A Genetic Algorithm for Reliability Evaluation of a Stochastic-Flow Network With Node Failure

A. Younes a_y_hamed@yahoo.com
Computer Science Department,
Faculty of Science, Sohag University,
Sohag, Egypt

M. R. Hassan m_r_hassan73@yahoo.com
Computer Science Department,
Faculty of Science, South Valley University,
Aswan, Egypt.

Abstract

The paper presents a genetic algorithm to compute the reliability of a stochastic- flow network in which each arc or node has several capacitis and may fail. I.e. Calculate the system reliability such that the maximum flow is not less than a given demand. The algorithm is based on generating all lower boundary points for the given demand and than the system reliability can be calculated in terms of such points. The proposed algorithm can be used for a network with large number of nodes and links. Also, the paper investigates the problems that are found in the solutions that obtained by using other previous methods.

Keywords: Genetic Algorithms, Stochastic-flow Network, System Reliability.

1. INTRODUCTION

The reliability of a computer network (in the case of no flow happen) is defind as probability that any source node can successfully communicate with any terminal node. This basically gives the probability that no node in a network is disconnected from the rest of the network and it doesn't yeild any information about the performance aspects, Ahuja and Kumar [1]. Rai and Aggarwal [2], preseneted a method for finding the terminal-pair reliability expression of a general network. Rai and Aggarwal [3], defined the network reliability for a computer-communication network and proposed a method based on spanning trees for its evaluation. Younes [4], presented an algorithm to find the spanning trees of a given network in terms of links for using them to compute the network reliability. The algorithm uses the connection matrix of a given network to find the spanning trees, and also is based on a relation that uses the probability of unions of the spanning trees to obtain the network reliability.

The system reliability of a flow network is the probability that the maximum flow of the network is not less than the given demand d. The capacity of each arc in this network, which is defined as the maximum flow passing the arc per unit time, has two levels 0 and positive integer. Lin [5], proposed a simple algorithm to generate all lower boundary points for a given demand d, and then the system reliability can be calculated in terms of such points. Lin [6], constructed a stochastic-flow network to model the information system. The studied problem is to evaluate the possibility (them mission reliability) that a given amount of multicommodity can be sent through an information network under the cost constraint. Lin [7], proposed a new performance index, the probability that the upperbound of the system capacity equals the demand vector subject to the budget constraint, to evaluate the quality level of a multicommodity limited-flow network. Statitsatin and Kapur [8], presented an algorithm to search for lower boundary points and used it for computing the exact reliability. Lin and Yeh [9], the network reliability under a components-assignment can be computed in terms of

minimal paths, and state-space decomposition. Subsequently, they proposed an optimization method based on a genetic algorithm.
In recent years, gentetic algorithms (GAs) have been applied to various problems in the network design ([10-16]). Lo and Chang [17], proposed a multiojective hybrid genetic algorithm to solve the capacitated multipoint network design problem.
In this paper, a gentetic algorithm (GA) is proposed to evaluate the reliability of a stochastic flow network. The proposed GA is based on minimal paths (MPs) to find all lower boundary points for d and then calculate the system reliability in terms of such points.
The paper is organized as follows: The assumptions and notation used given in Section 2. Section 3 describes the problem of calculating the network reliability. Section 4 presents the proposed algorithm for calculating the network reliability. The over all algorithm presented in section 5. In Section 6 shows how to use the proposed algorithm to calculate the reliability of a stochastic-flow network for two example networks and presents the discussion.

2. NOTATIONS and ASSUMPTIONS

Notations:

G(A, N, M) A stochastic-flow network with a set of arcs **A=** $\{a_i | 1 \le i \le n\}$, a set of nodes $N = \{a_i | n+1 \le i \le n+p\}$ and $M = \{M^1, M^2, ..., M^{n+p}\}$ with M^i (an integer) being the maximum capacity of each component a_i (arc or node).

X Capacity vector; X= $(x_1, x_2, …, x_{n+p})$.

F Flow vector; F $= (f_1, f_2, …, f_m)$.

MPs Minimal paths.

mp_j Is a minimal path no. j; j = 1, 2, …, m.

$\mathbf{L_j}$ Is the maximum capacity of mp_j; $L_j = \min\{M^i | a_i \in mp_j\}$.

V(X) The maximum flow under X; $V(X) = \max\{\sum_{j=1}^{m} f_j | F \in U_X\}$, where U_x = {F | F is feasible under X}.

R_d System reliability to the given demand d.

pop_size is the population size.

max_gen is the maximum number of generations.

p_m is the GA mutation rate.

p_c is the GA crossover rate.

Assumptions:

1- The capacity of each component a_i is an integer-valued random variable which takes values $0 < 1 < 2 < ... < M^i$ according to a given distribution..

2- Flow in G must satisfy th so-called flow-conservation law.
the processing elements, and edges denote the communication links.

3. The capacities of different components are statistically independent.

3. PROBLEM DESCRIPTION

Given the demand d, The system reliability R_d is defind by, Lin [5]:

$$\mathbf{R_d} = \Pr\{\mathbf{X} | \mathbf{V(X)} \ge \mathbf{d}\} \quad \text{…(1)}$$

Where X is a lower boundary point for d.
And X can be deduced from F $= (f_1, f_2, …, f_m)$ by using the following equation:

$$\mathbf{x_i} = \sum_{\mathbf{j=1}}^{\mathbf{m}} \{\mathbf{f_j} | \mathbf{a_i} \in \mathbf{mp_j}\} \text{ for each i = 1, 2, …, n+p.} \quad \text{…(2)}$$

So, the main purpose of the proposed GA in this paper is to find the set of all feasible solutions of F that satisfies the flowing two constraints:

$$\sum_{j=1}^{m} \{ f_j \mid a_i \in mp_j \} \le M^i \quad \text{for each } i = 1, 2, \ldots, n+p, \qquad \ldots(3)$$

$$\sum_{j=1}^{m} f_j = d . \qquad \ldots(4)$$

4. THE PROPOSED GENETIC ALGORITHM

This section describes the basic components of the proposed GA.

1. REPRESENTATION

If the network has m number of minimal paths, then the chromosome CH has m fields, each field represents the (current) flow on each path. I.e.

CH = $(f_1, f_2, \ldots, f_m)$, f_j is current flow on mp_j.

2. INITIAL POPULATION

The initial population is generated according to the following steps:
Step 1: Randomly generate a chromosome CH in the initial population in the form:

CH = $(f_1, f_2, \ldots, f_m)$

where $f_i \in \{0,1,\ldots,d-1\}$; d is the given demand.

Step 2: If the generated chromosome in step 1 doesn't satisfy eq. 4, discard it and go to step 1.
Step 3: Repeat steps 1 to 3 to generate pop_size chromosomes.

3. THE OBJECTIVE FUNCTION

The problem can be formulated as:

Find the set of all feasible solutions F
Such that equations 3 and 4 have been satisfied

4. Crossover Operator

In the proposed GA, one-cut point crossover (i.e. an integer value is randomly generated in the range (0, m-1) where m is the length of the chromosome) is used to breed two offsprings (two new chromosomes) from two parents selected randomly according to pc value, as shown in the flowing example (The network has 5 MPs):

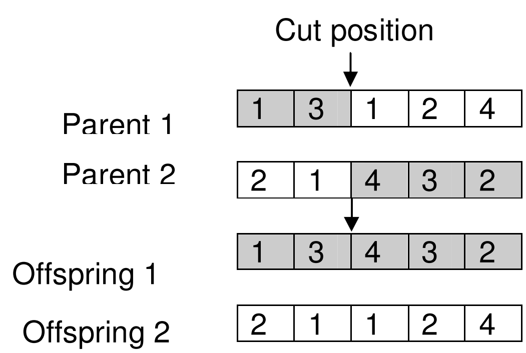

FIGURE 1: Single point crossover

5. MUTATION OPERATOR

A child undergoes mutation according to the mutation probability P_m .
Step 1: Generate a random number r_m, $r_m \in [0,1]$.

Step 2: If $r_m < P_m$, the chromosome is chosen to mutate and go to step 3; otherwise skip this chromosome.
Step 3: For each component of the child do:
Begin;
if $f_i >= 1$, then set $f_i = 0$.
Else
set $f_i = 1$.
End do.

Fig.2 shows the an example of performing the mutation operation on a given chromosome:

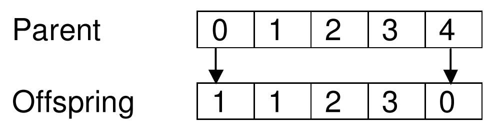

FIGURE 2: The mutation operator.

6. TERMINATION CONDITION
The execution of the GA is terminated when the number of generations exceeds the specified number of maximum generations or the set of all feasible solutions of F have been generated.

5. AN ALGORITHM FOR SYSTEM RELIABILITY EVALUATION

Given the demand **d**, then $\mathbf{R_d = Pr\{X | X \geq X^i}$ for a lower boundary point $\mathbf{X^i}$ for **d** $\}$ = $\mathbf{Pr\{\bigcup_{all\ lower\ boundary\ points\ X^i\ for\ d}\{X | X \geq X^i\}\}}$, i = 1, 2, …, q and q is the total number of lower boundary points for **d**. Applying an inclusion-exclusion rule to compute $\mathbf{Pr\{\bigcup_{all\ lower\ boundary\ points\ X^i\ for\ d}\{X | X \geq X^i\}\}}$, see [5]. The following algorithm is presented in this paper to calculate $\mathbf{R_d}$ according to the bove rules:-

Step 1: Generate all possible intersections for all lower boundary points X.

Step 2: Calculate the probability (accumulative probability) for each **X** and also for each intersection.

Step 3: Calculate $\mathbf{R_d}$ as follows:
Set $\mathbf{B_1 = \{X | X \geq X^1\},\ B_2 = \{X | X \geq X^2\},\ ...,\ B_q = \{X | X \geq X^q\}.}$
Apply inclusion-execlusion rule to calculate $\mathbf{R_d}$ by using the relation**:**

$$\mathbf{R_d = \sum_i Pr\{B_i\} - \sum_{i \neq j} Pr\{B_i \cap B_j\} + \sum_{i \neq j \neq k} Pr\{B_i \cap B_j \cap B_k\} - ... + + (-1)^{q-1} Pr\{B_1 \cap B_2 \cap ... \cap B_q\}}$$

6. THE OVERALL ALGORITHM

This section presents the proposed GA for computing the system reliability of a stochastic-flow network. The steps of this algorithm are as follows:

Step 1: Set the parameters: pop_size, max_gen, P_m , P_c, ns = 0, and set gen = 0.

Step 2: Generate the initial population, as described in section 4.1.

Step 3: To obtain chromosomes for the new population; select two chromosomes from the parent population according to P_c. Apply crossover, then mutate the new child according to P_m parameter.

Step 4: If the new child satisfies the two constraints (equations 3 and 4) and it doesn't equal to any pre generated child, then keep it and increase **ns**. If it fails to satisfy them, discard this child and reapply the mutation opereator to the original parent.

Step 5: Set gen =gen + 1;
If gen > max_gen, then go to step 3, otherwise goto step 3 /* to get a new generation.

Step 6: Report the set of all feasible solutions (equal to ns) F, and generate X from F using eq. 2.

Step 7: Suppose the result of step 6 is: $X^1, X^2, \ldots, X^q$. Then, obtain all lower boundary points for d by removing non-minimal ones in $X^1, X^2, \ldots, X^q$.

Step 8: Calculate R_d according to the algorithm given in section 5 and print out the results.

7. EXPERIMENTAL RESULTS & DISCUSSION

This section shows how to use the proposed GA to calculate the reliability of a stochastic-flow network for an example network with two different demands and presents the discussion of the obtained solutions.

1. EXPERIMENTAL RESULTS

To illustrate the proposed algorithm for computing the system reliability, consider the flowing example network shown in Fig. 3 taken from [5]. This network has 4 nodes and 6 arcs. The arcs are numbered from a_1 to a_6 and the nodes from a_7 to a_{10}. The same capacity distribution of each component given in [5] will be used to calculate R_5 (i.e. the demand d = 5).

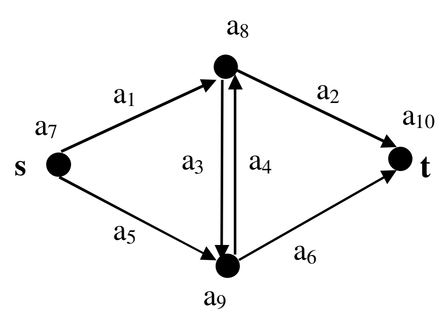

FIGURE 3: Computer network

There are 4 MPs :

$mp_1 = \{a_7, a_1, a_8, a_2, a_{10}\}$
$mp_2 = \{a_7, a_1, a_8, a_3, a_9, a_6, a_{10}\}$
$mp_3 = \{a_7, a_5, a_9, a_6, a_{10}\}$
$mp_4 = \{a_7, a_5, a_9, a_4, a_8, a_2, a_{10}\}$

The proposed algorithm will generate all lower boundary points for the demand **d = 5** as follows:

Step 1: Find the set of all feasible solution of **$F = (f_1, f_2, f_3, f_4)$** according to the constraints (3) and (4);

$$f_1 + f_2 \le 2, f_1 + f_4 \le 3, f_2 \le 2, f_4 \le 3, f_3 + f_4 \le 3, f_2 + f_3 \le 3, \quad ...(3)$$

$$f_1 + f_2 + f_3 + f_4 \le 6, f_1 + f_2 + f_4 \le 5, f_2 + f_3 + f_4 \le 4, f_1 + f_2 + f_3 + f_4 \le 5.$$

$$f_1 + f_2 + f_3 + f_4 \le 5. \quad ...(4)$$

There are four solutions:

(2 0 2 1), (1 1 2 1), (2 0 3 0) and (1 1 1 2)

Step 2: Transform F into X= ($x_1, x_2, \ldots, x_{10}$) according to eq. 2.

$x_1 = f_1 + f_2$, $x_2 = f_1 + f_4$, $x_3 = f_2$, $x_4 = f_4$, $x_5 = f_3 + f_4$, $x_6 = f_2 + f_3$,
$x_7 = f_1 + f_2 + f_3 + f_4$, $x_8 = f_1 + f_2 + f_4$, $x_9 = f_2 + f_3 + f_4$, $x_{10} = f_1 + f_2 + f_3 + f_4$.

The set of X that can be obtained from F is:

X^1 = (2 3 0 1 3 2 5 3 3 5)
X^2 = (2 2 1 1 3 3 5 3 4 5)
X^3 = (2 2 0 0 3 3 5 2 3 5)
X^4 = (2 3 1 2 3 2 5 4 4 5)

Step 3: Obtain all lower boundary points for **d = 5** by removing non-minimal ones in { X^1, X^2, X^3, X^4}, using the same algorithm in [5]. The only two vectors are lower boundary points for **d = 5:**

X^1 = (2 3 0 1 3 2 5 3 3 5) and X^3 = (2 2 0 0 3 3 5 2 3 5)

Step 4: Calculate the system reliability for the given demend **d = 5** according to the algorithm given in **Section 5**:

Set $B_1 = \{X | X \ge X^1\}$ and $B_2 = \{X | X \ge X^3\}$.

Apply inclusion-execlusion rule to calculate **R_5** by using the relation**:**

$$R_5 = \Pr\{B_1\} + \Pr\{B_2\} - \Pr\{B_1 \cap B_2\}$$
$$= \Pr\{X | X \ge (2,3,0,1,3,2,5,3,3,5)\} + \Pr\{X | X \ge (2,2,0,0,3,3,5,2,3,5)\}$$
$$- \Pr\{X | X \ge (2,3,0,1,3,3,5,3,3,5)\}$$

= 0.824242

Simailarly, The system reliability for the demend d = 4 is: R_4 = 0.889351. And, the results of F, X, and the set of lower boundary points for that demand is given as follows:
The set of all feasible solutions of F is:

(2, 0, 2, 0), (1, 1, 2, 0), (1, 0, 3, 0), (2, 0, 1, 1), (1, 1, 1, 1),
(0, 2, 1, 1), (1, 0, 2, 1), (0, 1, 2, 1), (1, 1, 0, 2), (0, 2, 0, 2),
(1, 0, 1, 2), (0, 1, 1, 2), (0, 1, 0, 3)

The set of X that can be obtained from F is:

X^1 = (2, 2, 0, 0, 2, 2, 4, 2, 2, 4)
X^2 = (2, 1, 1, 0, 2, 3, 4, 2, 3, 4)
X^3 = (1, 1, 0, 0, 3, 3, 4, 1, 3, 4)
X^4 = (2, 3, 0, 1, 2, 1, 4, 3, 2, 4)
X^5 = (2, 2, 1, 1, 2, 2, 4, 3, 3, 4)
X^6 = (2, 1, 2, 1, 2, 3, 4, 3, 4, 4)
X^7 = (1, 2, 0, 1, 3, 2, 4, 2, 3, 4)
X^8 = (1, 1, 1, 1, 3, 3, 4, 2, 4, 4)
X^9 = (2, 3, 1, 2, 2, 1, 4, 4, 3, 4)
X^{10} = (2, 2, 2, 2, 2, 2, 4, 4, 4, 4)
X^{11} = (1, 3, 0, 2, 3, 1, 4, 3, 3, 4)
X^{12} = (1, 2, 1, 2, 3, 2, 4, 3, 4, 4)
X^{13} = (1, 3, 1, 3, 3, 1, 4, 4, 4, 4)

The set of lower boundary points is:

X^1 = (2, 2, 0, 0, 2, 2, 4, 2, 2, 4),
X^2 = (2, 1, 1, 0, 2, 3, 4, 2, 3, 4)
X^3 = (1, 1, 0, 0, 3, 3, 4, 1, 3, 4),
X^4 = (2, 3, 0, 1, 2, 1, 4, 3, 2, 4)
X^7 = (1, 2, 0, 1, 3, 2, 4, 2, 3, 4),
X^{11} = (1, 3, 0, 2, 3, 1, 4, 3, 3, 4),

Note: For the studied network example, the parameters setting in the proposed algorithm are:
The population size (pop_size) = 50
The GA crossover rate (p_c) = 0.95
The GA mutation rate (p_m) = 0.05
The maximum number of generations (max_gen) =1000.

The following table summerizes the results of applying the proposed algorithm on another network example taken from [18]:

The demand (d)	The set of lower boundary points	The reliability
2	(0, 1, 0, 1, 2, 1, 2, 1, 2, 2) (1, 1, 0, 0, 1, 1, 2, 1, 1, 2) (2, 1, 1, 0, 0, 1, 2, 2, 1, 2) (1, 2, 0, 1, 1, 0, 2, 2, 1, 2) (1, 0, 1, 0, 1, 2, 2, 1, 2, 2,)	0.958636
3	(1, 1, 0, 0, 2, 2, 3, 1, 2, 3) (2, 3, 0, 1, 1, 0, 3, 3, 1, 3) (2, 0, 2, 0, 1, 3, 3, 2, 3, 3) (1, 0, 1, 0, 2, 3, 3, 1, 3, 3) (0, 2, 0, 2, 3, 1, 3, 2, 3, 3) (1, 3, 0, 2, 2, 0, 3, 3, 2, 3) (0, 1, 0, 1, 3, 2, 3, 1, 3, 3) (3, 2, 1, 0, 0, 1, 3, 3, 1, 3) (1, 2, 0, 1, 2, 1, 3, 2, 2, 3) (2, 1, 1, 0, 1, 2, 3, 2, 2, 3) (2, 2, 0, 0, 1, 1, 3, 2, 1, 3)	0.946564

TABLE 1: The results of network example taken from [18]

2. DISCUSSION

This section investigates the problem of the obtained solution to the above examples given in Lin[5].

For the given demand **d = 5** and according to [5], the set of all feasible solutions of F is:
(2 0 2 1), (1 1 2 1), (2 0 3 0), (1 1 1 2), (0 2 1 2) and (0 2 0 3)

But, the last two solutions (0 2 1 2) and (0 2 0 3) don't satisfy the costraint:

$$f_2 + f_3 + f_4 \leq 4 \, .$$

So, the two solutions (0 2 1 2) and (0 2 0 3) must be eliminated and the set of solutions become:

(2 0 2 1), (1 1 2 1), (2 0 3 0) and (1 1 1 2)

Which satisfy the constraints 3 and 4 and compatable with the sloution obtained by the proposed GA when comparing the set of lower boundary points and the reliability value.

8. CONCLUSION & FUTURE WORK

This paper presented a genetic algorithm to calculate the system reliability of a stochastic-flow network to given demand d. The algorithm is based on determining the set of all feasible solutions of the flow vector and generate the set of all lower boundary points for the given demand d and then calculate the reliability. Finally we illustrate the using of the proposed algorithm by calculating the reliability of a flow network to given sample network taken from literature. Also, The algorithm is efficeint and may be extended to compute the reliability of a flow network in two or multicommodity cases.

9. REFERENCES

1. Ahuja Sanjay P. and Kumar Anup “Reliability and Performance based measure for computer networks and distributed systems“, Proceedings of the IEEE SoutheastCon Conference, Charlotte, NC, 1993.

2. Rai Suresh and Aggarwal K. K., “An efficient method for reliability evaluation of a general network”, IEEE Transactions on Reliability, 27(3): 1978.

3. Aggarwal K. K. and Rai Suresh, “Reliability evaluation in computer communication networks”, IEEE Transactions on Reliability, 30(1): 1981.

4. Younes A., “Spanning trees and reliability of computer networks”, Egyptian Informatics Journal, 6(1): 2005.

5. Yi-Kuei Lin, “A simple algorithm for reliability evaluation of a stochastic-flow network with node failure”, Computers and Operations research, 28: 1277-1285, 2001.

6. Yi-Kuei Lin, “Reliability evaluation for an information network with node failure under cost constraint”, IEEE Transactions on Systems, Man. And Cybernetics, Part A: System and Humans, 37(2): 180-188, 2007.

7. Yi-Kuei Lin, “System reliability of a limited-flow network in multicommodity case”, IEEE Transactions on Reliability, 56(1): 17-25, 2007.

8. Satitsatian Sarintip and Kapur Kailash C., “An algorithm for lower reliability bounds of multistate two-terminal networks”, IEEE Transactions on Reliability, 55(2): 199-206, 2006.

9. Yi-Kuei Lin and Cheng-Ta Yeh, “Evaluation of Optimal Network Reliability Under Components-Assignments Subject to a Transmission Budget”, IEEE Transactions on Reliability, 59(3), 2010.

10. Altiparmark Fulya, Dengiz Berna and Smith Alice E.,“Reliability optimization of computer communication networks using genetic algorithms”, Proceedings of the 1998 IEEE International Conference on Systems, Man, and Cybernetics-Intelligent Systems For Humans In A Cyberworld, SMC’98, Hyatt Regency La Jolla, San Diego, California, USA, October 11-14, pp. 4676-4681, 1998.

11. Coit David W. and Smith Alice E., “Use of a genetic algorithm to optimize a combinatorial reliability design problem”, Proceeding of the Third IIE Research Conference, 467-472, 1994.

12. Coit David W. and Smith Alice E., “Penalty guided genetic search for reliability design optimization”, Accepted to Computers and Industrial Engineering, Special Issue on Genetic Algorithms Vol. 30(4): 1996.

13. Dengiz Berna, Altiparmark Fulya and Smith Alice E., “A genetic algorithm approach to optimal topological design of all terminal networks”, Intelligent Engineering Systems Through Artificial Neural Network, 5: 405-410, 1995.

14. Dengiz Berna, Altiparmak Fulya, Smith Alice E., “Local search genetic algorithm for optimization of highly reliable communications networks”, IEEE Transactions on Evolutionary Computation, 1: 179-188, 1997.

15. Dengiz Berna, Altiparmark Fulya and Smith Alice E., “Genetic algorithms design of networks considering all-terminal reliability”, The Sixth Industerial Engineering Research Conference Proceedings IERC’97, Miami Beach, Florida, USA, May 17-18, pp. 30-35, 1997.

16. Dengiz Berna and Aabbas C., “A simulated annealing algorithm for design of computer communication networks”, In Proceedings of world Multiconference on Cybernetics and Informatics-SCI 2001, Volume 5, 2001.

17. Lo Chi-Chun and Chang Wei-Hsin, “A multiobjective hybrid genetic algorithm for the capacitated multipoint network design problem”, IEEE Transactions on Systems, Man. And Cybernetics, Part B: Cybernetics, 30(3): 461-469, 2000.

18. Yi-Kuei Lin, “Two-commodity reliability evaluation for a stochastic-flow network with node failure”, Computers & Operations Research 29: 1927–1939, 2002.