E-Cordial Labeling of Some Mirror Graphs

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Abstract

Let *G* be a bipartite graph with a partite sets V_1 and V_2 and G' be the copy of *G* with corresponding partite sets V_1' and V_2'. The mirror graph *M(G)* of *G* is obtained from *G* and G' by joining each vertex of V_1 to its corresponding vertex in V_2 by an edge. Here we investigate E-cordial labeling of some mirror graphs. We prove that the mirror graphs of even cycle C_n, even path P_n and hypercube Q_k are E-cordial graphs.

Keywords: E-Cordial labeling, Edge graceful labeling, Mirror graphs.

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1. INTRODUCTION

We begin with finite, connected and undirected graph $G=\left(V\left(G\right),E\left(G\right)\right)$ without loops and multiple edges. For standard terminology and notations we refer to West[1]. The brief summary of definitions and relevant results are given below.

Definition 1.1
If the vertices of the graph are assigned values subject to certain condition(s) then it is known as *graph labeling*.

Most of the graph labeling techniques trace their origin to graceful labeling introduced independently by Rosa[3] and Golomb[4] which is defined as follows.

Definition 1.2
A function f is called graceful labeling of graph G if $f:V(G)\rightarrow\{0,1,2,\ldots,q\}$ is injective and the induced function $f^*(e=uv)=|f(u)-f(v)|$ is bijective. A graph which admits graceful labeling is called a *graceful graph*.

The famous Ringel-Kotzig graceful tree conjecture and illustrious work by Kotzig[5] brought a tide of labelig problems having graceful theme.

Definition 1.3
A graph G is said to be *edge-graceful* if there exists a bijection $f:E(G)\rightarrow\{1,2,\ldots,|E|\}$ such that the induced mapping $f^*:V(G)\rightarrow\{0,1,2,\ldots,|V|-1\}$ given by $f^*(x)=\sum f(xy)(mod\,|V|)$, $xy\in E(G)$.

Definition 1.4
A mapping $f:V(G)\rightarrow\{0,1\}$ is called *binary vertex labeling* of G and $f\left(v\right)$ is called the *label* of vertex v of G under f.

Notations 1.5

For an edge $e=uv$, the induced edge labeling $f^*: E(G)\rightarrow\{0,1\}$ is given by $f^*(e=uv)=|f(u)-f(v)|$. Let $v_f(0), v_f(1)$ be the number of vertices of G having labels 0 and 1 respectively under f and let $e_f(0), e_f(1)$ be the number of edges of G having labels 0 and 1 respectively under f^*.

Definition 1.6

A binary vertex labeling of graph G is called a *cordial labeling* if $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$. A graph G is called *cordial* if admits cordial labeling.

The concept of cordial labeling was introduced by Cahit[6]. He also investigated several results on this newly introduced concept.

Definition 1.7

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$ and let $f: E(G)\rightarrow\{0,1\}$. Define on $V(G)$ by $f(v)=\sum\{f(uv)*uv\in E(G)\}(mod\,2)$. The function f is called an *E-cordial* labeling of *G* if $|v_f(0)-v_f(1)|\leq 1$ and $|e_f(0)-e_f(1)|\leq 1$. A graph is called *E-cordial* if admits *E-cordial* labeling .

In 1997 Yilmaz and Cahit[7] introduced E-cordial labeling as a weaker version of edge-graceful labeling and having flavour of cordial labeling. They proved that the trees with *n* vertices, K_n, C_n are E-cordial if and only if $n\not\equiv 2(mod\,4)$ while $K_{m,n}$ admits E-cordial labeling if and only if $m+n\not\equiv 2(mod\,4)$.

Definition 1.8

For a bipartite graph G with partite sets V_1 and V_2. Let G' be the copy of G and V_1' and V_2' be the copies of V_1 and V_2. The *mirror graph* $M(G)$ of G is obtained from G and G' by joining each vertex of V_2 to its corresponding vertex in V_2' by an edge.

Lee and Liu[8] have introduced mirror graph during the discussion of $k-$graceful labeling. Devaraj[9] has shown that $M(m, n)$, the mirror graph of $K_{m,n}$ is E-cordial when $m+n$ is even while the generalized Petersen graph $P(n, k)$ is E-cordial when n is even.

In the following section we have investigated some new results on E-cordial labeling for some mirror graphs.

2. Main Results

Theorem 2.1 Mirror graph of even cycle C_n is E-cordial.

Proof: Let $v_1,v_2,\ldots,v_n$ be the vertices and $e_1,e_2,\ldots,e_n$ be the edges of cycle C_n where n is even and $G=C_n$. Let $V_1=\{v_2,v_4,\ldots,v_n\}$ and $V_2=\{v_1,v_3,\ldots,v_{n-1}\}$ be the partite sets of C_n. Let G' be the copy of G and $V_1'=\{v_2',v_4',\ldots,v_n'\}$ and $V_2'=\{v_1',v_3',\ldots,v_{n-1}'\}$ be the copies of V_1 and V_2 respectively. Let $e_1',e_2',\ldots,e_n'$ be the edges of G'. The mirror graph $M(G)$ of G is obtained from G and G' by joining each vertex of V_2 to its corresponding vertex in V_2' by additional edges $e_1^*,e_2^*,\ldots,e_{\frac{n}{2}}^*$.

We note that $|V(M(G))| = 2n$ and $|E(M(G))|=2n+\frac{n}{2}$. Let $f: E(M(G))\rightarrow\{0,1\}$ as follows:

<u>For $1\leq i\leq n$:</u>

$$f(e_i) = 1; \quad i \equiv 0,1(\bmod 4).$$
$$= 0; \quad otherwise.$$
$$f(e_i') = 1; \quad i \equiv 1,2(\bmod 4).$$
$$= 0; \quad otherwise.$$

For $1 \le j < \frac{n}{2}$:

$$f(e_j^*) = 1; \quad j \equiv 1(\bmod 2).$$
$$= 0; \quad otherwise.$$

For $j = \frac{n}{2}$:

$$f(e_j^*) = 0.$$

In view of the above defined labeling pattern f satisfies conditions for E-cordial labeling as shown in *Table 1*. That is, the mirror graph of even cycle C_n is E-cordial.

	vertex condition	*edge condition*
$n \equiv 0(mod 4)$	$v_f(0) = v_f(1) = n$	$e_f(0) = e_f(1) = n + \frac{n}{4}$
$n \equiv 2(mod 4)$	$v_f(0) = v_f(1) = n$	$e_f(0) + 1 = e_f(1) = n + \left\lceil \frac{n}{4} \right\rceil$

Table 1

Illustration 2.2: The E-cordial labeling for the mirror graph of cycle C_6 is shown in *Figure 1*.

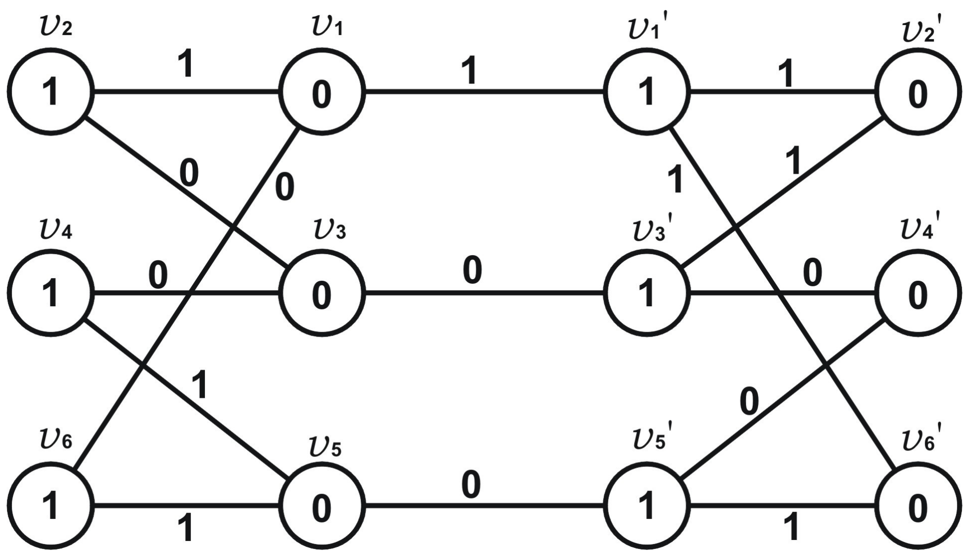

Figure 1

Theorem 2.3: Mirror graph of path P_n is E-cordial for even n.

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices and $e_1, e_2, \ldots, e_{n-1}$ be the edges of path P_n where n is even and $G = P_n$. P_n is a bipartite graph. Let $V_1 = \{v_2, v_4, \ldots, v_n\}$ and $V_2 = \{v_1, v_3, \ldots, v_{n-1}\}$ be the bipartition of P_n. Let G' be a copy of G and $V_1' = \{v_2', v_4', \ldots, v_n'\}$ and $V_2' = \{v_1', v_3', \ldots, v_{n-1}'\}$ be the copies of V_1 and V_2. Let $e_1', e_2', \ldots, e_{n-1}'$ be the edges of G'. The mirror graph $M(G)$ of G is obtained from G and G' by joining each vertex of V_2 to its corresponding vertex in V_2' by additional edges $e_1^*, e_2^*, \ldots, e_{\frac{n}{2}}^*$.

We note that $|V(M(G))| = 2n$ and $|E(M(G))| = 2(n-1) + \frac{n}{2}$. Let $f : E(M(G)) \to \{0,1\}$ as follows:

<u>For $1 \le i < n-1$:</u>

$$f(e_i) = 1; \quad i \equiv 0,1(\bmod 4).$$
$$= 0; \quad otherwise.$$

<u>For $i = n-1$:</u>

$$f(e_i) = 1.$$

<u>For $1 \le i \le n-1$:</u>

$$f(e_i') = 1; \quad i \equiv 0,3(\bmod 4).$$
$$= 0; \quad otherwise.$$

<u>For $1 \le j \le \frac{n}{2}$:</u>

$$f(e_j^*) = 1; \quad j \equiv 0(\bmod 2).$$
$$= 0; \quad otherwise.$$

In view of the above defined labeling pattern f satisfies the conditions for E-cordial labeling as shown in *Table 2*. That is, the mirror graph of path P_n is E-cordial for even n.

	vertex condition	edge condition
$n \equiv 0(mod 4)$	$v_f(0) = v_f(1) = n$	$e_f(0) = e_f(1) = n - 1 + \frac{n}{4}$
$n \equiv 2(mod 4)$	$v_f(0) = v_f(1) = n$	$e_f(0) = e_f(1) + 1 = n - 1 + \left\lceil \frac{n}{4} \right\rceil$

Table 2

Illustration 2.4: The E-cordial labeling for mirror graph of path P_8 is shown in *Figure 2.*

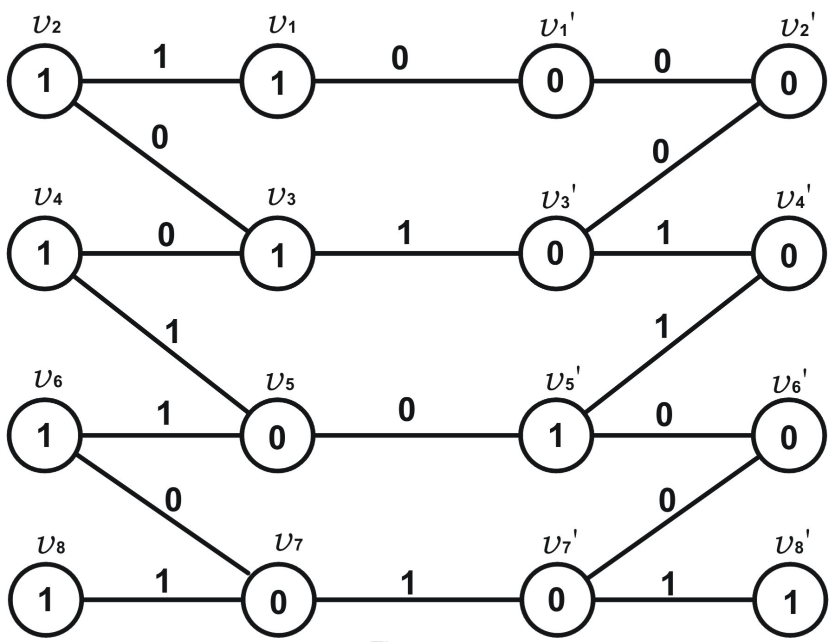

Figure 2.

Theorem 2.5: Mirror graph of hypercube Q_k is E-cordial.

Proof:
Let $G = Q_k$ be a hypercube with n vertices where $n = 2^k$. Let V_1 and V_2 be the bipartition of Q_k and G' be a copy of G with V_1' and V_2' be the copies of V_1 and V_2 respectively. Let $e_1, e_2, \ldots, e_m$ be the edges of graph G and $e_1', e_2', \ldots, e_m'$ be the edges of graph G' where $m = \frac{nk}{2}$. The mirror graph $M(G)$ of G is obtained from G and G' by joining each vertex of V_2 to its corresponding vertex in V_2' by additional edges $e_1^*, e_2^*, \ldots, e_{\frac{n}{2}}^*$ then $|V(M(G))| = 2n$ and $|E(M(G)| = \frac{n(2k+1)}{2}$.
Define $f : E(M(G)) \rightarrow \{0,1\}$ as follows:

<u>Case:1</u> $k \equiv 0(mod\, 2)$
Let $V_i = \{v_{i1}, v_{i2}, \ldots, v_{i\frac{n}{2}}\}$ and $V_i' = \{v_{i1}', v_{i2}', \ldots, v_{i\frac{n}{2}}'\}$ where $i = 1,2.$ All the edges incident to the vertices v_{1j} and v_{2j}' where $j \equiv 1(mod\, 2)$ are assigned the label 0 while the edges incident to the vertices v_{1j} and v_{2j}' where $j \equiv 0(mod\, 2)$ are assigned label 1.

<u>For $1 \le j \le \frac{n}{2}$:</u>

$$f(e_j^*) = 1; \quad j \equiv 0(\bmod 2).$$
$$= 0; \quad otherwise.$$

<u>Case:2</u> $k \equiv 1(mod\, 2)$

<u>For $1 \le i \le n$:</u>

$$f(e_i) = 1.$$

<u>For $1 \le i \le n$:</u>

$$f(e_i') = 0.$$

<u>For $1 \le j \le \frac{n}{2}$:</u>

$$f(e_j^*) = 1; \quad j \equiv 0(\bmod 2).$$
$$= 0; \quad otherwise.$$

In view of the above defined labeling pattern f satisfies the conditions for E-cordial labeling as shown in *Table 3*.

	vertex condition	*edge condition*
n	$v_f(0) = v_f(1) = n$	$e_f(0) = e_f(1) = \frac{n(2k+1)}{4}$

TABLE 3

That is, the mirror graph of hypercube Q_k is E-cordial.

Illustration:2.6
The E-cordial labeling for mirror graph of hypercube Q_3 is shown in *Figure 3*.

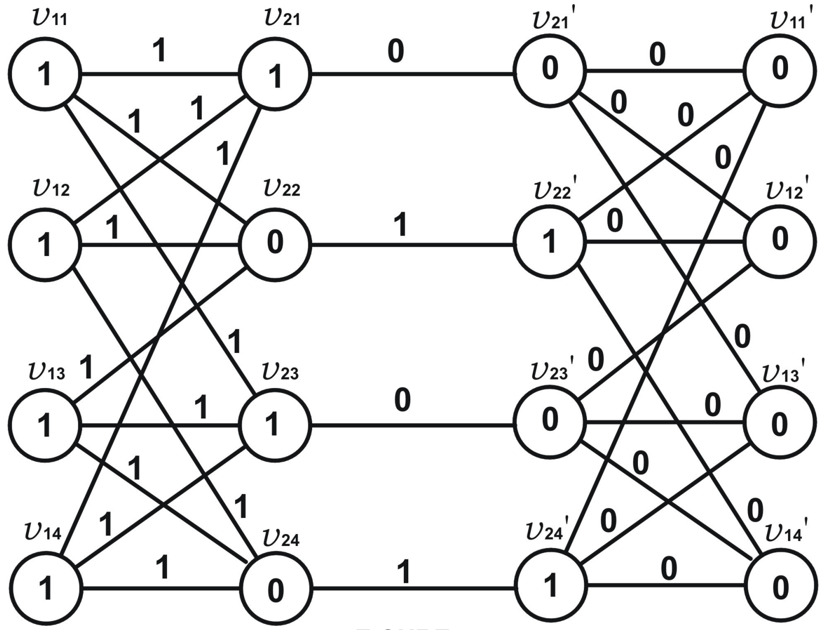

FIGURE 3

3. CONCLUDING REMARKS

Here we investigate E-cordial labeling for some mirror graphs. To investigate similar results for other graph families and in the context of different graph labeling problems is an open area of research.

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