On the State Observer Based Stabilization of T-S Systems with Maximum Convergence Rate

Abdallah SALEM abdallah.salem@isegf.rnu.tn
Laboratory Process Study and Control
Tunisia Polytechnic School,
BP. 743, 2078, La Marsa, Tunisia

Zohra KARDOUS zohra.kardous@enit.rnu.tn
Laboratory Process Study and Control
Tunisia Polytechnic School,
BP. 743, 2078, La Marsa, Tunisia

Naceur BENHADJ BRAIEK naceur.benhadj@ept.rnu.tn
Laboratory Process Study and Control
Tunisia Polytechnic School,
BP. 743, 2078, La Marsa, Tunisia

Abstract

This paper presents improved relaxed stabilization conditions and design procedures of state observer based controllers for continuous nonlinear systems in T-S model representation. First, the T-S model approach for nonlinear systems and some stabilization results are recalled. New stabilization conditions are obtained by relaxing those derived in previous works in this field. The asymptotic and exponential stabilization are considered with the maximization of the convergence rate. Design procedures for stabilizing T-S observer based controller using the concept of PDC (Parallel Distributed Compensation) and improved relaxed stabilization conditions are proposed.

Keywords: Continuous T-S systems, Observer based controller, PDC.

1. INTRODUCTION

The design of state feedback control, as well as the design of state observer, for nonlinear systems, has been actively considered during the last decades in many works using the Takagi-Sugeno (T-S) models [1], [2], [3], [4], [5].

The T-S model approach consists to construct nonlinear or complex dynamic systems that cannot be exactly modelled, by means of interpolating the behaviour of several LTI (Linear Time Invariant) submodels. Each submodel contributes to the global model in a particular subset of the operating space [2], [6], [7], [8].

Note that this modelling approach can be applied for a large class of physical and industrial processes as electrical machines and robot manipulators [9], [10], [11].

Recently, T-S observer based controller has attracted increasing attention, because it can provide a suitable solution to the control of plants that are complex and ill-defined and have immeasurable state variables [12], [13], [14], [15],

The T-S observer based controller has been considered to develop some systematic design algorithms to guarantee the stability and specific performances for the T-S model based systems [16], [17], [18].

The synthesis of the observer based controller can be considered as a convex problem and solved by Linear Matrix Inequalities (LMI) optimization techniques [19]. In spite of the advantages of LMI, the existence of a solution that satisfies the sufficient conditions is not guaranteed, especially, when the number of submodels increases or if many constraints are added such as control performance, the problem may become infeasible [20].

In attempt to avoid this situation, in some works relaxed stabilization conditions are derived to minimize the conservatism on LMIs [2], [3], [21]. However, the maximization of the convergence rate hasn't been considered.

This paper extends these works by proposing new relaxed conditions stabilization and design procedures for the observer based controller, using the concept of Parallel Distributed Compensation (PDC), with maximization of the convergence rate for the T-S model systems. An optimization tool is then used instead of LMIs.

This paper is organized as follows. Section 3 presents the structure of T-S models and recalls previous stability results. In Section 4, the observer design for T-S model is presented. In Section 5, we derive improved stabilization conditions and new design procedures of T-S observer based controller. To illustrate the proposed approaches a numerical example is considered in Section 6.

2. NOTATIONS

In this paper, we denote the minimum and maximum eigenvalues of a matrix X respectively by $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$, the symmetric positive definite matrix X by $X > 0$ (the symmetric positive semidefinite matrix X by $X \geq 0$) and the transpose of X by X^T.
The following notations are also considered:

$$\sum_{i,j}^{n} x_i x_j = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \ , \ \sum_{1\leq i<j}^{n} x_i x_j = \sum_{i=1}^{n}\sum_{i<j}^{n} x_i x_j \ \text{ and } \ \sum_{i,j,k,l}^{n} x_i x_j x_k x_l = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} x_i x_j x_k x_l \ .$$

3. T-S MODEL AND STABILITY RESULTS

3.1 T-S model representation

A T-S model is based on the interpolation of several LTI local models as follows [2], [22]:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{n} \mu_i\left(z(t)\right)\left(A_i x(t) + B_i u(t)\right) \\ y(t) = \sum_{i=1}^{n} \mu_i\left(z(t)\right) C_i x(t) \end{cases} \tag{1}$$

where n is the number of submodels, $x(t) \in \mathbf{R}^{p}$ is the state vector, $y(t) \in \mathrm{R}^{l}$ is the output vector, $u(t) \in \mathrm{R}^{m}$ is the input vector, $z(t) \in \mathbf{R}^{q}$ is the decision variables vector and $\mu_i\left(z(t)\right)$ is the activation function.
$A_i \in \mathbf{R}^{p\times p}$, $B_i \in \mathbf{R}^{p\times m}$ and $C_i \in \mathbf{R}^{l\times p}$ are respectively the state matrix, the input matrix and the output matrix.

Different classes of models can be considered with respect to the choice of the decision variables and the type of the activation function.
In this paper, all the decision variables of the T-S model (1) are assumed measurable.
Each linear consequent equation represented by $\left(A_i x(t)+B_i u(t)\right)$ is called "subsystem" or "submodel".
The normalized activation function $\mu_i\left(z(t)\right)$ corresponding to the i^th submodel is such that [6], [23], [24]:

$$\begin{cases}\sum_{i=1}^{n}\mu_i\left(z(t)\right)=1\\ \mu_i\left(z(t)\right)\geq 0 \quad \forall\ i\in\{1,...,n\}\end{cases} \tag{2}$$

3.2 Basic stabilization conditions

Let us consider the system (1) in its autonomous form, then we have:

$$\dot{x}(t)=\sum_{i=1}^{n}\mu_i\left(z(t)\right)A_i x(t) \tag{3}$$

Stabilization conditions of system (3) are derived using Lyapunov approach. So, the equilibrium of the T-S control system described by (3) is globally asymptotically stable if there exist a common positive definite matrix P such that [25]:

$$A_i^T P+PA_i<0 \quad \text{for } 1\leq i\leq n \tag{4}$$

4. OBSERVER DESIGN FOR T-S MODEL

In order to estimate the non measurable state variables of the T-S model (1), a T-S observer can be designed using PDC technique [7]. In this case, the global observer is obtained by interpolation of the local linear observers, associated to the different submodels.
For the T-S observer design, it is supposed that the decision variables are measurable and the T-S model of system (1) is locally detectable *i.e.* all the pairs $\left(A_i, C_i\right);\ i=1,...,n$ are detectable.
The T-S observer is written as follows [2], [3]:

$$\begin{cases}\dot{\hat{x}}(t)=\sum_{i=1}^{n}\mu_i\left(z(t)\right)\left[A_i\hat{x}(t)+B_i u(t)+L_i\left(y(t)-\hat{y}(t)\right)\right]\\ \hat{y}(t)=\sum_{i=1}^{n}\mu_i\left(z(t)\right)C_i\hat{x}(t)\end{cases} \tag{5}$$

where $\hat{x}(t)$ is the estimated state vector and the activation function is the same than that used in the T-S model verifying (2).
One considers the following state estimation error defined as :

$$\varepsilon(t)=x(t)-\hat{x}(t) \tag{6}$$

From (1) and (6), the state estimation error dynamic is described by the following equation:

$$\dot{\varepsilon}(t)=\sum_{i,j}^{n}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\left(A_i-L_iC_j\right)\varepsilon(t)=\sum_{i,j}^{n}\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)R_{ij}\varepsilon(t) \tag{7}$$

where :

$$R_{ij}=A_i-L_iC_j \tag{8}$$

The T-S observer is required to satisfy $\hat{x}(t)\to x(t)$ when $t\to\infty$, this condition is guaranteed when the error $\varepsilon(t)$ converges to zero.

5. OBSERVER BASED CONTROLLER DESIGN FOR T-S MODEL

When the estimated state $\hat{x}(t)$ is available, we can consider the global control law with PDC technique as follows:

$$u(t) = -\sum_{i=1}^{n} \mu_i\left(z(t)\right) K_i \hat{x}(t) \tag{9}$$

From (5), (6) and (9), one obtains:

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i,j}^{n} \mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\left(A_i - B_i K_j\right)\hat{x}(t) + \sum_{i,j}^{n} \mu_i\left(z(t)\right)\mu_j\left(z(t)\right) L_i C_j \varepsilon(t) \\ \hat{y}(t) = \sum_{i=1}^{n} \mu_i\left(z(t)\right) C_i \hat{x}(t) \end{cases} \tag{10}$$

The augmented system is given by:

$$\dot{X}(t) = \sum_{i,j}^{n} \mu_i\left(z(t)\right)\mu_j z(t) H_{ij} X(t) = \sum_{i=1}^{n} \mu_i^2\left(z(t)\right) H_{ii} X(t) + 2\sum_{i<j}^{n} \mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\left(\frac{H_{ij}+H_{ji}}{2}\right) X(t) \tag{11}$$

where:

$$X(t) = \begin{bmatrix} \hat{x}(t) \\ \varepsilon(t) \end{bmatrix},\ \mathrm{H}_{ij} = \begin{bmatrix} A_i - B_i K_j & L_i C_j \\ 0 & A_i - L_i C_j \end{bmatrix} = \begin{bmatrix} G_{ij} & L_i C_j \\ 0 & R_{ij} \end{bmatrix} \tag{12}$$

whith:

$$G_{ij} = A_i - B_i K_j \tag{13}$$

The equation (11) makes appear the dominant submodels characterized by the matrices H_{ii} and the coupled submodels characterized by the matrices $\left(\frac{H_{ij}+H_{ji}}{2}\right)$.

5.1 Asymptotic Stability

The T-S system described by (11) is globally asymptotically stable if there exist a common positive definite matrix P such that [2]:

$$\begin{cases} H_{ii}^T P + P H_{ii} < 0, & 1 \le i \le n \\ \left(\frac{H_{ij}+H_{ji}}{2}\right)^T P + P\left(\frac{H_{ij}+H_{ji}}{2}\right) \le 0, & 1 \le i < j \le n \\ \forall (i,j) / \mu_i\left(z(t)\right)\mu_j\left(z(t)\right) \ne 0, \forall t \end{cases} \tag{14}$$

One notes that the conditions (14) are conservative, because they require the stability of all the submodels (dominants and coupled). This result shows that the stabilization analysis of the T-S observer based controller system is reduced to a problem of finding a common matrix P. If *n* is large, it might be difficult to find a common P satisfying the conditions (14).
To reduce the conservatism, in the reference [2] relaxed conditions which require only the stability of the dominant submodels have been proposed. These conditions are recalled in the following theorem:

Theorem 1 [2]: Assume that the number of rules that fire for all *t* is less than or equal to s where $2 \le s \le n$. The equilibrium of the T-S system described by (11) is asymptotically stable in the large if there exist a common positive definite matrix P and a common positive semi definite matrix Q such that:

$$\begin{cases} H_{ii}^T P + P H_{ii} + (s-1) Q < 0, & 1 \le i \le n \\ \left(\frac{H_{ij}+H_{ji}}{2}\right)^T P + P\left(\frac{H_{ij}+H_{ji}}{2}\right) - Q \le 0, 1 \le i < j \le n \\ \forall (i,j) / \mu_i\left(z(t)\right)\mu_j\left(z(t)\right) \ne 0, \forall t \text{ and } s > 1 \end{cases} \tag{15}$$

5.2 Exponential stability

It is important to consider not only stabilization, but also other control performances such as speed of response, which is related to the decay rate, also called degree of stabilization and defined to be the largest $a > 0$ such that:

$$\lim_{t\to\infty} e^{at}\left\| X(t)\right\| = 0 \tag{16}$$

holds for all nonzero trajectories $X(t)$ of the system (11).
The condition (16) is equivalent to have:

$$\dot{V}\left(X(t)\right) \le -2aV\left(X(t)\right) \tag{17}$$

where :

$$V\left(X(t)\right) = X^T(t)PX(t) \tag{18}$$

is a quadratic Lyapunov function with $P > 0$.
The condition (17) has to be verified for all trajectories and leads to the inequality:

$$\left\|X(t)\right\| \le e^{-at}K(P)\left\|X(0)\right\| \tag{19}$$

where:

$$K(P) = \left(\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}\right)^{1/2} \tag{20}$$

and $a > 0$ is the minimum decay rate.

The inequality (19) guarantees the global exponential stability of (11).
In [3], conditions of global exponential stability of system (11) have been derived and the minimum decay rate of the system has been characterized. These results are recalled in the following theorem:

Theorem 2 [3]: Suppose that there exist a common positive definite matrix P and a common positive semi definite matrix Q such that:

$$\begin{cases} H_{ii}^T P + PH_{ii} + \left(s - \frac{1}{2}\right)Q < 0, \quad 1 \le i \le n \\ \left(\frac{H_{ij}+H_{ji}}{2}\right)^T P + P\left(\frac{H_{ij}+H_{ji}}{2}\right) - \frac{Q}{2} \le 0, \ 1 \le i < j \le n \\ \forall (i,j) / \mu_i\left(z(t)\right)\mu_j\left(z(t)\right) \ne 0, \forall t \text{ and } s > 1 \end{cases} \tag{21}$$

Then the closed loop T-S model described by (11) is globally exponentially stable.
The minimum decay rate in this case is:

$$a_c = \frac{\lambda_{\min}(Q)}{4\lambda_{\max}(P)} \tag{22}$$

Note that the conditions (15) of theorem 1 and those (21) of theorem (2) can be unified in the following form:

$$\begin{cases} H_{ii}^T P + PH_{ii} + (s-\beta)Q < 0, \qquad 1 \le i \le n \\ \left(\frac{H_{ij}+H_{ji}}{2}\right)^T P + P\left(\frac{H_{ij}+H_{ji}}{2}\right) - \beta Q \le 0, 1 \le i < j \le n \\ \forall (i,j) / \mu_i\left(z(t)\right)\mu_j\left(z(t)\right) \ne 0, \forall t \text{ and } s > 1 \end{cases} \tag{23}$$

with $\beta = 1 \text{ or } 0.5$.

Remark 1: When β takes 1, one obtains the asymptotic stability conditions (15) and when β is replaced by 0.5, one obtains the conditions (21).

5.3 Main results: generalized and improved relaxed stability conditions

Let us note that two questions arise about the unified conditions (23):

- When no solution exists for $\beta = 0.5$, could the conditions (23) be relaxed to obtain an exponential stability solution?

- How to maximize the minimum decay rate when the exponential stability is guaranteed?

In this work, we have been interested by these two points and we have proved that the conditions (23) can be extended for any β such that $0<\beta<1$. Then the following theorem can be stated:

Theorem 3: Assume that the number of submodels simultaneously activated is s such that $2\leq s\leq n$. The system described by (11) is globally exponentially stable, if there exist a common positive definite matrix P, a common positive semi definite matrix Q and a scalar $0<\beta<1$ such that:

$$\begin{cases} H_{ii}^{T}P+PH_{ii}+(s-\beta)Q<0, \qquad 1\leq i\leq n \\ \left(\dfrac{H_{ij}+H_{ji}}{2}\right)^{T}P+P\left(\dfrac{H_{ij}+H_{ji}}{2}\right)-\beta Q\leq 0,\ 1\leq i<j\leq n \\ \forall (i,j)/\,\mu_i\left(z(t)\right)\mu_j\left(z(t)\right)\neq 0, \forall t \ \text{ and } \ s>1 \end{cases} \tag{24}$$

Then, the minimum decay rate is:

$$v_c=(1-\beta)\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} \tag{25}$$

Proof. To prove the theorem 3, we use the following lemma 1:

Lemma 1 [2], [22]: Assume that the number of submodels simultaneously activated is s such that $2\leq s\leq n$, then:

$$\begin{aligned} &(s-1)\sum_{i=1}^{n}\mu_i^2-2\sum_{1\leq i<j}^{n}\mu_i\mu_j\geq 0 \\ &\text{and } \ s\sum_{i=1}^{n}\mu_i^2\geq 1 \ \text{ where } \sum_{i=1}^{n}\mu_i=1,\ \ \mu_i\geq 0 \end{aligned} \tag{26}$$

□

Multiplying the first term of (24) by μ_i^2, the second by $2\mu_i\mu_j$ and adding up all terms for $i=1$ to n, we get:

$$\sum_{i=1}^{n}\mu_i^2\left[H_{ii}^{T}P+PH_{ii}+(s-\beta)Q\right]+\sum_{1\leq i<j}^{n}\mu_i\mu_j\left[\left(H_{ij}+H_{ji}\right)^{T}P+P\left(H_{ij}+H_{ji}\right)-2\beta Q\right]<0$$

Since $\sum_{i,j}^{n}\mu_i\mu_jH_{ij}=\sum_{i=1}^{n}\mu_i^2H_{ii}+\sum_{1\leq i<j}^{n}\mu_i\mu_j\left(H_{ij}+H_{ji}\right)$, then the previous inequality is equivalent to :

$$\sum_{i,j}^{n}\mu_i\mu_j\left(H_{ij}^{\ T}P+PH_{ij}\right)+\left[(s-\beta)\sum_{i=1}^{n}\mu_i^2-2\beta\sum_{1\leq i<j}^{n}\mu_i\mu_j\right]Q<0$$

Assume that $H=\sum_{i,j}^{n}\mu_i\mu_jH_{ij}$, one obtains:

$$H^{T}P+PH+\left[s\sum_{i=1}^{n}\mu_i^2-\beta\left(\sum_{i=1}^{n}\mu_i^2+2\sum_{1\leq i<j}^{n}\mu_i\mu_j\right)\right]Q<0$$

or

$$H^{T}P+PH+\left(s\sum_{i=1}^{n}\mu_i^2-\beta\right)Q<0$$

From (26), we have $s\sum_{i=1}^{n}\mu_i^2\geq 1$, and then it comes out:

$$H^{T}P+PH+(1-\beta)Q\leq H^{T}P+PH+\left(s\sum_{i=1}^{n}\mu_i^2-\beta\right)Q<0$$

Consider the quadratic Lyapunov function $V(X) = X^T PX$ which is positive since $P > 0$. Then its derivative is given by:

$$\dot{V}(X) = \dot{X}^T PX + X^T P\dot{X} = X^T (H^T P + PH) X$$

Thus, it comes out : $\dot{V}(X) + (1-\beta) X^T QX < 0$.

In the other hand, we have:

$0 < \lambda_{\min}(P)\|X\|^2 \le X^T PX = V(X) \le \lambda_{\max}(P)\|X\|^2$ and $0 \le \lambda_{\min}(Q)\|X\|^2 \le X^T QX \le \lambda_{\max}(Q)\|X\|^2$

These two double inequalities yield the following one:

$$\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} V(X) \le X^T QX \le \frac{\lambda_{\max}(Q)}{\lambda_{\min}(P)} V(X)$$

Since $0 < \beta < 1$, we get:

$$\dot{V}(X) + (1-\beta)\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} V(X) \le \dot{V}(X) + (1-\beta) X^T QX < 0$$

Then, if there exist P and Q and a scalar β such that $0 < \beta < 1$ and the conditions (24) of theorem 3 are verified, the T-S system described by (11) is globally exponentially stable with minimum decay rate given by:

$v_c = \left[(1-\beta)\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}\right]$ where $\left[(1-\beta)\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}\right] > 0$ ❑

Remark 2: The generalized conditions (24) of theorem 3 are:

- less conservative than those of theorem 2 (corresponding to $\beta = 0.5$) if $0.5 < \beta < 1$;
- more conservative than those of theorem 2 if $\beta < 0.5$.

and the minimum decay rate can reach important values greater than that obtained for $\beta = 0.5$.

5.4 Proposed procedures for observer based exponential stabilization of T-S system

The observer based stabilization of T-S systems can be leaded using two procedures : the first one is based on the separation principle to synthesise the observer and the controller gains and the second one aims to the maximisation of the decay rate.

5.4.1 Separation principle based procedure

Using the separation principle [3], [18], the conditions of theorem 3 (or those of theorem 2 when $\beta = 0.5$) are developed to determine the state feedback and the observer gains. The resulted procedure is summarized as the separated inequalities presented in the following theorem 4:

Theorem 4: If there exist positive symmetric definite matrices P_1, P_2, Q_1 and Q_2 such that:

$$\begin{cases} P_1 > 0, P_2 > 0, Q_1 > 0, Q_2 > 0 \\ G_{ii}^T P_1 + P_1 G_{ii} + (s-\beta) Q_1 < 0,\ 1 \le i \le n \\ \left(\dfrac{G_{ij}+G_{ji}}{2}\right)^T P_1 + P_1\left(\dfrac{G_{ij}+G_{ji}}{2}\right) - \beta Q_1 \le 0, 1 \le i < j \le n \\ R_{ii}^T P_2 + P_2 R_{ii} + (s-\beta) Q_2 < 0,\ 1 \le i \le n \\ \left(\dfrac{R_{ij}+R_{ji}}{2}\right)^T P_2 + P_2\left(\dfrac{R_{ij}+R_{ji}}{2}\right) - \beta Q_2 \le 0, 1 \le i < j \le n \\ \forall (i,j) / \mu_i(z(t))\mu_j(z(t)) \ne 0, \forall t \text{ and } s > 1 \end{cases} \tag{27}$$

then one can always find a quadratic Lyapunov function which prove the global exponential stability of the augmented system (11).

Proof. Consider the Lyapunov function $V(X) = X^T PX$ with the following structure of P and Q :

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & \sigma P_2 \end{bmatrix},\ Q = \begin{bmatrix} Q_1 & 0 \\ 0 & \sigma Q_2 \end{bmatrix} \tag{28}$$

with $P_1 > 0,\ Q_1 > 0,\ P_2 > 0,\ Q_2 > 0$ and $\sigma \in \mathbb{R}^{+*}$.

To prove (27), one can proceed with the same proof in [3] and obtains that $\sigma \geq \max(\sigma_1, \sigma_2)$ where:

$$\sigma_1 = \frac{\lambda_{\min}\left[P_1 L_i C_i \left[R_{ii}^T P_2 + P_2 R_{ii} + (s-\beta)Q_2\right]^{-1} (L_i C_i)^T P_1\right]}{\lambda_{\max}\left(G_{ii}^T P_1 + P_1 G_{ii} + (s-\beta)Q_1\right)}$$

$$\sigma_2 = \frac{\lambda_{\min}\begin{bmatrix} P_1\left(L_i C_j + L_j C_i\right)\begin{bmatrix}\left(R_{ij} + R_{ji}\right)^T P_2 + \\ P_2\left(R_{ij} + R_{ji}\right) - 2\beta Q_2\end{bmatrix}^{-1} \\ \left(L_i C_j + L_j C_i\right)^T P_1 \end{bmatrix}}{\lambda_{\max}\left[\left(G_{ij} + G_{ji}\right)^T P_1 + P_1\left(G_{ij} + G_{ji}\right) - 2\beta Q_1\right]} \tag{29}$$

In order to simplify the resolution of the bilinear inequalities (27) we consider the following variables change:

$$X_1 = P_1^{-1},\ M_i = K_i X_1,\ Y_1 = P_1^{-1} Q_1 P_1^{-1} \text{ and } N_i = P_2 L_i \tag{30}$$

Then, one obtains the following Generalized Eigenvalues Problem (GEVP) in X_1, Y_1, Q_2, P_2 , M_i , N_i and β :

$$\begin{cases} X_1 > 0,\ P_2 > 0,\ Y_1 > 0,\ Q_2 > 0 \\ X_1 A_i^T - M_i^T B_i^T + A_i X_1 - B_i M_i + (s-\beta)Y_1 < 0,\ \ 1 \leq i \leq n \\ X_1\left(A_i + A_j\right)^T + \left(A_i + A_j\right)X_1 - M_j^T B_i^T - M_i^T B_j^T - B_i M_j - B_j M_i - 2\beta Y_1 \leq 0, 1 \leq i < j \leq n \\ A_i^T P_2 + P_2 A_i - C_i^T N_i^T - N_i C_i + (s-\beta)Q_2 < 0,\quad 1 \leq i \leq n \\ \left(A_i + A_j\right)^T P_2 + P_2\left(A_i + A_j\right) - C_j^T N_i^T - C_i^T N_j^T - N_i C_j - N_j C_i - 2\beta Q_2 \leq 0,\ 1 \leq i < j \leq n \end{cases} \tag{31}$$

Note that for a given scalar β , the constraints (31) are Linear Matrix Inequalities (LMI) in X_1, Y_1, Q_2 , P_2, M_i and N_i .

5.4.2 Maximization of the decay rate

From the generalized exponential stability conditions of theorem 3, one can look for a control law (9) maximizing the minimum decay rate v_c . This problem can be solved with respect to the scalar β such that $0.5 < \beta < 1$, the common positive definite matrix P , the common positive semi definite matrix Q , the state feedback gains $K_i, i = 1,...,n$ and the observer gains $L_i, i = 1,...,n$ as follows:

$$\begin{cases} \text{Maximize } v_c = \left[(1-\beta)\dfrac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} \right] \text{ subject to:} \\ 0.5 < \beta < 1,\ P > 0,\ Q \geq 0 \\ \mathrm{H}_{ij} = \begin{bmatrix} A_i - B_i K_j & L_i C_j \\ 0 & A_i - L_i C_j \end{bmatrix},\ 1 \leq i \leq n \text{ and } 1 \leq j \leq n \\ H_{ii}^T P + P H_{ii} + (s-\beta) Q < 0, \qquad 1 \leq i \leq n \\ \left(\dfrac{H_{ij} + H_{ji}}{2} \right)^T P + P \left(\dfrac{H_{ij} + H_{ji}}{2} \right) - \beta Q \leq 0, 1 \leq i < j \leq n \\ \forall (i,j) / \mu_i(z(t)) \mu_j(z(t)) \neq 0, \forall t \text{ and } s > 1 \end{cases} \tag{32}$$

The maximization problem (32) can be solved using the optimization tools of MATLAB as the *fmincon.*

6. NUMERICAL EXAMPLE

We consider the T-S system composed by two subsystems studied in [21] and characterized by:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \mu_i(z(t)) (A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{2} \mu_i(z(t)) C_i x(t) \end{cases} \tag{33}$$

where:

$$x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix},\ A_1 = \begin{bmatrix} 2 & -10 \\ 1 & 0 \end{bmatrix},\ A_2 = \begin{bmatrix} 15 & -10 \\ a & 0 \end{bmatrix},$$

$B_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ B_2 = \begin{bmatrix} b \\ 0 \end{bmatrix}$ and $C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

$$\mu_1 = \begin{cases} 1 - \dfrac{|x_1(t)|}{3}, \forall x_1(t) \in [-3 \quad 3] \\ 0 \qquad \text{otherwise} \end{cases}, \qquad \mu_2 = 1 - \mu_1$$

a and b are the system parameters.

We assume that the number of the submodels simultaneously activated is $s = 2$.

With this example of multimodel system, we will show that the derived result in this paper can be used for two different goals:

- the enlargement of the system parameters variation area in which the stability of the multimodel system is guaranteed;
- the improvement of the decay rate of the system.

6.1 Enlargement of the system parameters variation area with the guaranteed stabilization

We study the multimodel stabilization of the system (33) with respect to the (a,b) parameters variation. The applied feedback control law is given by:

$$u(t) = -(\mu_1 K_1 + \mu_2 K_2) \hat{x}(t) \tag{34}$$

where K_1 and K_2 are the local feedback gains determined such that the poles of the local controlled subsystems are placed to the values -1 and -2.

The local observers gains L_1 and L_2 are determined such that the poles of the local estimation error dynamic are placed to the values -2 and -4.

Thus we have $K_1 = [5 \quad -8]$, $L_1 = \begin{bmatrix} 8 \\ 0.2 \end{bmatrix}$ and the gains K_2 and L_2 depend on the values of *a* and *b* parameters.

To determine the (a,b) area in which the stability of the controlled T-S system (33) is guaranteed, we vary the parameters *a* and *b* ($a > 0$ and $b > 0$) and then we verify the stability conditions of theorem 2 $(\text{for } \beta = 0.5)$ and those of theorem 3 $(\text{for } 0.5 < \beta < 1)$.

The figures 1 and 2 show the feasible (a,b)-area corresponding to the conditions given respectively by theorem 2 and theorem 3. The mark (*) indicates the stability conditions feasibility.

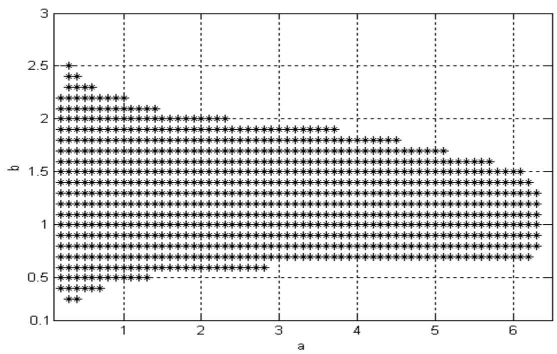

FIGURE 1: Feasibility (a,b)-area for the stability conditions of theorem 2 ($\beta = 0.5$).

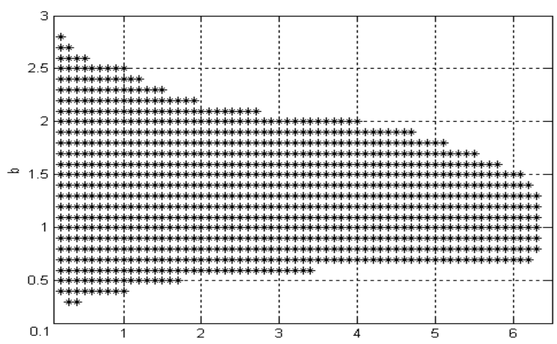

FIGURE 2: Feasibility (a,b)-area for the stability conditions of theorem 3 ($\beta = 0.9$).

From the figures 1 and 2, it can be noted that the theorem 3 $(0.5 < \beta < 1)$ leads to relaxed conditions compared to those of theorem 2. Indeed the feasibility (a,b)-area of the figure 2 corresponding to the application of theorem 3 is clearly larger than that of the figure 1 obtained by the application of theorem 2.

6.2 Maximization of the decay rate

We consider now the system (33) with $a = b = 2$, and we search to maximize the decay rate $v_c = \left[(1-\beta) \dfrac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)} \right]$ subject to the optimization problem (32).

The maximization is leaded with respect to P and Q matrices, the feedback gains K_1 and K_2 and the observers gains L_1 and L_2 for different values of β. The obtained values are the following :

- For $\beta = 0.5$

$$K_1 = \begin{bmatrix} 55.53 & 31.19 \end{bmatrix},\ K_2 = \begin{bmatrix} 46.27 & 18.492 \end{bmatrix},\ L_1 = \begin{bmatrix} 18.69 \\ -0.722 \end{bmatrix},\ L_1 = \begin{bmatrix} 45.088 \\ -3.542 \end{bmatrix}$$

$$v_{c\,\max} = 0.046$$

$$P = \begin{bmatrix} 50.621 & 39.337 & -19.409 & -0.338 \\ 39.337 & 482.69 & -19.278 & 70.198 \\ -19.409 & -19.278 & 36.821 & 20.259 \\ -0.338 & 70.198 & 20.259 & 125.31 \end{bmatrix},\ Q = \begin{bmatrix} 186.62 & -99.644 & -12.877 & 10.071 \\ -99.644 & 196.66 & 14.082 & -10.328 \\ -12.877 & 14.082 & 95.853 & -0.0137 \\ 10.071 & -10.328 & -0.0137 & 94.084 \end{bmatrix}$$

- For $\beta = 0.55$

$$K_1 = \begin{bmatrix} 38.732 & 27.381 \end{bmatrix},\ K_2 = \begin{bmatrix} 79.457 & 15.051 \end{bmatrix}\ ,\ L_1 = \begin{bmatrix} 13.062 \\ -0.463 \end{bmatrix},\ L_2 = \begin{bmatrix} 40.503 \\ -0.645 \end{bmatrix}$$

$$v_{c\,\max} = 0.122$$

$$P = \begin{bmatrix} 15.972 & 11.274 & -3.075 & 2.150 \\ 11.274 & 582.36 & -13.46 & 24.851 \\ -3.075 & -13.46 & 44.326 & 31.224 \\ 2.150 & 24.851 & 31.224 & 417.78 \end{bmatrix},\ Q = \begin{bmatrix} 387.58 & 1.366 & 4.069 & 54.824 \\ 1.366 & 318.68 & 2.304 & 1.968 \\ 4.0694 & 2.304 & 367.87 & 24.644 \\ 54.824 & 1.968 & 24.644 & 371.51 \end{bmatrix}$$

It comes out from these results that the decay rate has been improved $(v_{c\max} = 0.122)$ with the parameter $\beta = 0.55$ in comparison with that obtained for $\beta = 0.5$ $(v_{c\max} = 0.046)$. This conclusion shows the importance of the new proposed stability conditions corresponding to $0.5 < \beta < 1$ and given by theorem 3.

The figure 3 shows the simulation results (x_1 and x_2 state variables behaviour) of the system (33) controlled with the multimodel law (34) in both cases of local gains: the gains values corresponding to $\beta = 0.5$ and the gains values corresponding to $\beta = 0.55$.

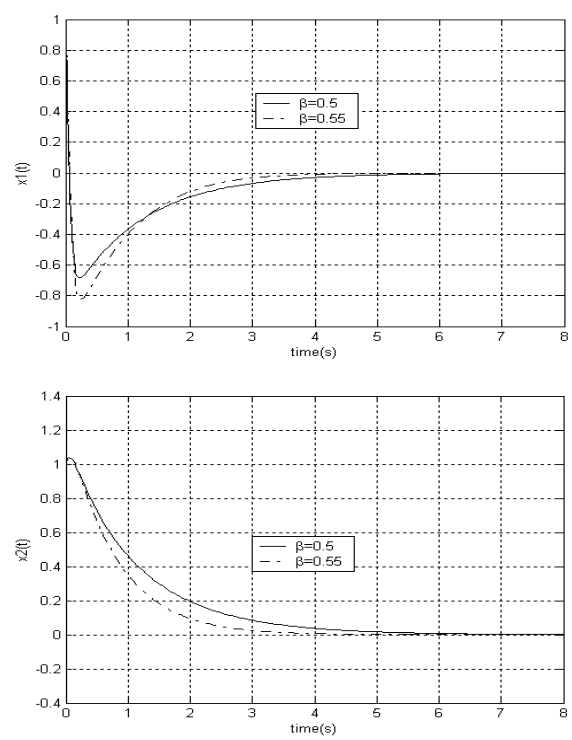

FIGURE 3: State variables behaviour for $\beta = 0.5$ and $\beta = 0.55$.

It appears on the simulation curves that the dynamic of the system is faster for the control law corresponding to $\beta = 0.55$, which confirm the conclusion obtained with the comparison study of the decay rates.

7. CONCLUSION

In this paper, improved approaches are suggested for the quadratic stabilization of observer based controlled T-S systems.

These approaches which aim to relax some results reached in previous works can be applied to the stabilization of nonlinear systems represented by T-S models, using the concept of parallel distributed compensation.

Two design procedures of the improved stabilization synthesis have been proposed. The first one is based on the separation principle between the controller and observer gains determination and it is formulated as a Generalized Eigenvalues Problem. The second one aims to the maximization of the decay rate of the exponential stability of the controlled system and it leads to an optimization problem.

A comparison study of the results derived in this work with previous ones has shown the importance of the proposed approaches for the enlargement of the availability domain of the stabilization conditions and the performance improvement of the stabilized nonlinear systems.

8. REFERENCES

1. N. Benhadj Braiek, F. Rotella. “State observer design for a class of nonlinear system”. Journal of Systems Analysis, Modelling and Simulation (SAMS), 17:265-277, 1995

2. K. Tanaka, T. Ikeda, H. O. Wang. “Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs”. IEEE Trans. on Fuzzy Systems, 6:250-265,1998

3. M. Chadli, D. Maquin and J. Ragot. “Observer-based controller for the Takagi-Sugeno models”. In Proceeding of the IEEE International Conference of SMC. Tunisia, 2002

4. J. Zhang and M. Fei. “Analysis and design of robust fuzzy controllers and robust fuzzy observers of nonlinear systems”. In Proceeding of the 6th World Compress on Intelligent Control and Automation. China, 2006

5. R. Murray-Smith and T. A. Johansen. “Multiple model approaches to modelling and control”, Edition Taylor and Francis, (1997)

6. T. Takagi, M. Sugeno. “Fuzzy identification systems and its applications to modeling and control”. IEEE Trans. Syst. Man Cybern., 15:116-132,1985

7. H.O. Wang, K. Tanaka, M. F. Griffin. “An approach to fuzzy control of nonlinear systems: Stability and design issues”. IEEE Trans. Fuzzy Systems, 4:14-23, 1996

8. C.-S. Ting. “Stability analysis and design of Takagi-Sugeno systems”. Information Sciences, 176:2817-2845, 2006

9. B.B. Kook, W. Chul Ham. “Adaptative Control of Robot Manipulator Using Fuzzy Compensator”', IEEE Transactions on Fuzzy Systems, 8:718-737, 2000

10. A. Salem, A. S. Tlili, N. Benhadj Braiek. “On the Polytopic and Multimodel State Observers of Induction Motors”', Journal of Automation and Systems Engineering (JASE), 2(4): 235-247, 2008

11. A. Salem, Z. Kardous, A.S. Tlili and N. Benhadj Braiek. “Observateur Multimodèle avec Stabilité Garantie de Systèmes à Paramètres Variants: Application à une Machine Asynchrone”. Cinquième Conférence Internationale d’Électrotechnique et d’Automatique JTEA 2008. Hammamet, Tunisie, 02-04 Mai , 2008

12. Ch-Sh Ting. “An adaptive fuzzy observer-based approach for chaotic synchronization”. International Journal of Approximate Reasoning, 39(1):97-114, 2005

13. A. Sala, C. Arino. “Asymptotically necessary and sufficient conditions for stability and performance in fuzzy control: Applications of Polya’s theorem”. Fuzzy Sets and Systems, 158:2671-2686, 2007

14. S.-W. Kau, C.-M. Yang, H.-W. Chen, W.- J. Xie, H.-J. Lee and C.-H. Fang. “Stabilization and Observer-Based H_∞ Control for T-S Fuzzy Systems: An Improved LMI Approach”. In Proceeding of the European Control Conference. Greece, 2007

15. J. Zhang, Y. Xia, P. Shi and J. Qiu. “H_∞ output feedback control design for discrete-time fuzzy systems”. Journal of Optimal Control Application and Methods, Published Online: 23 Oct 2008

16. X. Liu, Q. Zhang. “New approaches to H_∞ controller designs based on fuzzy observers for fuzzy systems via LMI”. Automatica, 39:1571-1582, 2003

17. C. Lin, Q.-G. Wang, T. H. Lee. “Improvement on observer-based H_∞ control for T-S fuzzy systems”. Automatica, 41:1651-1656, 2005

18. X. Ma, Z. Sun, Y. He. “Analysis and design of fuzzy controller and fuzzy observer”. IEEE Trans. Fuzzy Syst., 6:41-51, 1998

19. J. Kim, D. Park. “LMI-based design of stabilizing fuzzy controllers for nonlinear systems described by Takagi-Sugeno model”. Fuzzy Sets Syst., 122:73-82, 2003

20. L. Luoh. “New Stability analysis of fuzzy systems with robust approach”. Math. Comput. Simul., 59:335-340, 2002

21. E. Kim, H. Lee. “New approaches to relaxed quadratic stability condition on fuzzy control systems”. IEEE Transactions on Fuzzy Systems, 8:523-533, 2000

22. M. Chadli, D. Maquin and J. Ragot. “Stabilisation of Takagi-Sugeno models with maximum convergence rate”. In Proceeding of the IEEE International Conference on Fuzzy Systems. Hungary, 2004

23. Z. Kardous, N. Benhadj Braiek and A. Elkamel. “On the residue formulation in multimodel control”. In Proceeding of the IEEE International Conference on SMC. Tunisia, 2002

24. Z. Kardous, N. Benhadj Braiek, A. Elkamel and P. Borne, “On the quadratic stabilization in discrete multimodel control”. Conference on Control Applications CCA’2003. Istanbul, 2003

25. K. Tanaka, M. Sugeno. “Stability analysis and design of fuzzy control systems”. Fuzzy Sets Systems, 45:136-156, 1992