An Approach of Human Emotional States Classification and Modeling from EEG

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Abstract

In this paper, a new approach is proposed to model the emotional states from EEG signals with mathematical expressions based on wavelet analysis and trust region algorithm. EEG signals are collected in different emotional states and some salient features are extracted through temporal and spectral analysis to indicate the dispersion which will unify different states. The maximum classification accuracy of emotion is obtained for DWT analysis rather than FFT and statistical analysis. So DWT analysis is considered as the best suited for mathematical modeling of human emotions. The emotional states are modeled with different mathematical expressions using the obtained coefficients from trust region algorithm that can be compared with the sub-band wavelet coefficients of different states. The proposed approach is verified with the adjusted R-square percentage and the sum of square errors. The adjusted R- square percentage of the mathematical modeled states are 78.4% for relax, 77.18% for motor action; however for memory, pleasant, enjoying music and fear they are 93%, 95.6%, 97.7% and 91.5% respectively. The proposed system is reliable that can be applied for practical real time implementation of human emotion based systems.

Keywords: EEG, Emotional State, Emotion Modeling, DWT, FFT, Trust-region Algorithm.

1. INTRODUCTION

In the present context, emotions are considered to be relatively short-duration intentional states that entrain changes in motor behavior, physiological changes, and cognitions in which the brain estimates with previously acquired experiences expressed with mathematical structural formulas and different cognitive activities. It is subjective and non-deterministic. The same stimulus can create different emotion for different time. Study with human behavioral activities and different brain functions have been an active research area for recent researchers, where researches have been carried out on facial expressions [1] or on the recognition of emotions according to the internal changes of emotional activity of brain signal [2]. The human brain is the part of the body that regulates almost all of the human emotional activity. The lack of mismatch between the ready-made inner stereo-type prepared in advance by the brain and the changing circumstances with different time and situations views different emotions [3]. Research in the field of emotional states modeling with mathematical expressions is not only challenging due to the interdisciplinary aspects of the topic, but it is also hindered by the fact that human cognition, emotion and behavior are investigated by researchers from different disciplines. This leads to dissimilar research methods, goals, specifications and results. A mathematical description of research results from the area of emotion studies will support and facilitate the study and development of affective systems [4].

The focus of this paper is to establish the possibility of successful mathematical modeling of the highly complex phenomenon representing human emotional states. To explain any emotional state with mathematics is really complex because the functional activity of human brain is related with time and the environments and also the frequency components are the unique parameters which primarily related with emotional activity. So, for expressing of emotional states time associated with its frequency information is the most feasible which have made more interest to model the states with different mathematical expressions and functions based on the discrete wavelet transformation and different time-frequency based coefficients.

In case of signal acquisition the following aspects should be considered: (i) proper positioning of electrodes and depth of placement of electrodes on the scalp (ii) time duration of video clips and (iii) proper design of signal acquisition protocol [5]. Many researchers have discussed the recording of EEG signals, removing noise technique for estimating ERP signals and signal analysis for better understanding of neural correlations [6]. An emotion recognition system using EEG signal from music induced activity is discussed in [7] in which emotion detection is analyzed from alpha EEG powers related to left hemisphere, right hemisphere regions of brains. Authors in [8, 9] have extracted spectral features for mental state detection by applying Gabor functions and wavelet transform. Authors in [10] classified different emotional states and used statistical analysis to find the emotion stimulation effect. In this paper we propose an emotion recognition technique using multiclass support vector machine. Beside this the best analysis technique is selected according to the classification rate of human emotion recognition.

Modeling of human behavior and characterization of emotional states with mathematical expressions is a new research topic now a day which is being experimented in this paper. Authors in [3] applied a model of information processing by memory by human adaptation to emotional factors. They tried to structure a mathematical formula according to the strength and actual need of human behavior. The appraisal theory model is focused in [4] to predict emotions and determine the appropriate system behavior to support Human-Computer-Interaction. The use of mathematical modeling was investigated as a unifying language to translate the coherence of appraisal theory and found that the mathematical category theory supports the modeling of human emotions according to the appraisal theory model. Al Mejrad in [11] detected mental behavior from physiological signals, feature extraction through wavelet transform, data reduction, and features classification using various classification methods. Swangnetr et.al in [12] proposed a new computational algorithm for accurate patient emotional state classification in interaction with nursing robots during medical service. Gratch et.al in [13] discussed a detailed domain-independent model for deriving emotional state that can be modeled by virtual humans such as facial expressions, dialogue management, planning, reacting, and social understanding. The use of trust region algorithm for non-stationary EEG signal is discussed in [14, 15]. In [16, 17, 18, 19] effective channel selection technique of BIOPAC automated MP36 system for EEG signal acquisition was proposed for emotion recognition using statistical and frequency based measures of EEG signals and recognized emotional states using SVM classifier. An emotional cognitive architecture CELTS was investigated for showing that the emotional component of the architecture is an essential element of CELTS value as a cognitive architecture in [20]. The use of multi-modal sensor data discussed in [21] implied difficulties in the fusion process which may arise and must be treated predictions regarding the expressed emotion based on different databases. So in this paper we proposed the mathematical modeling of emotional states which act as a unifying language of mental behavior evaluation in case of real time application and applicable for diagnostic purpose.

The rest of the paper organized as different sections: Section 2 describes the proposed approach for modeling of human emotion. A brief review on the data collection process of the categorized emotional states, signal analysis in time, frequency and time-frequency domain and short description on classifier and algorithm are presented here. Section 3 shows the results and verification of our proposed modeling approach. Section 4 describes the conclusion on these results.

2. PROPOSED APPROACH FOR MODELING OF EMOTION

To design an EEG based emotion model, effective feature selection and accurate classification are the two important factors in order to improve the performance. The useful features need to be extracted from the EEG signals with appropriate signal processing. These extracted features are the basic inputs to the classifier for identification of different emotions from EEG signal. The extracted features include the statistical value from DWT and FFT analysis as well as statistical measures of raw EEG signal. In this research work the most important stages of a fully implemented emotional state evaluation based on some salient feature extraction of collected EEG signal are presented. The block diagram of the proposed approach of emotion classification and modeling is shown in FIG. 1.

2.1 Categorization of Emotional States

In this work the emotional states are created with different environmental impacts during audio visual activities and the categorized emotional states are: (i) Relax (RLX); (ii) Memory related (MR) task; (iii) Motor Action (MA); (iv) Pleasant (PLS); (v) Fear (FR) and (vi) Enjoying Music (EM).

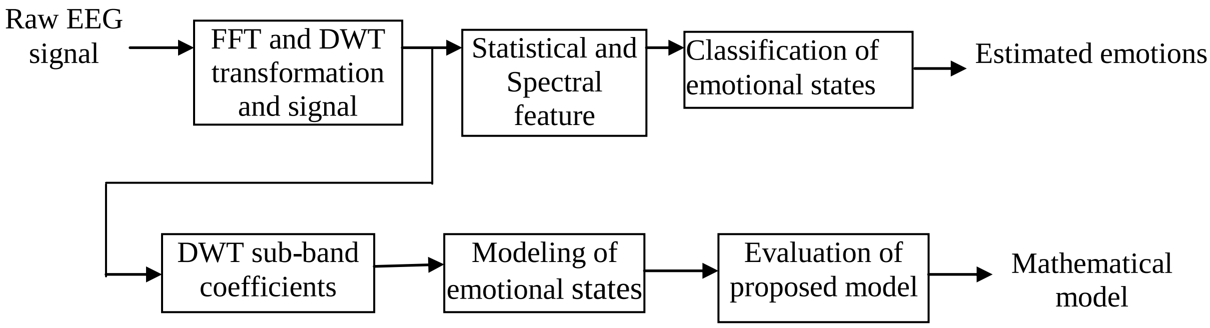


FIGURE 1: Block diagram of the proposed approach for mathematical modeling.

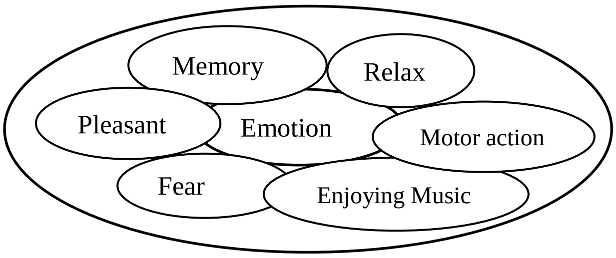


FIGURE 2: Categorization of different emotional states.

Figure 2 illustrates the six categories of emotional states in this proposed work which need to be modeled with specific mathematical expressions with different coefficients.

2.2 EEG Signal Collection and Analysis

The EEG signal for various emotional states are collected such that data of one step has no effect on other step as proper interval was given in case of data acquisition. For EEG signal collection electrode lead set (SS2L), disposable vinyl electrodes (EL503), BIOPAC data acquisition unit (MP36 and MP150) with cable and power are used. For EEG measurement, the electrodes are placed in occipital lobe region which give the variation of EEG signal.

In this research, the time domain characteristics are analyzed by taking some statistical measures on the raw EEG signal as well as the spectral components are also analyzed using discrete Fourier transform and discrete wavelet transform to identify and represent the non-stationary characteristics of EEG the signal which is particularly important for recognition of human emotional states. The pathological condition of bio-signals can sometimes be diagnosed more easily when the frequency contents of the signal are analyzed [2]. In this work some global statistical features such as amplitude (maximum, minimum, peak to peak value), mean, median, skewness, kurtosis and standard deviation have been extracted from the raw EEG signal to discriminate the predetermined emotional states.The non-Gaussianity of the EEG signals can be checked by measuring or estimating some higher-order moments such as skewness, kurtosis. Skewness shows the degree of asymmetry in a distribution (away from normal Gaussian distribution) of EEG signal and Kurtosis indicates the degree of peakedness in the distribution of EEG signal. Data sets with low kurtosis tend to have a flat top near the mean rather than a sharp peak [22].EEG signals are transformed in to frequency domain and different spectral features are extracted. The real value and imaginary value of transformed FFT data, phase angle and power spectral density of the signal are considered as features of frequency domain analysis. The time-frequency analysis can be applied to extract the wavelet coefficients of discrete time non-stationary EEG signals. The EEG signal, consisting of many data points, can be compressed into a few parameters which characterize the behavior of the EEG signal [23]. The DWT decomposition of the signal is simply obtained by successive high pass and low pass filtering of the time domain signal as shown in Eqs. (1) & (2). The original signal $x_{EEG}[n]$is first passed through a half band highpass filter g[n] and a lowpass filter $\mathrm{h[n]}$. The signal can therefore be subsampled by 2, simply by discarding every other sample.

$$y_{high}[k] = \sum_n x_{EEG}[n].g[2k-n] \tag{1}$$

$$y_{low}[k] = \sum_n x_{EEG}[n].h[2k-n] \tag{2}$$

Where,$x_{EEG}[n]$is the discrete time EEG signal, $y_{high}[k]$ and $y_{low}[k]$are the outputs of the high-pass and low-pass filters, the number of data samples $n = 256$ for each emotional states. In this paper, the Daubechies4 wavelet function (“db4”) is used for extracting the statistical feature from the EEG signal.

2.3 SVM Classifier and Trust Region Algorithm

In this paper, the human emotions have been classified into the defined classes using SVM. The learning and classification component consists of a training module and a classification module for different human emotions. The training data of the defined features of the emotional states are separated into defined classes and all the features are used as the input of multiclass SVM (MCSVM). In case of classifying mental states through SVM, the radial basis function kernel is used and the scaling range was taken $-1 \text{ to } 3$ for statistical features, $0 \text{ to} 15$ for FFT and $-1 \text{ to } 1$ for DWT. For nonlinear optimization and mathematical formulation of emotional states trust region methods are applicable [15]. It optimizes the unconstrained minimization problem by minimizing a function,$f(x)$, where the function takes vector arguments and returns scalars. In this algorithm, the approximate model is only “trusted” in a region near the current iterate. A model algorithm for unconstrained optimization problem is given in Eqs. [3] & [4]. At the $k-th$ iteration, the trial step is computed by solving the equations.

$$\min_{d\epsilon\Re^n} g_K^T d + \frac{1}{2} d^T G_K d = \emptyset_K(d) \tag{3}$$

$$s.t. \|d\|_2 \leq \Delta_K \tag{4}$$

Where $g_K = \nabla f(x_k)$ is the gradient at the current iterate x_K, B_K is a $n\times n$ symmetric matrix which approximates the Hessian of$f(x)$ and $\Delta_K > 0$ is a trust region radius. Let s_K be a solution of Eqs. (3) & (4). The predicted reduction $Pred_k$ is defined by the reduction in the approximate model,$\varphi_K(0) - \varphi_K(s_K)$.Unless $X_K$is a stationary point and $B_K$is positive semidefinite, $Pred_k$is always positive. The actual reduction, $Ared_K = f(X_K) - f(X_K + S_K)$is the reduction in the objectivefunction. The ratio between the actual reduction and the predicted reduction $r_K = Ared_K/Pred_K$ plays a very important role in the algorithm. This ratio is used to decide whether the trial step is acceptable and to adjust the new trust region radius [8].Mathematically the trust region sub-problem is typically stated.

$$min\left\{\frac{1}{2}s_K^T Hs + s_K^T g_K, \|Ds\| \leq \Delta \right\} \quad (5)$$

Where g_K is the gradient of f at the current point x_K, H is the Hessian matrix (the symmetric matrix of second derivatives), D is a diagonal scaling matrix, Δ is a positive scalar, and $//.//$ is the 2-norm. A sketch of unconstrained minimization using trust-region ideas is:

(i) Formulate the two-dimensional trust-region sub-problem.
(ii) Solve equation 5 to determine the trial step s.
(iii) If $f(X_K + S_K) < f(X_K)$, then $X_K = X_K + S_K$
(iv) Adjust Δ.

These four steps are repeated until convergence. In particular, it is decreased if the trial step is not accepted, i.e.,$f(X_K + S_K) \geq f(X_K)$.In this work, this algorithm is efficiently used to obtain the coefficients for the proposed mathematical modeling of emotional states.

3. RESULTS AND DISCUSSIONS ON MATHEMATICAL MODELING OF EMOTIONAL STATES

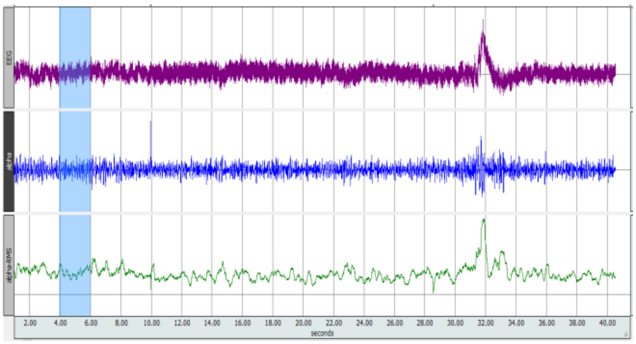
In this section, the collected data sets, their wavelet transform and FFT transform have been discussed with feature extraction. Then emotional states are classified with the extracted statistical, FFT and DWT features. Then the emotional states are modeled with mathematical expressions with the wave band coefficients.

3.1 Wavelet and FFT Transformation of Raw EEG Data

To implement an effective EEG based system a proper mathematical background emotional states is needed. So a proper suitable data collection protocol BIOPAC Electroencephalography II signal has been used to capture the raw EEG signal [19].The captured raw data and the transformed DWT and FFT data of EEG signal at relax and pleasant condition are shown in Figs. 3 and 4 respectively. In the Figs. 3(b) and 4(b) the EEG signals are separated into its different sub-bands by using DWT. In case of Fourier transformation as shown in Figs. 3(c) and 4(c) the difference of the dominant frequencies can easily be seen [17].

3.2 Feature Extracted from EEG Signal for DWT Analysis

The recognition system of different emotional states has been performed using different features. The statistics over the extracted wavelet coefficients provide compact representation that shows all the physical parameters of the EEG signal in different frequency bands. The extracted features after transforming the raw signals into DWT and FFT are shown in Table 1 and Table 2 respectively. The transformed (FFT) signal shows the deviations in amplitude and frequency in different mental states. The maximum and minimum value with different mental states for FFT and DWT is shown in Figs. 5(a) & 5(b) respectively. The variations of maximum and minimum FFT magnitude cannot differentiate the mental states clearly whereas the maximum and minimum values of 4th level sub-band coefficients clearly detect different mental states. The wavelet approach decomposes the EEG signal into its frequency components according to its range while maintaining the time resolution. So certain frequency domain properties in different mental states can be obtained in particular time series. The features are accurately captured and localized in both time-frequency contexts for the estimation of mental states that cannot be obtained by FFT. In case of mathematical modeling of emotional states it is very essential to know which features are more efficient for classifying the emotional states effectively. Table 3 shows the classification rate of different emotional states using SVM classifier and it is highest for DWT features. So in our proposed method the combination of wavelet features have proved the better consideration for emotion classification so wavelet coefficients are considered for the mathematical modeling of emotional states. Comparison of Emotion classification with other related works are given in Table 4.



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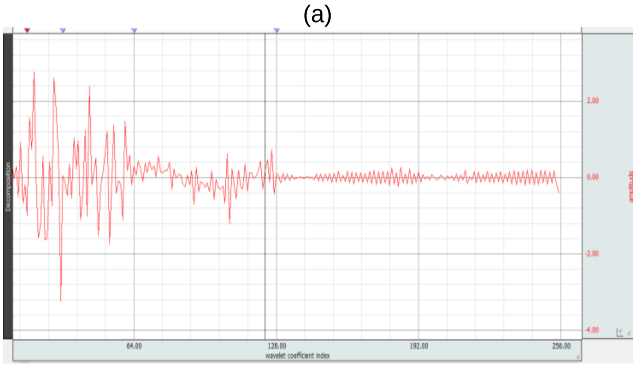

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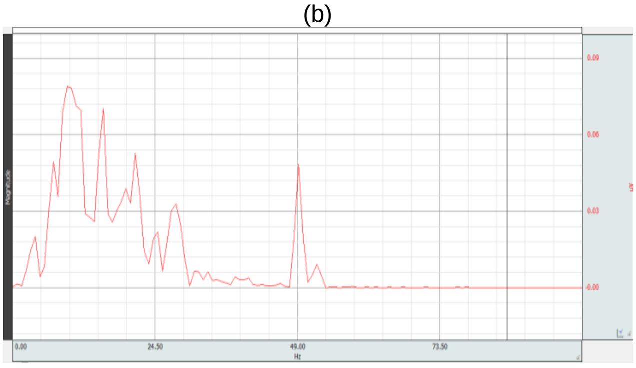
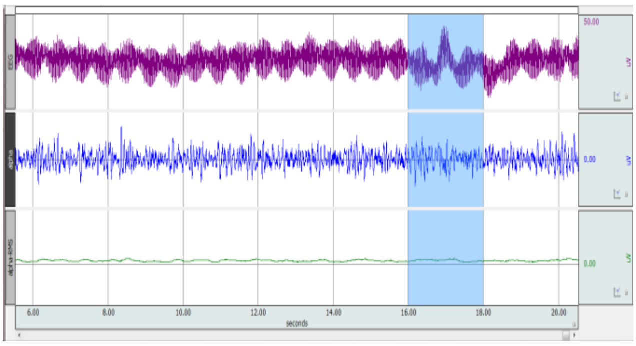
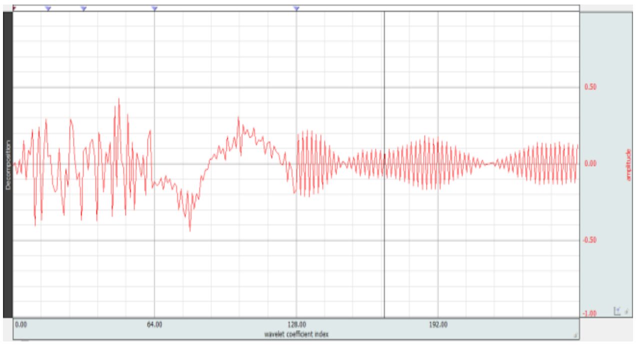


(c)

FIGURE 3: The EEG signal at relax condition, (a) Raw data, (b) DWT signal, (c) FFT signal.



(a)



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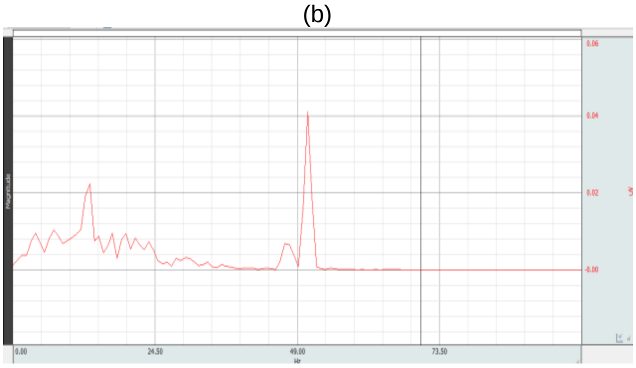


(c)

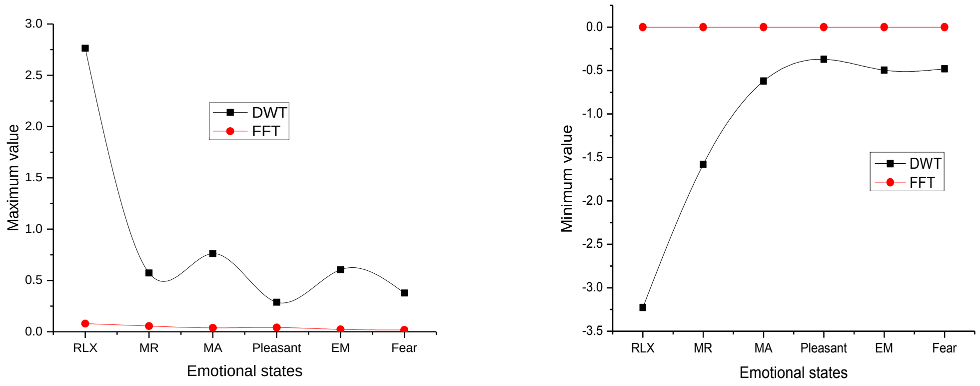
FIGURE 4: The EEG signal at pleasant condition, (a) Raw data, (b) DWT signal, (c) FFT signal.

Mental states	Extracted features	Sub-bands Coefficients of Wavelet				
		D_1	D_2	D_3	D_4	A_4
RLX	Maximum	0.26783	0.74121	2.38683	2.76374	0.911114
	Minimum	-0.39378	-1.22042	-1.74029	-3.22820	-0.99828
	Mean	0.00266	0.00071	0.10691	0.00464	-0.10520
	Standard deviation	0.13894	0.34938	0.89386	1.63026	0.61962
MR	Maximum	0.26752	0.38553	1.04740	0.57264	0.60856
	Minimum	-0.28120	-0.77295	-1.69796	-1.57946	-0.48784
	Mean	-0.00207	-0.01164	-0.02152	-0.14150	0.03588
	Standard deviation	0.16761	0.23181	0.47988	0.49296	0.32039
MA	Maximum	0.34934	0.24737	0.42874	0.76200	0.88096
	Minimum	-0.14148	-0.23056	-0.296	-0.62133	-0.94222
	Mean	0.00193	0.03021	0.01156	0.03337	0.04697
	Standard deviation	0.09510	0.10439	0.19989	0.50958	0.46321
PLS	Maximum	0.21722	0.30657	0.42664	0.28811	0.28867
	Minimum	-0.21972	-0.44256	-0.37298	-0.36932	-0.40428
	Mean	-9.43E-005	-0.01330	0.00733	-0.05271	0.01227
	Standard deviation	0.11884	0.1636	0.20334	0.18184	0.18529
EM	Maximum	0.11503	0.29909	0.38840	0.60569	0.62840
	Minimum	-0.11414	-0.37688	-0.39099	-0.49465	-0.41847
	Mean	0.00044	0.02273	-0.04357	0.02281	-0.00032
	Standard deviation	0.06306	0.11584	0.17089	0.27774	0.29538
FR	Maximum	0.07622	0.20831	0.33731	0.37702	0.2775
	Minimum	-0.08182	-0.12551	-0.43828	-0.48087	-0.27239
	Mean	-8.55E-005	0.01839	-0.01236	-0.04448	-0.00795
	Standard deviation	0.03457	0.07698	0.18909	0.22184	0.17976

TABLE 1: Extracted Features of Six Exemplary Records from Seven Mental States by DWT.

Mental States	Maximum Amplitude(µV)	Frequency at Maximum value (Hz)	Minimum Amplitude(µV)	Frequency at Minimum value (Hz)
RLX	0.07909	9.375	3.42E-005	65.62
MR	0.05571	12.5	9.54E-005	97.65625
MA	0.03757	5.46875	1.37E-005	58.59375
PLS	0.04102	50.78125	3.46E-006	76.56250
EM	0.02324	7.03125	66.25E-006	71.09375
Fear	0.01752	14.84375	3.21E-006	78.90625

TABLE 2: Extracted Features of Six Exemplary Records from Seven Mental States by FFT.



(a) (b)

FIGURE 5: Comparison of results obtained from DWT and FFT at different mental states. (a) Maximum Amplitude, (b) Minimum Amplitude.

Features	Classification Accuracy					
	RLX	**MR**	**MA**	**EM**	**Pleasant**	**Fear**
Statistical	36.67%	44.19%	37.8%	42.5%	30.56%	38.2%
FFT	55.30%	58.15%	45.6%	64.1%	73.19%	65.10%
DWT	66.84%	67.69%	52.3%	72.1%	82.35%	79.22%
All (Both spatial and temporal)	34.5%	78.12%	37.8%	57.7%	72.43%	55.65%

TABLE 3: Accuracy of different emotional states using SVM.

References	Emotional sates	Classification accuracy (%)	Features Used	Classifier
Takahashi [30]	Thought	46.7	Absolute level of heart and respiration rate, BP, skin potential response, blink rate	Artificial neural network (ANN)
	Memory Related task	80.8		
	Motor Action	66.0		
Saadat Nasehi [9]	Happiness	58.76	Gabor-PCA-based features of EEG signal	PNN
	Fear	56.79		
Proposed	Relax	66.84	Wavelet based features	Support Vector Machine (SVM)
	Memory related Task	67.69		
	Motor Action	52.3		
	Enjoying Music	72.1		
	Pleasant	82.35		
	Fear	79.22		

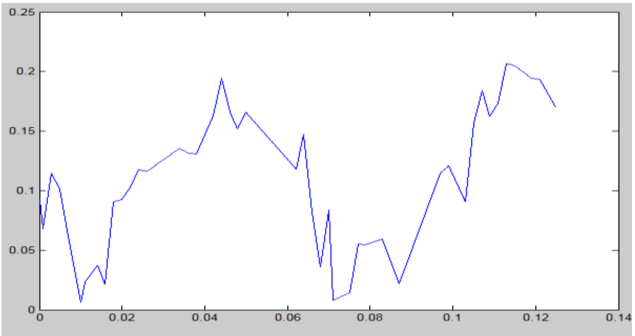
TABLE 4: Comparison Table of Emotion classification other related works.

3.3 Mathematical Modeling of Human Emotions

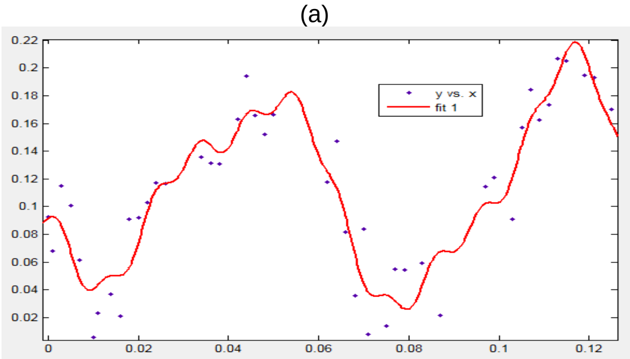
Emotion is a dynamic process that needs to understand and its changes in different environmental conditions must be analyzed [25]. In case of practical orientation of emotional states and also in case of real time implementation to interact with cognition and behavior, a mathematical background is essential which can be used as a model of significant emotional states. Authors in [26] used selected video materials and data logging process to present human emotional reactions model, which was implemented by numerical methods. Many authors used cybernetic approach to model the artificial emotion through the use of different theories of psychology [27]. Authors in [28] proposed a mathematical model by combining a queuing network-based encoding model with a brain-computer interface (BCI) model. Different machine learning based such as ANN and fuzzy based approaches has been adopted to classify emotion [29]-[30]. In our proposed approach the DWT analysis is applied for mathematical modeling of different emotional states. The DWT analysis derives the wavelet coefficients which can map the EEG data into consequent emotional states. The sub-band coefficients (D_1) of 128 point EEG data are plotted as the reference one of individual emotion. The mathematical expression of this state is plotted with the coefficients obtained from trust region. The actual wave shape of the wavelet coefficients for relax state is shown in Fig. 6(a) and Fig. 6(b) shows the wave shape of the mathematical expression which has modeled this state based on the coefficients as shown in Table 5.

Equation (6) shows the mathematical expression of relax state which can model this state on the basis of the amplitude, frequency and phase constant of this sine wave series.

$$f_{RLX}(x) = \sum_{i=1}^{6} a_i \sin{(b_i x + c_i)}, \text{ where } x \in N \qquad (6)$$



(a)



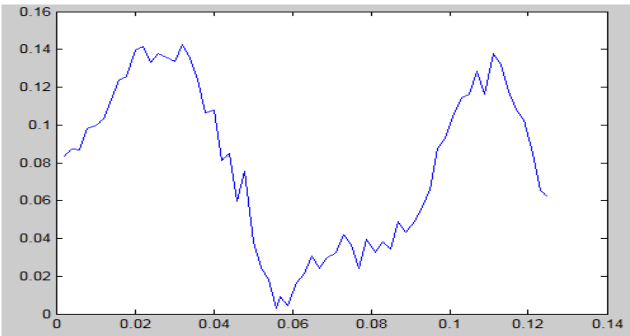
(b)

FIGURE 6: (a) Plot of Detail wavelet coefficients (D_1) of relax state;
(b) Plot of mathematical modeling for relax state.

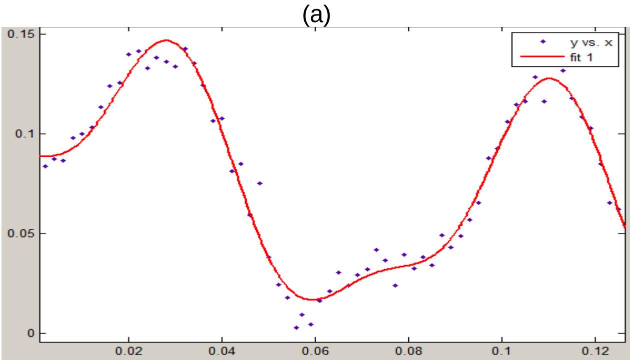
Coefficients for modeling emotional states	**Emotional states**					
	Relax	**MA**	**Memory**	**Pleasant**	**Fear**	**EM**
a_1	0.3385	0.1336	0.4645	0.1389	0.0424	0.0761
b_1	24.78	30.5	31.43	25	14.74	14.74
c_1	-0.8322	-0.3293	-0.6147	0.2056	0.0372	1.464
a_2	0.2593	0.1196	0.3392	0.2258	0.0105	0.00731
b_2	38.32	56.93	41.82	90.7	253.2	126.5
c_2	1.224	0.9673	1.827	2.473	-1.282	-1.118
a_3	24.29	0.01793	0.08284	0.2012	0.0217	0.0778
b_3	135.1	164.8	102.7	78.38	220.8	43.07
c_3	0.6373	2.306	0.9824	0.03406	-3.465	3.885
a_4	0.223		0.06213	6.58	0.9438	0.00376
b_4	142.5		152.7	229.7	108.3	1297
c_4	3.303		-2.475	-1.471	0.8613	0.2212
a_5	0.03524			6.574	0.0045	
b_5	189.5			229.8	355.5	
c_5	3.436			1.667	0.0370	
a_6	0.007478				0.9345	
b_6	604.6				108.8	
c_6	-0.03913				3.997	
a_7					0.0112	
b_7					297.9	
c_7					-1.922	

TABLE 5: Coefficients of the mathematical expressions required for modeling the emotional states.

Here, *N* is a set of 128 real numbers of the sub-band coefficients. The wave shape of motor action state of the wavelet coefficients are shown in the Fig. 7(a) and the wave shape of the mathematical expression of the emotional state is shown in the Fig. 7(b).

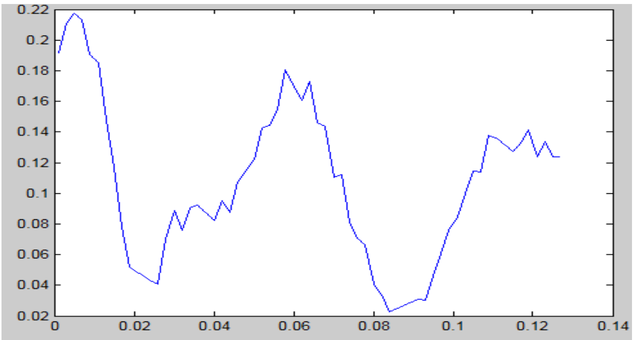


(a)

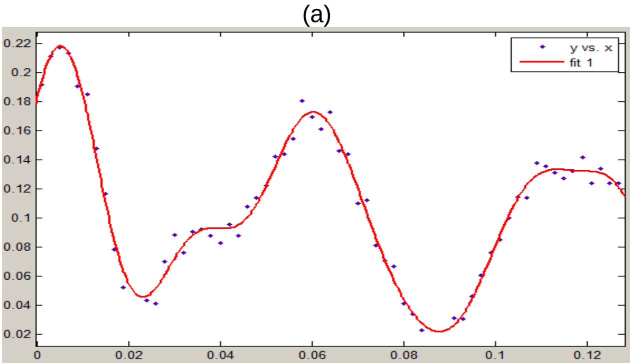


(b)

FIGURE 7: (a) Plot of Detail wavelet coefficients (D_1) of motor action (MA);
(b) Plot of mathematical model of motor action (MA).

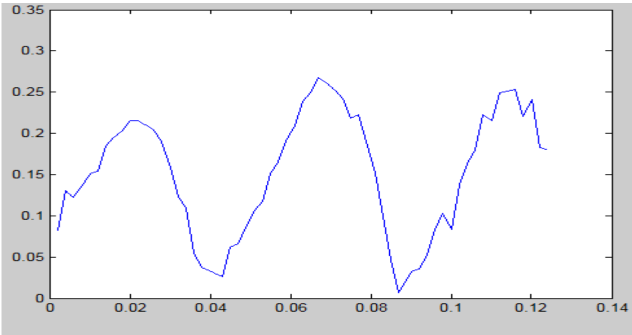


(a)

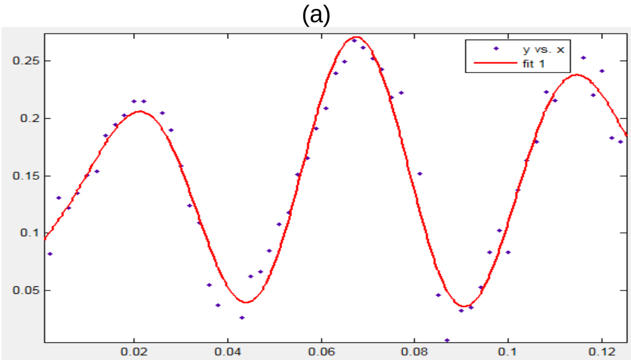


(b)

FIGURE 8: (a) Plot of Detail wavelet coefficients (D_1) of pleasant state;
(b) Plot of mathematical expressions for pleasant state modeling.



(a)



(b)

FIGURE 9: (a) Plot of Detail wavelet coefficients (D_1) of memory related task;
(b) Plot of mathematical expressions for memory related task.

The mathematical modeling of motor action (MA) is given in Eq. (7).

$$f_{MA}(x) = \sum_{i=1}^{3} a_i \sin(b_i x + c_i), \text{ where } x \in N \tag{7}$$

The graphical representation of pleasant state and Memory related task with actual coefficients and modeled state with derived coefficients are shown in Figs. 8(a) & 9(a) and 8(b) & 9(b) respectively. The mathematical model of pleasant state and memory related (MR) task is given in Eq. (8) & Eq.(9).

$$f_{PLS}(x) = \sum_{i=1}^{5} a_i \sin(b_i x + c_i), \text{ where } x \in N \tag{8}$$

$$f_{MR}(x) = \sum_{i=1}^{4} a_i \sin(b_i x + c_i), \text{ where } x \in N \tag{9}$$

Figs. 10(a) &11(a) represents the actual wave shapes with wavelet coefficients and 10(b) & 11(b) represent the plot of mathematical expression which modeled the fear state and EM state respectively. The mathematical model of fear state and EM state is given in Eqs. (10) & (11) respectively.

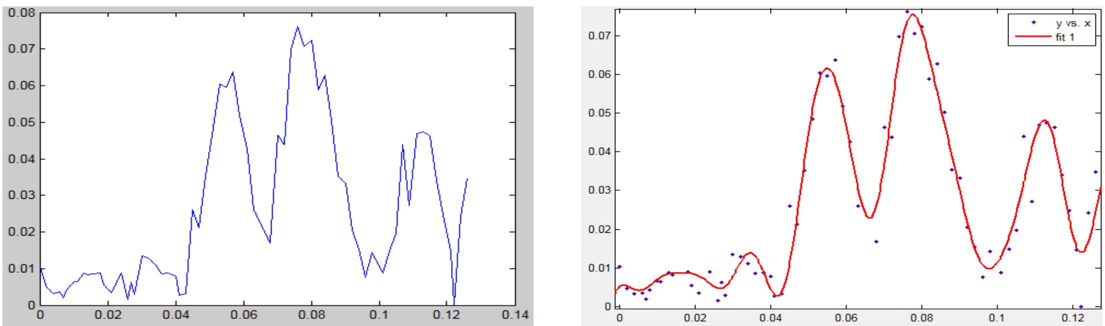
$$f_{FR}(x) = \sum_{i=1}^{7} a_i \sin(b_i x + c_i), \text{ where } x \in N \tag{10}$$

$$f_{EM}(x) = \sum_{i=1}^{4} a_i \sin(b_i x + c_i), \text{ where } x \in N \tag{11}$$

Where a_i,$b_i$and $c_i$are the coefficients of the summation of sinusoidal series which represents the amplitude, frequency and phase constants respectively in which i is the iteration of sine series for each sine series expressions in specific emotional states.

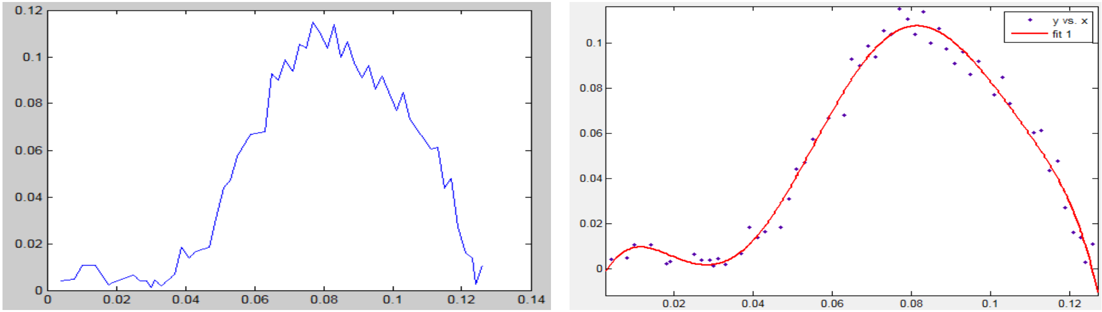
In Figs. 6(a) ~11(a) the plot of detailed wavelet coefficients for the specific emotional states are shown. Where x-axis represents the sampled value of EEG data and the y-axis represents the detailed wavelet coefficients (D_1) of the transformed EEG data. In Figs. 6(b) ~11(b) the dotted values are the actual wavelet transformed EEG coefficients and the continuous shape of the plot is the acquired shape based on mathematical modeling of the specific mental states. The modeling of emotions is based on the different coefficients as shown in Table 5 for different emotional states. In Table 5, a is the amplitude, b is the frequency, and c is the phase constant for each sine wave term. i is the number of terms in the series and $1 \leq i \leq 8$. Depending upon these values the different emotional states are modeled with different mathematical equation.

In Table 6 R-square computes the coefficient of determination (R-square) value from actual data and modeled data. The larger the R-squared is, the more variability is explained for the mathematical modeling. R-squared increases with added predictor variables and the adjusted R-squared adjusts for the number of predictor variables in the model. R-square also outputs the root mean squared error (RMSE) and sum of squared errors (SSE) for convenience. From the values of SSE it can be determined that how much the proposed model is valid for each emotional state.



(a) (b)

FIGURE 10: (a) Plot of Detail wavelet coefficients (D_1) of fear state; (b) Plot of mathematical expressions which models the fear state.



(a) (b)

FIGURE 11: (a) Plot of Detail wavelet coefficients (D_1) of enjoying music (EM) state; (b) Plot of mathematical expressions which has modeled the enjoying music (EM) state.

In case of our modeling the adjusted R-square values for relax, MA, memory, pleasant, EM and fear states are 0.7849, 0.7718, 0.9327, 0.9559, 0.9771 and 0.9149 respectively which can be further improved by modifying the parameters on which the fitting of the curve depends. Among the all states the pleasant and the enjoying music (EM) states are the best mathematical model in this analysis and their deviations from the proposed model is 0.001742 and 0.00213 respectively. Experimental result shows that the modeled mathematical expressions represent the sum of sinusoidal series which varies with number terms of sine series, *i* and the value of the coefficients amplitude, frequency and phase constant of each sine series. Here, i=6 represents the relax state. The term of sine series, *i* = 3, 5, 7, 4, 4 represents the MA, pleasant, fear, memory states and EM states respectively according to different coefficients a_i, b_i, c_i.

From the simulation result the mathematical model of the emotional state has been developed. The obtained coefficients and the developed mathematical expression can be applied when emotional states will be modeled. When the DWT coefficients are obtained from the EEG data at different emotional states then using these expressions we can express them in a unifying language. There will be no need to use the machine learning algorithm to find what states the subjects were belong to. From the modeled curve the physiologist will be able to estimate the mental state of the subjects.

Emotional states	R-square	Adjusted R-square	SSE	RMSE
Relax	0.868	0.7849	0.02007	0.02726
MA	0.8007	0.7718	0.03713	0.02598
Memory	0.9461	0.9327	0.01556	0.01881
Pleasant	0.9669	0.9559	0.001742	0.004063
EM	0.9719	0.9771	0.00213	0.00659
Fear	0.9407	0.9149	0.00192	0.006461

TABLE 6: List of correlation showing the adjustment between original and modeled curves with their errors for 256 data samples.

3.4 Performance Evaluation of Our Proposed Model

This mathematical model is for 256 data samples of each state. The raw EEG signals are analyzed with different time and frequency window and transformed in to 256 data samples. If 512 data samples are taken the coefficients can be obtained according to the trust region algorithm and applied with the sum of sinusoidal series expressions and can be verified from their R-square and Adjusted R-square value. Table 7 shows the result of correlation for 512 data samples which verify our proposed model with reference one for the 256 samples of data.

The sum of square error (SSE) and root mean square error (RMSE) shows the effectiveness of our proposed mathematical model as shown in Figs. 12(a) & 12(b) for 256 and 512 data samples of different emotional states respectively. Table 8 shows the value of adjusted R-square and errors for 128 number of data samples. Figure 13(a) & 13(b) shows the plot of

SSE and RMSE for 256 and 128 data samples respectively. From Table 5, 6 and 7 it is shown that when the model is for 512 data samples the SSE values from 0.5% to 5% and for 128 samples it varies from 0.2% to 5%. For RMSE it is between 0.7% to 4% and 0.2% to 4% for 512 samples and 128 samples respectively. From the adjusting percentage of the predictor variables in the model and the percentage errors the efficiency of our proposed model can be justified.

Emotional states	R-square	Adjusted R-square	SSE	RMSE
Relax	0.766	0.724	0.0501	0.0451
MA	0.821	0.794	0.036	0.0262
Memory	0.853	0.785	0.0356	0.0257
Pleasant	0.945	0.872	0.0057	0.0154
EM	0.913	0.887	0.0136	0.00784
Fear	0.924	0.902	0.0142	0.00847

TABLE 7: List of correlation showing the adjustment between original and modeled curves with their errors for 512 data samples.

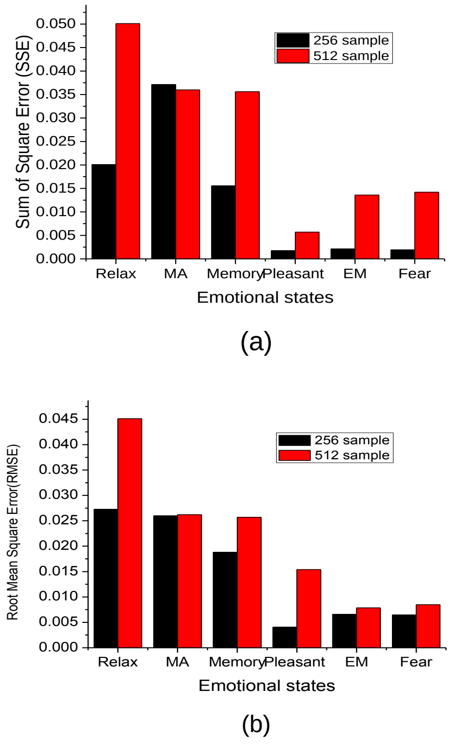


FIGURE 12: (a) Plot showing the SSE value for 256 and 512 data samples, (b) Plot showing the RMSE value for 256 and 512 data samples.

Emotional states	R-square	Adjusted R-square	SSE	RMSE
Relax	0.826	0.801	0.0432	0.034
MA	0.785	0.773	0.054	0.0432
Memory	0.975	0.946	0.0031	0.0047
Pleasant	0.924	0.912	0.0211	0.0256
EM	0.966	0.946	0.0048	0.0035
Fear	0.975	0.962	0.00293	0.00245

TABLE 8: List of Correlation showing the adjustment between original and modeled curves with their errors for 128 data samples.

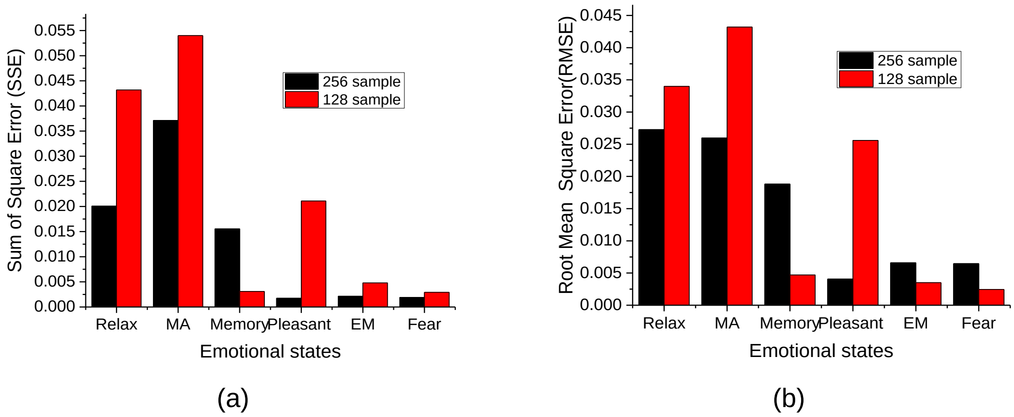


FIGURE 13: (a) Plot showing the SSE value for 256 and 512 data samples, (b) Plot showing the RMSE value for 256 and 128 data samples.

4. CONCLUSION

This paper focus on the impact of emotional states on EEG signals in different environmental conditions using different wavelet functions, spectral components and statistical measures. For this purpose, different features are extracted and applied on SVM for classification. The classification rates of different emotional states are higher for DWT analysis than FFT and statistical analysis which makes this good candidate for further mathematical modeling of emotions. The emotional states are modeled with the terms of sinusoidal series based on the amplitude, frequency and phase constants. These proposed models were compared with the actual coefficients values in terms of SSE and RMSE values. The sum of square error percentage of the predicted model are 2%, 3.7%, 1.5%, 0.17%, 0.2% and 0.19% for the relax, MA, memory, pleasant, EM and fear state respectively for 256 data samples and also verified with 512 and 128 data samples. These percentage errors of different states and the graphical representation of the modeled emotional states justify the efficacy of our proposed approach. Finally, this work developed an approach for the modeling of human emotional states with proper mathematical expressions which can further extend for practical implementation of emotion based systems. In our future work we can use these parameters to develop emotion engine architecture for real time implementation of emotion based systems. In case of pattern recognition, diagnostic decision making with lower computational complexity and low cost patient monitoring system this work can be further extended which will be much effective for real time application.

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