On Reliability Analysis of Fault-tolerant Multistage Interconnection Networks

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Abstract

The design of a suitable interconnection network for inter-processor communication is one of the key issues of the system performance. The reliability of these networks and their ability to continue operating despite failures are major concerns in determining the overall system performance. In this paper a new irregular network IABN (Irregular Augmented Baseline) has been proposed. IABN is designed by modifying existing ABN (Augmented Baseline) network. ABN is a regular multi-path network with limited fault tolerance. IABN provides three time more paths between any pair of source-destination in comparison to ABN. The reliabilities of the IABN and ABN multi-stage interconnection networks have been calculated and compared in terms of the Upper and Lower bounds of Mean time to failure (MTTF). The proposed network IABN provides much better fault-tolerance and reliability at the expanse of little more cost than ABN.

Keywords: Multistage Interconnection Networks, Reliability, Augmented Baseline Network, Irregular Augmented Baseline Network, Fault-tolerance.

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1. INTRODUCTION

Advances in LSI and VLSI technology are encouraging greater use of multiple-processor systems with processing elements to provide computational parallelism and memory modules to store the data required by the processing elements. Interconnection Networks (INs) play a major role in the performance of modern parallel computers. Many aspects of INs, such as implementation complexity, routing algorithms, performance evaluation, fault-tolerance, and reliability have been the subjects of research over the years. There are many factors that may affect the choice of appropriate interconnection network for the underlying parallel computing environment [5,6]. Though crossbar is the ideal IN for shared memory multiprocessor, where N inputs can simultaneously get connected to N outputs, but the hardware cost grows astronomically. Multistage Interconnection Networks (MINs) are recognized as cost-effective means to provide programmable data paths between functional modules in multiprocessor systems [1]. These networks are usually implemented with simple modular switches, employing two-input two-output switching elements. Most of the MINs proposed in the literature have been constructed with 2x2 crossbar switches as basic elements, and have $n = \log_2 N$ switching stages with each stage consisting of N/2 elements, which makes the cost of this network as O(NlogN), as compared to $O(N^2)$ for a crossbar [4]. The pattern of interconnection may be uniform or non-uniform, which classifies the MINs to be regular or irregular respectively [7]. In the case of irregular networks, the path length varies from any input to any output, in contrast with regular networks, where it is the same [10]. Fault-tolerance in an interconnection network is very important for its continuous operation over a relatively long period of time [8]. Fault-tolerance is the ability of the network to operate even in the presence of faults, although at a degraded performance. Permutation capability and other issues related to MINs have also been widely covered, but little attention has been paid to the reliability of these networks. Reliability [8] of a system is the probability that it will perform its intended function satisfactorily for a given time under stated operating conditions. Reliability can be measured in terms of Mean Time to Failure (MTTF). The MTTF of a MIN is defined as the expected time elapsed before some source is disconnected from some destination [3]. The analysis is based on the lower and upper bounds of the network reliability. This paper is organized as follows : Section 2 describes the structure and design of ABN and proposed network IABN. Section 3 provides the reliability analysis of ABN. Section 4 discuss the reliability analysis of IABN. Finally conclusions are given in Section 5.

2. STRUCTURE AND DESIGN OF NETWORKS

2.1 ABN (Augmented Baseline Network)

An ABN of size N x N consists of, two identical groups of N/2 sources and N/2 destinations. The switches in the last stage are of size 2 x 2 and the remaining switches in stages 1 through n-3 ($n=\log_2 N$) are of size 3 x 3 [9]. In each stage, the switches can be grouped into conjugate subsets, where a conjugate subset is composed of all switches in a particular stage that lead to the same subset of destinations. The switches, which communicate through the auxiliary links, form a conjugate loop. The conjugate loops are formed in such a way that the two switches, which form a loop, have their respective conjugate switches in a different loop. This pair of loops are called conjugate loops [1]. Each source is linked to both the groups via multiplexers. An ABN of size 16 x 16 is illustrated in Fig. 1.

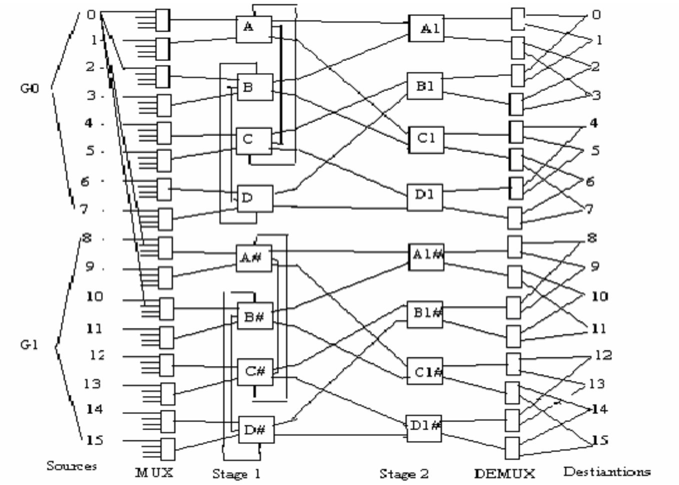

FIGURE 1: An ABN of size 16 X 16.

2.2 IABN (Irregular Augmented Baseline Network)

An Irregular Augmented Baseline Network is an augmented baseline network with one additional stage, additional auxiliary links and increased size of demultiplexers. An IABN of size N x N consists of two identical groups of N/2 sources and N/2 destinations. Each group consists of a multiple path modified baseline network of size N/2. The switches in the last stage are of size 2 x 2 and the remaining switches in stages 1 through n-2 ($n=\log_2 N$) are of size 3 x 3. In each stage, the switches can be grouped into conjugate subsets, where a conjugate subset is composed of all switches in a particular stage that lead to the same subset of destinations. The modified baseline network achieves the multiple path property by permitting two switches in the same conjugate subset that are not a conjugate pair to communicate through auxiliary links. Each source is linked to both the groups via multiplexers. There is one 4 x 1 MUX for each input link of a switch in stage 1 and one 1 x 4 DEMUX for each output link of a switch in stage n-1. Each group consisting of a modified baseline network of size N/2 plus its associated MUXs and DEMUXs is called a subnetwork. Thus an IABN consists of two identical sub-networks which are denoted by G^i. For example, in Figure 1, switches A, B, C, D belonging to stage 1 of a subnetwork (G^i) form a conjugate subset, switches A and B form a conjugate pair, and switches A and C form a conjugate loop.

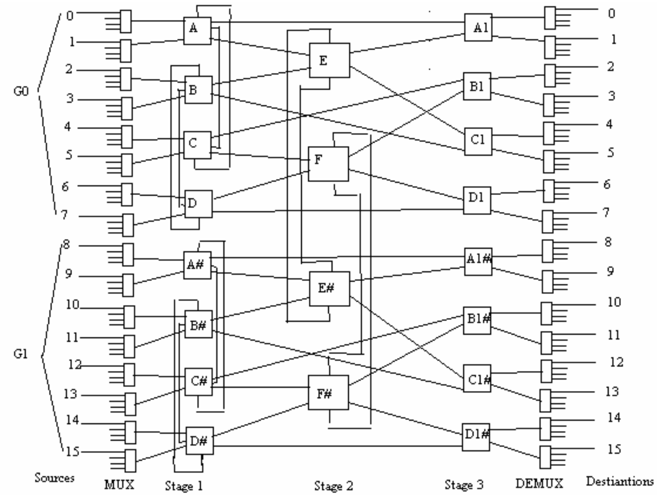

FIGURE 2 : An IABN of size 16 X 16.

2.3 Routing tag for ABN and IABN

A source selects a particular subnetwork (G^i) based upon the most significant bit of the destination. Each source is connected to two switches (primary and secondary) in a subnetwork. Let the source S and destination D be represented in binary code as:

$$S=s_0,\ s_1,\ldots,s_{n-2},s_{n-1}$$
$$D=d_0,\ d_1,\ldots,d_{n-2},d_{n-1}$$

(i) Source S is connected to the ($s_1,\ldots,s_{n-2}$) primary switch in both the sub-networks through the multiplexers.
(ii) Source S is also connected to the [$\{(s_1,\ldots,s_{n-2})+1\}$mod N/4] secondary switch in both the sub-networks through the multiplexers.

3. RELIABILITY ANALYSIS OF ABN

3.1 Upper bound (optimistic)

In ABN each source is connected to two multiplexers in each sub-network, and each switch has a conjugate. So if we assume that the ABN is operational as long as one of the two multiplexers attached to a source (in a particular sub-network) is operational and as long as a conjugate pair (loop or switch) is not faulty, then we will permit as many as one half of the components to fail and the ABN may still be operational. This permits a simple reliability block diagram of the optimistic (upper) bound as shown in Figure 3.

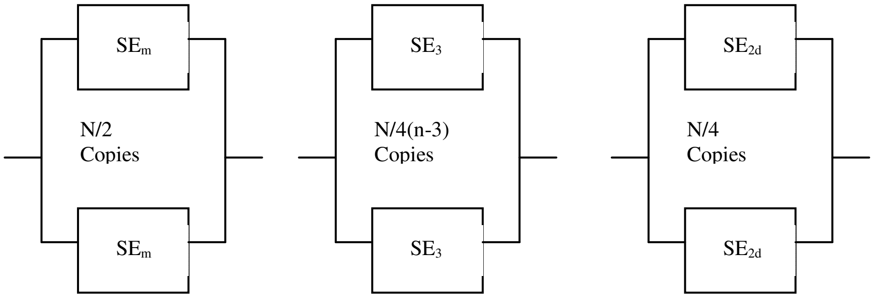

FIGURE 3: Reliability block diagram of ABN for MTTF upper bound.

The expression for the upper bound of the ABN reliability is:

$R_{ABN\text{-}ub}(t) = f1 * f2 * f3$

$$f1 = \left[1 - \left(1 - e^{-\lambda_m t}\right)^2\right]^{\left(N/2\right)}$$

$$f2 = \left[1 - \left(1 - e^{-\lambda_3 t}\right)^2\right]^{N/4(n-3)}$$

$$f3 = \left[1 - \left(1 - e^{-\lambda_{2d} t}\right)^2\right]^{\left(N/4\right)}$$

Where,

$\lambda_m = \lambda$, $\lambda_3 = 2.25\lambda$, $\lambda_{2d} = 2\lambda$

$$MTTF_{ABN\text{-}ub} = \int_0^\infty R_{ABN\text{-}ub}(t).dt$$

3.2 Lower bound (pessimistic)

At the input side of the ABN, the routing scheme does not consider the multiplexers to be an integral part of a 3 x 3 switch. For example, as long as at least one of the two multiplexers attached to a particular switch is operational, the switch can still be used for routing. Hence, if we group two multiplexers with each switch in the input side and consider them a series system (SE_{3m}), then we will have a conservative estimate of the reliability of these components. Their aggregate failure rate will be $\lambda_{3m} = 4.25\lambda$. Finally these aggregated components and the switches in the intermediate stages can be arranged in pairs of conjugate loops. To obtain the pessimistic (lower) bound on the reliability of ABN, we assume that the network is failed whenever more than one conjugate loop has a faulty element or more than one conjugate switch in the last stage fails. The reliability block diagram is shown in Figure 4.

$R_{ABN\text{-}lb}(t) = f1 * f2 * f3$

$$f1 = \left[1 - \left(1 - e^{-2\lambda_m t}\right)^2\right]^{\left(N/8\right)}$$

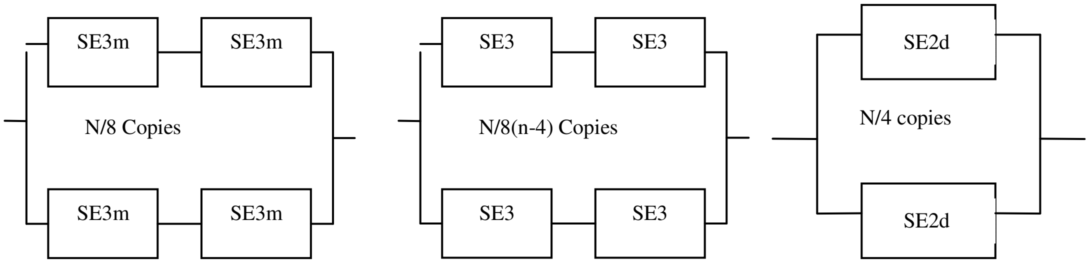

FIGURE 4: Reliability block diagram of ABN for MTTF lower bound.

$$f2 = \left[1-\left(1-e^{-2\lambda_3 t}\right)^2\right]^{N/8(n-4)}$$

$$f3 = \left[1-\left(1-e^{-\lambda_{2d} t}\right)^2\right]^{\left(N/4\right)}$$

Where,

$\lambda_{3m=}4.25\lambda$, $\lambda_{3=}2.25\lambda$, $\lambda_{2d=}2\lambda$

$$\mathrm{MTTF}_{ABN\text{-}lb}= \int_0^\infty R_{ABN\text{-}lb}(t).dt$$

4. RELIABILITY ANALYSIS OF IABN

4.1 Upper bound (optimistic)

In IABN each source is connected to two multiplexers in each sub-network, and each switch has a conjugate. So if we assume that the IABN is operational as long as one of the two multiplexers attached to a source (in a particular sub-network) is operational and as long as a conjugate pair (loop or switch) is not faulty, then we will permit as many as one half of the components to fail and the IABN may still be operational. This permits a simple reliability block diagram of the optimistic (upper) bound as shown in Figure 5.

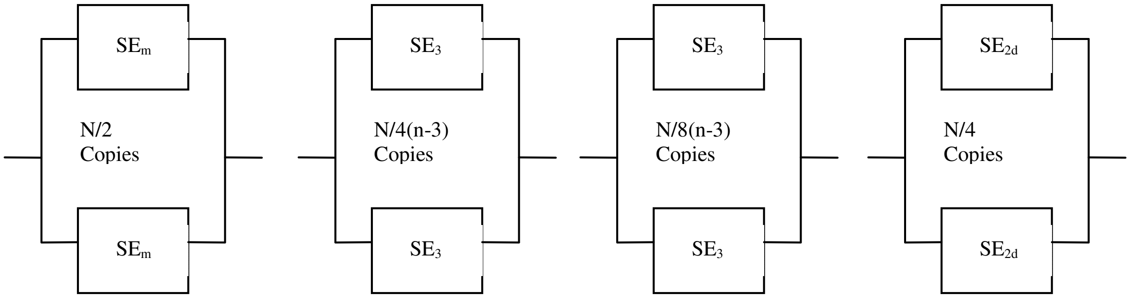

FIGURE 5: Reliability block diagram of MABN for MTTF upper bound.

$R_{IMABN\text{-}ub}(t)$= f1 * f2 * f3*f4

$$f1 = \left[1-\left(1-e^{-\lambda_m t}\right)^2\right]^{\left(N/2\right)}$$

$$f2 = \left[1 - \left(1 - e^{-\lambda_3 t}\right)^2\right]^{N/4(n-3)}$$

$$f3 = \left[1 - \left(1 - e^{-\lambda_3 t}\right)^2\right]^{N/8(n-3)}$$

$$f4 = \left[1 - \left(1 - e^{-\lambda_{2d} t}\right)^2\right]^{\left(N/4\right)}$$

Where $\lambda_{m}=\lambda$, $\lambda_{3}=2.25\lambda$, $\lambda_{2d}=3\lambda$

$$MTTF_{IABN\text{-}ub} = \int_0^\infty R_{IABN\text{-}ub}(t).dt$$

4.2 Lower bound (pessimistic)

At the input side of the IABN, the routing scheme does not consider the multiplexers to be an integral part of a 3 x 3 switch. For example, as long as at least one of the two multiplexers attached to a particular switch is operational, the switch can still be used for routing. Hence, if we group two multiplexers with each switch in the input side and consider them a series system (SE_{3m}), then we will have a conservative estimate of the reliability of these components. Their aggregate failure rate will be $\lambda_{3m} = 4.25\lambda$. Finally these aggregated components and the switches in the intermediate stages can be arranged in pairs of conjugate loops. To obtain the pessimistic (lower) bound on the reliability of IABN, we assume that the network is failed whenever more than one conjugate loop has a faulty element or more than one conjugate switch in the last stage fails. The reliability block diagram is shown in Figure 6.

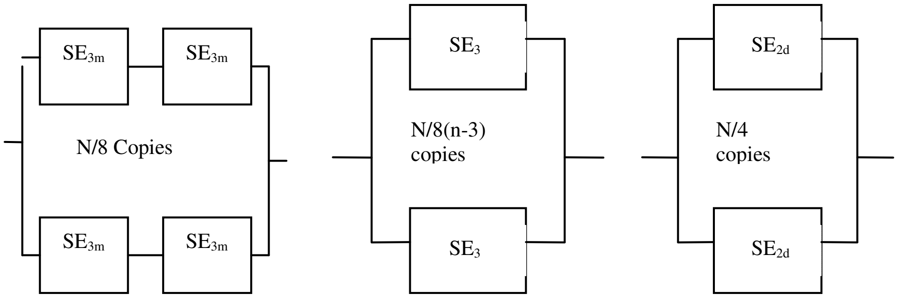

FIGURE 6: Reliability block diagram of MABN for MTTF lower bound.

$R_{IABN\text{-}lb}(t) = f1 * f2 * f3$

$$f1 = \left[1 - \left(1 - e^{-2\lambda_{3m} t}\right)^2\right]^{\left(N/8\right)}$$

$$f2 = \left[1 - \left(1 - e^{-\lambda_3 t}\right)^2\right]^{N/8(n-3)}$$

$$f3 = \left[1 - \left(1 - e^{-\lambda_{2d} t}\right)^2\right]^{\left(N/4\right)}$$

Where $\lambda_{3m}=4.25\lambda$, $\lambda_{3}=2.25\lambda$, $\lambda_{2d}=3\lambda$

$$MTTF_{IABN\text{-}lb} = \int_0^\infty R_{IABN\text{-}lb}(t).dt$$

The results of the MTTF Reliability equations have been shown in Table 1.

LogN	**ABN**		**IABN**	
	Lower Bound	*Upper Bound*	*Lower Bound*	*Upper Bound*
4	4.934369	5.141202	5.665934	5.847616
5	4.717386	4.923061	5.437529	5.556905
6	4.508375	4.71246	5.223375	5.302414
7	4.30494	4.507272	5.018943	5.068058
8	4.105551	4.306118	4.821243	4.846257
9	3.909194	4.108067	4.628248	4.632892
10	3.71518	3.912467	4.438573	4.425513

TABLE 1: MTTF of ABN and IABN for different network size

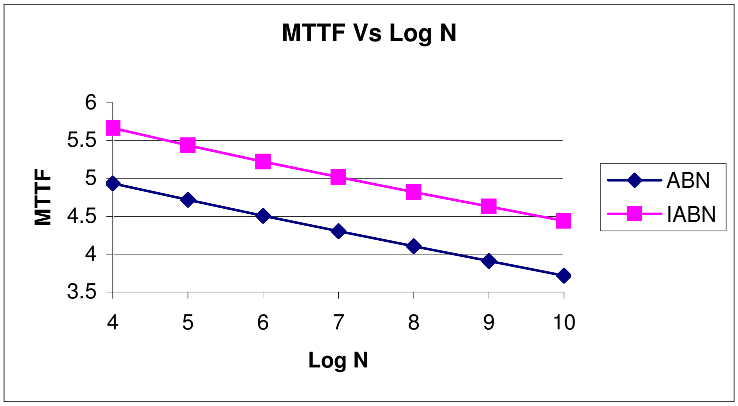

FIGURE 7: MTTF (Lower Bound) comparison of ABN and IABN

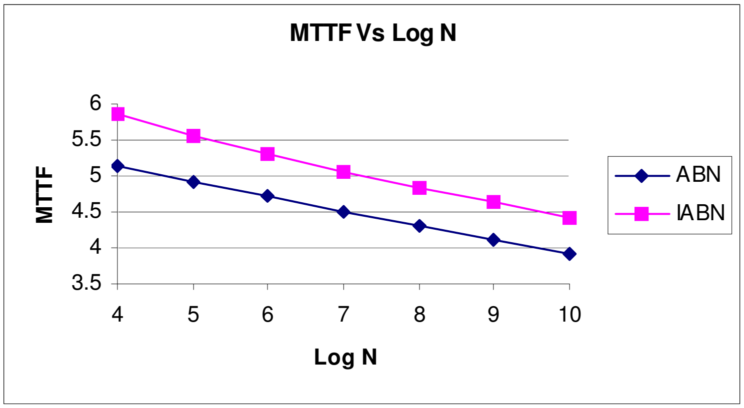

FIGURE 8: MTTF (Upper Bound) comparison of ABN and IABN

5. CONCLUSION

An Irregular Augmented Baseline Network (IABN) is designed from regular Augmented Baseline Network (ABN) have one extra stage. IABN is a dynamically re-routable and provides multiple paths of varying lengths between a source-destination pair. It has been found that in an IABN, there are six possible paths between any source-destination pair, whereas ABN has only two paths with same length. Thus proposed IABN is more fault-tolerant. The reliability analysis shows that IABN has better performance than ABN for both lower and upper bounds.

6. REFERENCES

1. Aggarwal R., Aggarwal H. and Kaur L., “On Bandwidth analysis of Irregular Fault-tolerant Multistage Interconnection networks”, International Review on Computers and Software, Vol. 3, No. 2, March 2008, pp. 199-202.

2. Aggarwal H., and Bansal P.K., Routing and Path Length Algorithm for cost effective Modified Four Tree Network, IEEE TENCON, 2002, pp. 293-294.

3. Bansal P.K., Joshi R.C. and Singh K., “On a fault-tolerant Multi-stage interconnection network”, International Journal of Electronics and Electrical Engineering, Vol 20, No.4, 1994,, pp. 335-345.

4. Bansal P.K.,Joshi R.C, Singh K. and Siroha G.P., “Fault-tolerant Augmented Baseline Multistage Interconnection Network”, International Conference IEEE TENCON 91, INDIA, Aug. 1991, pp. 200-204.

5. Bhuyan Laxmi N., Yang Qing and Agrawal Dharma P., “Performance of Multiprocessor Interconnection Networks”, IEEE Computer, Vol. 22, Feb. 1993, pp. 25-37.

6. Duato Jose, Yalamanchili Sudhakar and Ni Lionel, “Interconnection Networks: An Engineering Approach”, IEEE Computer Society, 1997.

7. Lubazewski M. and Coutois B., ”A Reliable Fail-safe System”, Parallel and Distributed Systems, IEEE Computer Society, Vol. 47, No. 2, Feb. 1998, pp. 236-241.

8. Nitin, “On analytic Bounds of Regular and Irregular Fault-tolerant Multistage Interconnection Networks”, International Conference on PDPTA, June 26-29, 2006.

9. Sadawarti Harsh and Bansal P.K., “Fault tolerant Irregular Augmented Shuffle Network” WSEAS International Conference on Computer Engineering and Applications, Gold Coast, Australia, 17-19, Jan, 2007.

10. Sengupta J. and Bansal P.K, “Reliabilty and performance measures of regular and irregular multi-stage interconnection networks”, International Conference IEEE TENCON, 2000, pp. 531-536.

11. Sengupta J. and Bansal P.K., “Performance Analysis of Regular and Irregular Dynamic MINs”, International Conference IEEE TENCON 99, Sept. 1999, Cheju Island, Korea, pp. 427-430.

12. Sharma Sandeep, Kalhon K.S., Bansal P.K. and Singh Kawaljeet, “Improved Irregular Augmented Shuffle Exchange Multistage Interconnection Network”, International Journal of Computer Science and Security, Vol. 2, No. 3, 2008, pp. 28-33.