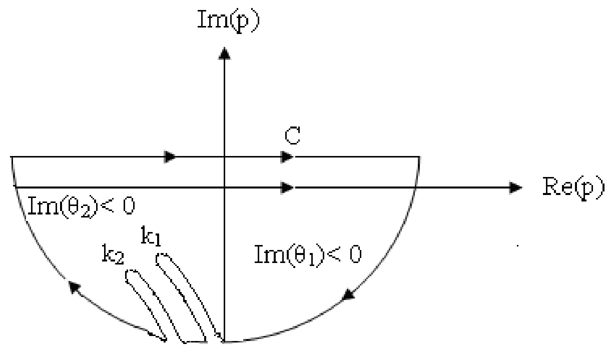

FIGURE 2: The contour of integration in complex p-plane

Now integrating equation (45) along two sides of branch cut, we get

$$v_{2,1}(x,z)=\frac{i}{2\pi}\int_0^\infty [\{\bar{v}_2(p,z)\}_{\theta_1=i\bar{\theta}_1}-\{\bar{v}_2(p,z)\}_{\theta_1=-i\bar{\theta}_1}]e^{-xk_1''}e^{-tx}dt$$

$$=\frac{-1}{\pi}\int_0^\infty\left[\frac{\xi(t)\gamma(2k_1''t)^{1/2}\cosh\bar{\theta}_2\,(z+H)}{\bar{\theta}_2^2\sinh^2\bar{\theta}_2\,H+\gamma^2\bar{\theta}_1^2\cosh^2\bar{\theta}_2\,H}+\frac{\eta(t)\gamma(2k_1''t)^{1/2}\cosh\bar{\theta}_2\,(z+H)}{\bar{\theta}_2^2\sinh^2\bar{\theta}_2\,h+\gamma^2\bar{\theta}_1^2\cosh^2\bar{\theta}_2\,h}\right]$$

$$\times e^{-k_1''x}e^{-tx}dt$$

$$=-e^{-k_1''x}\int_0^\infty t^{1/2}\psi(t)\,e^{-tx}dt, \qquad (46)$$

where,

$$\psi(t)=\frac{-\gamma(2k_1'')^{1/2}}{\pi}\left[\frac{\xi(t)\cosh\bar{\theta}_2\,(z+H)}{\bar{\theta}_2^2\sinh^2\bar{\theta}_2\,H+\gamma^2\bar{\theta}_1^2\cosh^2\bar{\theta}_2\,H}+\frac{\eta(t)\cosh\bar{\theta}_2\,(z+h)}{\bar{\theta}_2^2\sinh^2\bar{\theta}_2\,h+\gamma^2\bar{\theta}_1^2\cosh^2\bar{\theta}_2\,h}\right] \qquad (47)$$

and

$$\xi(t)=\left[\frac{T\bar{\theta}_2^2}{k_{1N}-k_1-it}+\frac{\bar{v}_{2+}'(k_2)K_+(k_2)(k_2-k_1-it)}{2k_2}+\frac{i\bar{v}_{2-}'(-p_{2m})R_+(p_{2m})\delta\bar{\theta}_2^2R_+(-k_1-it)}{K_+(p_{2m})}\right.$$

$$-2iA\theta_{2N}\bar{\theta}_2^2\sin\theta_{2N}\delta.\sum_{n=1}^{\infty}\frac{1}{(k_{1N}^2+p_n^2)(ip_n-k_1-it)K_+(ip_n)}+\frac{\bar{v}_{2+}'(k_2)(k_2+k_1+it)}{2k_2K_+(k_2)}$$

$$\left.+\sum_{n=1}^{\infty}\frac{i\bar{v}_{2+}'(ip_n)\bar{\theta}_2^2}{p_n(ip_n-k_1-it)K_+(ip_n)}-\frac{iA\sin\theta_{2N}\delta(k_2+k_1+it)}{\theta_{2N}K_+(k_2)}\right]K_-(-k_1-it)\frac{\sinh\bar{\theta}_2\,\delta}{\bar{\theta}_2\delta} \qquad (48)$$

$$\eta(t)=\Bigg[\frac{N\bar{\theta}_2^2}{k_{1N}+k_1+it}+\frac{\bar{v}_{2-}^{;}(-k_2)K_+(k_2)(k_2+k_1+it)}{2k_2}-\frac{\bar{v}_{2+}^{;}(p_{2m})R_+(p_{2m})\delta\bar{\theta}_2^2R_-(-k_1-it)}{K_+(p_{2m})}$$

$$-2iA\theta_{2N}\bar{\theta}_2^2\sin\theta_{2N}\delta.\sum_{n=1}^{\infty}\frac{1}{(k_{1N}^2+p_n^2)(ip_n+k_1+it)K_+(ip_n)}+\frac{\bar{v}_{2-}^{;}(-k_2)(k_2-k_1-it)}{2k_2K_+(k_2)}$$

$$+\sum_{n=1}^{\infty}\frac{\iota\bar{v}_{2-}^{;}(-ip_n)\bar{\theta}_2^2}{p_n(ip_n+k_1+it)K_+(ip_n)}-\frac{iA\sin\theta_{2N}\delta(k_2-k_1-it)}{\theta_{2N}K_+(k_2)}\Bigg]\frac{1}{K_-(-k_1-it)}. \tag{49}$$

Now using the result of Ewing et al. [5],

$$\int_0^{\infty}t^{1/2}\psi(t)e^{-tx}dt=\frac{\psi(0)\Gamma(3/2)}{x^{3/2}}+\frac{\psi'(0)\Gamma(5/2)}{x^{5/2}}+\frac{\psi''(0)\Gamma(7/2)}{x^{7/2}}+\dots\dots\dots \tag{50}$$

where $\Gamma(x)$ is Gamma function. Neglecting first and higher order derivatives of $\psi(t)$ at $t=0$ and using equations (46) and (50), we obtain

$$v_{2,1}(x,z)=\frac{\psi(0)\Gamma(3/2)}{x^{3/2}}e^{-k_1''x} \tag{51}$$

where,

$$\psi(0)=\frac{-\gamma(2k_1'')^{1/2}}{\pi}\left[\frac{\xi(0)\cos\theta_2''(z+H)}{\theta_2''^2\sin^2\theta_2''H}+\frac{\eta(0)\cos\theta_2''(z+h)}{\theta_2''^2\sin^2\theta_2''h}\right] \tag{52}$$

and $\theta_2''=\sqrt{k_2^2-k_1^2}$.

The equation (51) represents the scattered waves due to the presence of a rigid barrier $-H\le z\le -h,\ \ x=0$ in the crustal layer $-H\le z\le 0$ and the amplitude of the scattered waves is given by equation (52).

2.3 Reflected and Transmitted Waves

The incident Love waves are not only scattered but they are reflected also by the surface irregularity. For finding the reflected component, we evaluate the integral in equation (45) in upper half plane when $x<0$. There is a pole at $p=k_{1N}$ and the corresponding wave is given as

$$v_{2,2}(x,z)=A_m\cos\theta_{2N}(z+H)e^{-ik_{1N}x},\quad x<0,\ -H\le z\le -h, \tag{53}$$

where,

$$A_m = \left[\frac{iK_+(k_2)\bar{v}^{\prime}_{2^-}(-k_2)(k_{1N}-k_2)}{2k_2} - \frac{i\bar{v}^{\prime}_{2^+}(p_{2m})R_-(k_{1N})R_+(p_{2m})}{K_+(P_{2m})}\theta_{2N}^2\delta - \frac{i\bar{v}^{\prime}_{2^-}(-k_2)(k_2+k_{1N})}{2k_2K_+(k_2)} \right.$$

$$-2A\theta_{2N}^3\sin\theta_{2N}\delta.\sum_{n=1}^{\infty}\frac{1}{(k_{1N}^2+p_n^2)(k_{1N}-ip_n)K_+(ip_n)} + \sum_{n=1}^{\infty}\frac{\bar{v}^{\prime}_{2^-}(-ip_n)\theta_{2n}^2}{p_n(k_{1N}-ip_n)K_+(ip_n)}$$

$$\left. -\frac{A\sin\theta_{2N}\delta(k_2+k_{1N})}{\theta_{2N}K_+(k_2)}\right]\frac{\sin\theta_{2N}\delta K_+(k_{1N})}{\theta_{2N}\delta\cos\theta_{2N}H\left[\frac{d}{dp}f_1(p)\right]_{p=k_{1N}}} \tag{54}$$

and

$$\left[\frac{d}{dp}f_1(p)\right]_{p=k_{1N}} = k_{1N}\left[\frac{\theta_{1N}H+\gamma}{\theta_{1N}}\cos\theta_{2N}H + \frac{1+\gamma\theta_{1N}H}{\theta_{2N}}\sin\theta_{2N}H\right] \tag{55}$$

The amplitude of reflected Love wave is given by taking modulus of A_m. Now we evaluate the integral (45) in lower half plane when $x>0$. There is a pole at $p=-k_{1N}$ and the corresponding wave is given by

$$v_{2,3}(x,z) = B_m\cos\theta_{2N}(z+H)e^{ik_{1N}x}, \quad x>0, \ -H\le z\le 0 \tag{56}$$

where,

$$B_m = \left[-\frac{iK_+(k_2)\bar{v}^{\prime}_{2^+}(k_2)(k_2-k_{1N})}{2k_2} - \frac{i\bar{v}^{\prime}_{2^-}(-p_{2m})R_+(-k_{1N})R_+(p_{2m})}{K_+(p_{2m})}\theta_{2N}^2\delta - \frac{i\bar{v}^{\prime}_{2^+}(k_2)(k_2+k_{1N})}{2k_2K_+(k_2)} \right.$$

$$-2A\theta_{2N}^3\sin\theta_{2N}\delta.\sum_{n=1}^{\infty}\frac{1}{(k_{1N}^2+p_n^2)(k_{1N}-ip_n)K_+(ip_n)} - \sum_{n=1}^{\infty}\frac{\bar{v}^{\prime}_{2^+}(ip_n)\theta_{2n}^2}{p_n(ip_n-k_{1N})K_+(ip_n)}$$

$$\left. -\frac{A\sin\theta_{2N}\delta(k_2+k_{1N})}{\theta_{2N}K_+(k_2)}\right]\frac{\sin\theta_{2N}\delta\, K_+(k_{1N})}{\theta_{2N}\delta\cos\theta_{2N}H\left[\frac{d}{dp}f_1(p)\right]_{p=-k_{1N}}} \tag{57}$$

and

$$\left[\frac{d}{dp}f_1(p)\right]_{p=-k_{1N}} = -\left[\frac{d}{dp}f_1(p)\right]_{p=k_{1N}}. \tag{58}$$

Equation (56) gives the transmitted waves and their amplitudes are given by taking the modulus of B_{m}.

2.4 Numerical Computations and Discussion of Results

The incident Love waves are scattered as well as reflected due to the presence of rigid barrier in the surface layer of earth. The scattered Love waves move with the speed of waves in the half space and not with the speed of waves in the layer. The mathematical calculations have been done by taking $h = 0.49 km.,\ H = 0.51 km.,\ \gamma = 2,\ v_2 / v_1 = 3/4,\ k_{1N} = k_{2,}\ \ z = -H$ and considering $k_2 \delta$ very small. The graph of amplitude versus the wave number of the scattered waves has been plotted in figure 3.

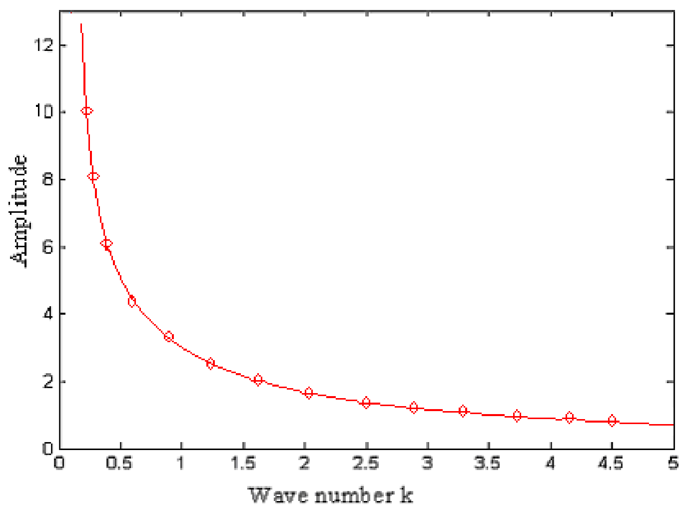

FIGURE 3: Variation of amplitude versus wave number k of scattered waves

The graph indicates that the amplitude of the scattered waves depends on the wave number and hence on the wavelength of the scattered wave. Also the scattered waves given in equation (51) are of the form $\frac{e^{-k_1'' x}}{x^{3/2}}$ which shows that the scattered waves decrease for large values of x and they behave as decaying cylindrical waves at the distant points. The reflected and transmitted waves are respectively given by equations (53) and (56) and their amplitudes are given in equations (54) and (57) respectively. The graph showing the variation of amplitude versus wave number of reflected Love waves by taking the barriers of different sizes is shown in figure 4.

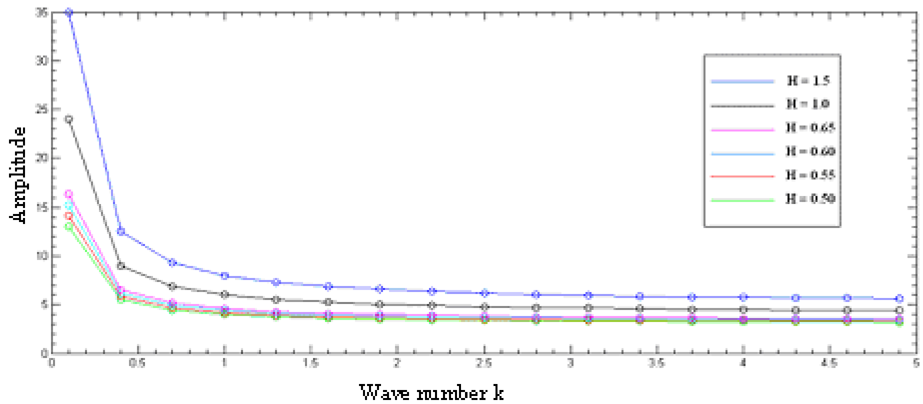

FIGURE 4: Variation of amplitude versus wave number for different values of H

For computation and graphical purpose we have fixed h = 0.49 km. and graphs has been plotted by taking H = 0.50, 0.55, 0.60, 0.65, 1.00, 1.50 km. Taking H = 0.50 km., the amplitude attains saturation at 3.0010 km. and for H= 0.55 km., it fixes at 3.0030 km. If we take H = 1.5 km., the amplitude becomes approximately stable at 5.0032 km. It is clear from comparison that amplitude of the reflected Love waves depends upon size of barrier to some extent.

3. CONCLUSIONS AND FUTURE WORKS

The results indicate that amplitude of the scattered waves decreases very rapidly with the slower increment in the value of wave number which signifies that as the wave number increases, the amplitude decreases at a faster rate but reduces to zero after a very long time. This is the practical reason why the scattered Love waves are considered one of the most destructive seismic waves during earthquake. The amplitude of reflected Love waves falls of rapidly as the wave number increases and then it decreases at a very slow rate with the increase in wave number and becomes stable at a particular value showing that the reflected Love waves take a very long time to dissipate. The theory presented in this paper also indicates that the behavior of reflected Love waves depends on the size of irregularity. In particular, the results show that larger is the size of barrier, larger is the amplitude of reflected wave resulting into more energetic reflected Love waves. This explains why the regions with more irregularities in earth surface face frequent earthquakes with high intensity.

We derived the approximate numerical solution for the case that the thickness difference for the surface layer is small as compared with the wavelength, leaving the scope for larger difference in future. The barrier may also be considered as a horizontal barrier instead of vertical one and the problem may be discussed accordingly.

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