Measurement and Comparison of Productivity Performance Under Fuzzy Imprecise Data

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Abstract

The creation of goods and services requires changing the expended resources into the output goods and services. How efficiently we transform these input resources into goods and services depends on the productivity of the transformation process. However, it has been observed there is always a vagueness or imprecision associated with the values of inputs and outputs. Therefore, it becomes hard for a productivity measurement expert to specify the amount of resources and the outputs as exact scalar numbers. The present paper, applies fuzzy set theory to measure and compare productivity performance of transformation processes when numerical data cannot be specified in exact terms. The approach makes it possible to measure and compare productivity of organizational units (including non-government and non-profit entities) when the expert inputs can not be specified as exact scalar quantities. The model has been applied to compare productivity of different branches of a company.

Keywords: Productivity; Performance Measure; Efficiency; Fuzzy Set Theory.

1. INTRODUCTION

Any for-profit or non-profit organization requires a set of input resources in order to operate and survive. In return, it provides goods or value-adding services for its clients or stake-holders. The efficiency with which it consumes the resources to provide those services, is measured by the productivity of the organization. The notion of productivity therefore, focuses on exploring the relationship between the results achieved and the resources expended to achieve those results. In its basic form, the productivity is measured by the ratio of outputs (often goods or services) to the input resources (such as labor, capital, management, materials, energy etc).

Two most common measures of productivity are total measure and partial measure. Total measure includes all the input resources used in achieving the desired outputs whereas partial measure focuses on an incomplete list of input factors. If a partial measure focuses on one factor only (e.g. output per labor hour), it is referred to as single factor productivity measure whereas including more than one factor gives multi factor productivity. Sometimes, the use of single factor productivity can be misleading when there is a tradeoff involved among multiple inputs. For example, an organization may procure a better and more expensive software or technology that require less manual processing by its staff. Thus, it is possible to increase labor productivity but at the expense of increased technological costs. Therefore, if an improvement in the single factor productivity has been achieved, it is important to carefully examine the factors responsible for it or alternatively, have a more holistic approach towards productivity in terms of multi or total productivity measure. The main reasons for process productivity measurement are to monitor and control the organizational performance, judge the effectiveness of our decisions and to create a metric that causes behavioral change among the employees leading towards a productive unit.

Measuring productivity is not an easy task mainly because both output as well inputs are difficult to measure or count in a meaningful way. At first instance, the determination of the output seems quiet straightforward but due to problems in measuring the quality in service sector and the prohibitive costs of surveys; it becomes difficult to specify the exact amount of satisfactory output. Frequent service offerings, price and fees fluctuations, service aggregation are some of the other issues that further add to the problem. Personnel, capital and management are considered to be the critical inputs to enhance productivity. Inappropriate time standards, disparity in employee skills and motivation levels, flexibility in over and underutilization of budgets, technological changes, economies of scale, unaccounted hidden costs and the difficulties in measuring the efforts of management, all these factors make it increasingly more difficult to ascertain the systems inputs in precise numerical terms. Thus determining both the system inputs as well as the results achieved is an onerous task and it is highly unlikely that an expert would be able to specify them in precise quantities. The fundamental flaw in the traditional approaches is that the imprecision of parameters is ignored. If such imprecision has not been incorporated into the productivity measurement model, it may result in misrepresentation of a situation which further leads to erroneous results. A model that explicitly incorporates the effects of such vagueness may is appropriate under these conditions. Fuzzy set theory has proved to be a very valuable tool to handle this type of imprecision or vagueness in data.

2. LITERATURE REVIEW

Miller and Rao [1] analyzed profit-linked productivity models at the firm level. The issue of productivity measurement under multiple criteria has been explored in Ray and Sahu [2] and the sensitivity of productivity measurement in a multi-product setting has been discussed in Ray and Sahu [3]. Garrigosa and Tatje [4] performed comparative study between profits and productivity as the two measures for performance. Sudit [5] discussed various productivity measures applicable in diverse settings. Chiou et al. [6] utilized quality function deployment in an approach that measures the productivity of technology in product development process. Agrell and West [7] critically examined a set of relevant properties that a productivity index must satisfy in order to assess the performance of a decision-making unit. Neely et al. [8] and Singh et al. [9] provided fairly detailed reviews of the previous research on productivity measurement. Suwignjo et al. [10] made use of tools such as cognitive maps, cause-effect diagrams, tree diagrams as well as analytical hierarchical process to quantify the effects of performance factors. Odeck [11] analyzed the efficiency and productivity growth of vehicle inspection services using DEA piecewise linear function and Malmquist indices. Ylvinger [12] presented multi-input and multi-output generalized structural efficiency measures based on linear programming DEA models to estimate the relative performance of an industry. Hannula [13] mentioned the trade-off between validity and practicality of productivity measures, and presented a practical method expressing total productivity as a function of partial productivity ratios with acceptable validity at an organizational unit level. Raa [14] presented an approach to quantify the inconsistency in aggregating the firm productivities through allocative efficiency and excess marginal productivities. Chavas and Mechemache [15] investigated the measures for technical, efficiency, allocative efficiency and price efficiency which can be conveniently summed into an overall efficiency measure. Cooper et al. [16] provided a fairly comprehensive account of applications of data envelop analysis (DEA) in performance measures. Majority of these publications do not address the vagueness or imprecision in data.

There are quite a few publications that explore the imprecise nature of the input-output data in productivity and efficiency measures. Chen et al. [17] applied fuzzy pattern recognition clustering techniques to determine productivity characters and a business unit is diagnosed through these characters. Joro et al. [18] showed that the DEA formulation to identify efficient units is similar to the multi-objective linear programming model based on the reference point approach to generate efficient solutions. Triantis and Girod [19] proposed a three stage approach to measure the technical efficiency in a fuzzy parametric programming environment by expressing input and output variables in terms of their risk-free and impossible bounds. Girod and Triantis [20] illustrated the implementation of a fuzzy set-based methodology that can be used to accommodate the measurement inaccuracies using risk-free and impossible bounds to represent the extremes for fuzzy input and output. Triantis and Eeckaut [21] used fuzzy pair-wise dominance to measure the distance of a production plan from a frontier. Cooper et al. [22] provided imprecise data envelop analysis (IDEA) that permits a mixture of imprecise and exact data. Cooper et al. [23] further extended it for assurance region and cone-ratio concepts by placing bounds on variables rather than data values. The approach is applicable to bounded data and data sets satisfying ordinal relations and has been illustrated through an application to branch offices of a telecommunication company in Korea. Cooper et al. [24] removed a limitation of IDEA and assurance region IDEA which required access to actually attained maximum values in the data, by introducing a dummy variable for normalization of maximal values. Despotis and Smirlis [25] developed an approach to transform a non-linear DEA model to a linear programming equivalent, on the basis of the original data set, by applying transformations only on the variables. Despotis and Smirlis [25] model allows post-DEA discriminating among the efficient units by endurance indices and is an alternative to Cooper et al. [22]. Zhu [26] reviewed and compared two different approaches dealing with imprecise DEA; one using scale transformations and the second using variable alterations through an efficiency analysis. Zhu [26] presented these two approaches as improvements over Cooper et al.[23]. Triantis [27] proposed a fuzzy DEA approach to compute fuzzy non-radial technical efficiency measures. Kao and Liu [28] provided a fuzzy DEA procedure by transforming it into a crisp DEA model using the α-cut concept of fuzzy set theory and the resulting efficiency measures are provided in terms of fuzzy sets. Kao and Liu [29] applied a maximizing–minimizing set method for fuzzy efficiency ranking of 24 university libraries in Taiwan. Lertworasirikul [30] and Lertworasirikul et al. [31] proposed two main approaches; a possibility approach and a credibility approach to resolve the problem of ranking fuzzy sets in fuzzy DEA models. León et al. [32] developed fuzzy versions of the classical BCC-DEA model by using ranking methods based on the comparison of α-cuts. Entani et al. [33] and Wang et al. [34] changed fuzzy input - output data into intervals using α-level sets and suggested two interval-DEA models. Dia [35] fuzzy-DEA model requires the decision maker to specify an aspiration level and a safety α-level in order to transform it into a crisp DEA model. Kao and Liu [36] transformed fuzzy input and output data into intervals by using α-level sets and fuzzy extension principle and built a family of crisp DEA models for the intervals. Soleimani-damaneh et al. [37] addressed some computational and theoretical pitfalls of the fuzzy DEA models and provided a fuzzy DEA model to produce crisp efficiencies for DMUs with fuzzy input and output data. You et al. [38] presented a fuzzy multiple objective programming approach to imprecise data envelopment analysis (IDEA) with an increased discriminating power than available from Cooper et al. [22]. Wang et al. [39] proposed two new fuzzy DEA models constructed from the perspective of fuzzy arithmetic and the models are applied to evaluate the performances of eight manufacturing enterprises in China.

As evident from this literature survey, most of the existing approaches that deal with imprecise nature of data, present several variations of the DEA approach in a fuzzy environment. DEA-based approaches are optimization approaches in the sense that they identify the best set of weights to identify the maximum achievable efficiency for an organizational unit, rather than identifying its true efficiency. Secondly, DEA-based approaches provide a relative measure of efficiency amongst a set of decision making units (DMU's). These approaches compare the DMU's input and output against a composite input and output. If a particular DMU uses more inputs than the composite, it is termed as inefficient and vice-versa. As a potential drawback, if one DMU has substantially higher performance than others, most of the DMU's (except the one with exceptional performance) are likely to be termed as inefficient. Similarly, a DMU with an exceptionally low performance may render other DMU's as efficient, not because of their own performances but due to the relative nature of the measurement. Furthermore, when new DMU's enter or leave the system (e.g. a new member joining or leaving a supply chain), efficiencies need to be re-evaluated. This establishes the need to have an approach that measures real productivity of a system in an absolute sense and in an environment involving imprecision and vagueness of data. This is one area where the present paper intends to contribute.

The next section deals with some basic concepts of fuzzy set theory that have been used to develop the proposed framework to model productivity. The subsequent section presents a fuzzy set theoretic model for multi factor productivity. The proposed model is illustrated through an application to 13 branches of a credit union. The computational experience and some important observations drawn from this experience are discussed. Finally, concluding remarks and some directions for further research are presented.

3. BASIC FUZZY CONCEPTS

Since its inception by Lofti Zadeh [40], fuzzy logic has revolutionized the business world with its ability to model the imprecise decision making situations. This section presents some basic concepts in fuzzy set methodology that have been utilized to develop the proposed model in this paper. For details of these concepts, the reader is referred to Kaufmann and Gupta [41] and Zimmermann [42].

3.1 Fuzzy Set and Membership Function

A fuzzy set A in X is characterized by a membership function, $\mu_A(x)$ which associates with each element in X, a real number in the interval [0,1] with the value of $\mu_A(x)$ at x representing the "grade of membership" of x in A.

3.2 Triangular Fuzzy Number (TFN)

A triangular fuzzy number is a linear approximation for a normal and convex fuzzy set and is represented by the triplet (a_1, a_2, a_3). Under this linear approximation, the membership function for triangular fuzzy number can be expressed as follows:

$$\mu_A = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x < a_2 \\ \dfrac{x - a_3}{a_2 - a_3}, & a_2 \le x < a_3 \\ 0, & x > a_3 \end{cases}$$

(1)

3.3 α-cut of a Fuzzy Set

As shown in Figure 1, α-cut of a fuzzy set denoted by A_α, is a subset of its domain that allows us to represent a fuzzy set in a confidence interval form, as $A_\alpha = [\, a_1^\alpha, a_3^\alpha \,]$. Using equation (1), the triangular fuzzy number approximation in its α-cut confidence interval form can be expressed as follows:

$$A_\alpha = [\, a_1^\alpha, a_3^\alpha \,] = [\, a_1 + (a_2 - a_1)\,\alpha, \;\; a_3 + (a_2 - a_3)\,\alpha] \qquad (2)$$

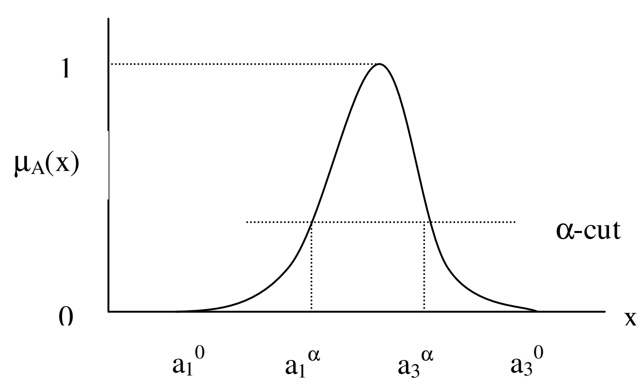

FIGURE 1: α-cut of a fuzzy set

4. FUZZY SET MODEL FOR PRODUCTIVITY

Having stated the fuzzy concepts to develop this methodology, we now present the proposed fuzzy model for multiple factor productivity. The present appear does not specifically deal with the issue of aggregating the inputs and outputs. The issue of heterogeneity of outputs and inputs and their aggregation through appropriate weights has been addressed by various authors. For more details, the reader is referred to [43], [44], [45], [46], [47]. The present paper assumes that these conversion weights are input to the model and therefore, they are known.

Most transformation processes have positive inputs and outputs, but without any loss of generality, it is possible to define fuzzy sets over negative real number domains. Although, in order to avoid a zero denominator in productivity measure, we make the assumption that aggregate sum of inputs is non-zero which is a fairly standard assumption.

4.1 Notation

P^k = Fuzzy measure of the productivity of the k^{th} unit.
O_i^k = Triangular fuzzy number representing the i^{th} output of the k^{th} unit. (in case of single output measure, O^k could be used).
I_j^k = Triangular fuzzy number representing the j^{th} input of the k^{th} unit.
w_j^k = Unit conversion factor or weight for the j^{th} input of the k^{th} unit.
z_i^k = Unit conversion factor or weight for the i^{th} output of the k^{th} unit.
k = 1,2, 3,…….,m. i = 1,2, 3,……..,p. j = 1,2, 3,……..,n.

Triangular fuzzy numbers O^k and I_j^k can be specified in its triplet form as

$O_i^k = (z_i^k * o_{i1}^k,\ z_i^k * o_{i2}^k,\ z_i^k * o_{i3}^k)$
$I_j^k = (w_j^k * i_{j1}^k,\ w_j^k * i_{j2}^k,\ w_j^k * i_{j3}^k)$,
where $o_{i1}^k, o_{i2}^k, o_{i3}^k, i_{j1}^k, i_{j2}^k, i_{j3}^k \in R^+$

4.2 The Model

The fuzzy measure of multifactor productivity can be expressed by the following equation

$$P^k = \frac{O^k}{\sum_{j=1}^{n} I_j^k} \quad for\ k = 1,2,3,....,m \tag{3}$$

Following equation (2), we can express the triangular fuzzy numbers O^k and I_j^k in their confidence interval form as follows:

$$O^k = [\ o_1^k + \alpha(\ o_2^k - o_1^k\ ),\ \ o_3^k + \alpha(\ o_2^k - o_3^k\ )] \tag{4}$$

$$I_j^k = [\ i_{j1}^k + \alpha(\ i_{j2}^k - i_{j1}^k\ ),\ \ i_{j3}^k + \alpha(\ i_{j2}^k - i_{j3}^k\ )] \quad for\ j = 1,2,....,n \tag{5}$$

For two fuzzy sets A and B, A=[a_1, a_3] and B=[b_1, b_3] with $a_1, a_3, b_1, b_3 \in R$, the addition of these sets, A (+) B, is given as (Kaufmann and Gupta 1988):
A (+) B = [a_1, a_3] (+) [b_1, b_3] = [$a_1 + b_1$, $a_3 + b_3$]

(6)

This addition operator can be conveniently extended for three or more sets.

Making use of the summation result (6) for fuzzy sets, the summation of the process inputs of equation (5) is expressed as follows:

$$\sum_{j=1}^{n} I_j^k = \left[\sum_{j=1}^{n} (i_{j1}^k + \alpha (i_{j2}^k - i_{j1}^k)), \quad \sum_{j=1}^{n} (i_{j3}^k + \alpha (i_{j2}^k - i_{j3}^k))\right] \tag{7}$$

Substituting equations (4) and (7) into equation (3), we obtain the productivity measure for the k[th] process as follows:

$$P^k = \frac{[\, o_1^k + \alpha (o_2^k - o_1^k), \quad o_3^k + \alpha (o_2^k - o_3^k)]}{[\, \sum_{j=1}^{n} (i_{j1}^k + \alpha (i_{j2}^k - i_{j1}^k)), \quad \sum_{j=1}^{n} (i_{j3}^k + \alpha (i_{j2}^k - i_{j3}^k))\,]}, \quad for\ k = 1,2,....,m$$

(8)

For two fuzzy sets A and B, where A=[a_1, a_3] and B=[b_1, b_3] with a_1, a_3 , b_1, $b_3 \in$ R, the division of these sets, A / B, is given as

A/ B = [a_1, a_3] / [b_1, b_3]

= [min (a_1 / b_1 , a_1 / b_3 , a_3 / b_1 , a_3 / b_3), max (a_1 / b_1 , a_1 / b_3 , a_3 / b_1 , a_3 / b_3)]

This general formula is applicable when inputs and outputs are not necessarily positive real numbers, however, if the sets A and B have been defined over R^+ , as is the case in most physical transformation processes, then it can be shown that the division operator simplifies to

A / B = [a_1, a_3]/ [b_1, b_3] = [a_1 / b_3 , a_3 / b_1]

(9)

Since all the output and input resource estimates belong to R^+, we apply the simplified division operator (9) to equation (8) to compute:

$$P^k = \left[\frac{o_1^k + \alpha (o_2^k - o_1^k)}{\sum_{j=1}^{n} (i_{j3}^k + \alpha (i_{j2}^k - i_{j3}^k))}, \quad \frac{o_3^k + \alpha (o_2^k - o_3^k)}{\sum_{j=1}^{n} (i_{j1}^k + \alpha (i_{j2}^k - i_{j1}^k))} \right]$$

(10)

Equation (10) expresses multifactor productivity measure in its interval form. Next, we express this productivity in its triplet form by finding its α-cut's at α=0 and α=1 levels. The above equation (10) reduces to its triplet form as follows:

$$P^k = \left[\frac{o_1^k}{\sum_{j=1}^{n}(i_{j3}^k)} \; , \; \frac{o_2^k}{\sum_{j=1}^{n}(i_{j2}^k)} , \; \frac{o_3^k}{\sum_{j=1}^{n}(i_{j1}^k)} \right]$$
(11)

It is worth noting that equation (11) is a more general approach for computing multifactor productivity. The traditional approach can be deduced as a special case of this more general approach.

4.3 Traditional Approach as a Special Case

If there were no imprecision involved in specifying the model parameters, then the three triplets for inputs and outputs will coincide as follows:

$o_1^k = o_2^k = o_3^k = o^k$ and $i_{j1}^k = i_{j2}^k = i_{j3}^k = i_j^k$

Putting this into equation (11) implies

$$P^k = \left(\frac{o^k}{\sum_{j=1}^{n}(i_j^k)} \right)$$

, which is the traditional crisp measure of productivity.

4. COMPARISON OF PRODUCTIVITY

The productivity measure in itself has a limited usefulness unless it is compared with the productivity of another organizational unit or industry average. This could be modeled through the removal concept of a fuzzy set at reference point z = 0. The removal of a fuzzy set that follows a possibility distribution and it amounts to finding an expected value representation of a fuzzy set. Such a transformation is necessary because we often work in a fuzzy decision environment where we have to deal with vague information and data, but we are often required to take crisp decisions. As defined in Kauffman and Gupta (1988), the removal of a fuzzy number is the average of its LHS removal R_l (A, z) and its RHS removal R_r (A, z), where $z \in R$. Therefore, removal essentially measures the average distance of a fuzzy number from a reference point z. It can be shown that at reference point z=0, the removal of fuzzy productivity measure is given by:

$$R(P^k, \; z = 0) = \left(\frac{1}{4} \right) \left(\frac{o_1^k}{\sum_{j=1}^{n}(i_{j3}^k)} + \frac{2 \cdot o_2^k}{\sum_{j=1}^{n}(i_{j2}^k)} + \frac{o_3^k}{\sum_{j=1}^{n}(i_{j1}^k)} \right)$$

(12)

If we have two organizational units with their respective removals as $R(P^1, z = 0)$ and $R(P^2, z = 0)$. Unit 1 will be more productive than unit 2 if $R(P^1, z = 0)$ > $R(P^2, z = 0)$. This concept can be used to compare the productivities of a number of processes or business units by ranking them in the descending order of removals.

5. MODEL ILLUSTRATION

We illustrate the usefulness of our model by evaluating thirteen branches of a credit-union ([48]) in terms of their productivity performance. Without loss of generality, we consider each branch to have four relevant inputs: number of personnel or staff, number of computers or IT infrastructure, area of the branch in square footage and the administrative costs. The output measure is daily transactional volume. For unit and time aggregation purposes, these inputs are converted to common monetary unit by applying the following conversion weights from the problem situation: $50,000 per staff member including perks and benefits, $7,000 per computer or other technological equipment, $2,500 per square foot and on an average 300 business days in a year. For each input and output, the decision maker assigns his best estimate of the value i.e. an estimate in which he has maximum belief. Furthermore, the decision maker also provides a range outside which the input or output values are not likely to lie. For the credit union branches, there is no ambiguity involved in specifying the number of IT infrastructure and branch areas, so these variables are treated as non-fuzzy but the proposed fuzzy model is general enough to handle both fuzzy and non-fuzzy inputs. This estimation of inputs in terms of triangular fuzzy numbers is presented in Table 1.

Branch #	Input 1 (Personnel)	Input 2 (IT)	Input 3 (Space) (1000's sq ft)	Input 4 (Expenses) ($100,000)
1	(4,6,8)	(8,8,8)	(4,4,4)	(147.30, 628.95, 1036.56)
2	(6,7,8)	(8,8,8)	(2.56, 2.56, 2.56)	(411.44, 716.50, 957.01)
3	(7,9,11)	(10,10,10)	(1.34,1.34,1.34)	(287.92,428.13,603.01)
4	(8,10,12)	(12,12,12)	(1.5,1.5,1.5)	(242.77,722.95,945.69)
5	(4,6,8)	(9,9,9)	(1.68,1.68,1.68)	(433.55,692.00, 807.29)
6	(5,7,9)	(7,7,7)	(3.75,3.75,3.75)	(455.71,777.62,1162.68)
7	(7,9,11)	(10,10,10)	(3.31,3.31,3.31)	(780.11,1145.37,1711.52)
8	(6,8,10)	(7,7,7)	(1.5,1.5,1.5)	(149.69,755.97,1358.58)
9	(8,10,12)	(8,8,8)	(1.6,1.6,1.6)	(610.24,1019.76,1829.51)
10	(6,8,10)	(9,9,9)	(1.72,1.72,1.72)	(216.25,712.23,945.11)
11	(9,10,11)	(7,7,7)	(1.92,1.92,1.92)	(396.64,905.15,1249.84)
12	(7,9,11)	(8,8,8)	(4.43,4.43,4.43)	(231.06,749.94,1398.88)
13	(6,8,10)	(10,10,10)	(2.5,2.5,2.5)	(177.56,778.15,1444.62)

TABLE 1: Input Data for the 13 Credit Union Branches

The model given in equations (11, 12) was used to compute the fuzzy productivity and the fuzzy removal of productivity (i.e. its crisp equivalent) for the thirteen branches. The relevant output, fuzzy and crisp productivities for this data set are provided in Table 2.

Branch #	Output (Transaction Volume)	Fuzzy productivity	Crisp equivalent of productivity
1	(55830,56570,57318)	(0.147, 0.232,0.688)	0.325
2	(36740,36800,36852)	(0.107,0.141,0.231)	0.155
3	(38004,38446,38783)	(0.177,0.247,0.357)	0.257
4	(35469,35685,36017)	(0.107,0.140,0.379)	0.191
5	(52927,53869,54817)	(0.186,0.219,0.344)	0.242
6	(70254,72446,78574)	(0.167,0.248,0.427)	0.273

7	(32585,35856,37443)	(0.054,0.087,0.130)	0.090
8	(42900,45027,47270)	(0.092,0.169,0.744)	0.294
9	(85399,86221,87220)	(0.137,0.243,0.400)	0.255
10	(46924,47142,47316)	(0.142,0.186,0.540)	0.263
11	(36652,40482,44298)	(0.084,0.127,0.296)	0.158
12	(39582,39594,39620)	(0.078,0.137,0.344)	0.174
13	(56144,57484,58816)	(0.111,0.204,0.724)	0.31

TABLE 2: Output Data and Productivity Performance Measurement Results

6. EXPERIMENTAL RESULTS

The main objective in this experimental analysis was to test the sensitivity of productivity measure w.r.t. right hand side fuzziness, left hand side fuzziness and both types of fuzziness. We first reduced the left triplet range for a sample productivity data set in decrements of 5%, then its right triplet range in decrements of 5% and finally reduced both triplet ranges simultaneously in decrements of 5%. The results of experimental analysis are summarized in the following Table 3.

Percent Decrement	Percentage change in productivity w.r.t. crisp measure while reducing LHS triplet	LHS triplet	Both triplets
5	4.91	5.345	4.880
10	4.46	5.315	4.445
15	4.04	5.300	4.010
20	3.635	5.285	3.620
25	3.245	5.270	3.245
30	2.870	5.270	2.885
35	2.510	5.255	2.555
40	2.150	5.255	2.240
45	1.820	5.240	1.955
50	1.490	5.240	1.685
55	1.175	5.240	1.430
60	0.875	5.255	1.205
65	0.590	5.255	0.995
70	0.305	5.270	0.800
75	0.035	5.285	0.620
80	-0.235	5.300	0.470
85	-0.490	5.315	0.320
90	-0.730	5.345	0.200
95	-0.970	5.360	0.095
100	-1.210	5.390	0.005

TABLE 3: Results of Experimental Analysis

A number of interesting observations can be drawn from the experimental experience.

- First of all, the fuzzy measure of productivity performance is robust in the sense that errors in specifying the triangular fuzzy numbers do not result in a large change in productivity measure. The maximum change found was 5.39% for the experimental case.
- Changes in the RHS triplet range does not give a large change in the productivity suggesting that productivity change is not significantly affected by the RHS fuzziness. LHS triplet fuzziness was found to have a more profound effect on productivity measures. Furthermore, LHS triplet ranges can result in both underestimation and overestimation in productivity measure w.r.t. crisp measure. Therefore, much more careful attention should be given in selecting the LHS triplet ranges.
- Finally, as we change both the ranges together, the fuzzy measure of productivity approaches the traditional crisp measure asymptotically. At the point of maximum belief, productivity measure from the fuzzy formula becomes the same as the crisp measure from the traditional formula. This further reinforces the point that

traditional measure of productivity is a special case of a more general fuzzy measure of productivity given in the present paper.

7. CONCLUDING REMARKS

The present paper recognizes that measurements of system inputs and outputs for productivity measurement is a difficult task resulting in vagueness or imprecision in data. The paper proposes an approach based on fuzzy set theory to model this type of vagueness. The proposed approach provides a general model for productivity measurement. The traditional single and multi factor productivity measures can be deduced as its special cases. The paper further presents a method to compare the productivity performance across different organizations using the fuzzy removal concept. It may be noted that the approach is equally applicable for non-profit and non-government organizations. The approach is illustrated with the help of a case example from a credit union, and the computational experience and important observations are also discussed.

The literature dealing with the imprecise nature of data mainly consists of variations of DEA model under fuzzy logics. These approaches are relative in nature and identify the maximum achievable efficiency for an organizational unit amongst a set of similar decision making units within an organization. An absolute and real treatment of efficiency is needed for organizations that consist of seemingly unrelated organization units or entities. The relative approach is an intra-organization approach but for competitive reasons, an inter-organization approach is preferable. That is one area where approaches such as the current one, could be potentially useful.

8. REFERENCES

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