New PID Tuning Rule Using ITAE Criteria

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Abstract

This paper demonstrates an efficient method of tuning the PID controller parameters using the optimization rule for ITAE performance criteria. The method implies an analytical calculating the gain of the controller (K_c), integral time (T_i) and the derivative time (T_d) for PID controlled systems whose process is modeled in first order lag plus time delay (FOLPD) form. Firstly A mat lab program with objective function is written to find the optimum value for the PID controller parameters which can achieve most of the systems requirements such as reducing the overshoot, maintaining a high system response, achieving a good load disturbances rejection and maintaining robustness. The objective function is selected so as to minimize the integral of Time Absolute Error (ITAE) performance index. Using crave fitting technique, equations that define the controller parameters is driven. A comparison between the proposed tuning rules and the traditional tuning rules is done through the Matlab software to show the efficiency of the new tuning rule.

Keywords: ITAE criteria; AMIGO; Z-N tuning rule; PID;

1. INTRODUCTION

Controlling the process is the main issue that rises in the process industry. It is very important to keep the process working probably and safely in the industry, for environmental issues and for the quality of the product being processed. In order for the controllers to work satisfactorily, they must be tuned probably. Tuning of controllers can be done in several ways, depending on the dynamics desired strengths of the system, and many methods have been developed and refined in recent years. The proportional-integral-derivative (PID) controller is widely used in the process industries. The main reason is their simple structure, which can be easily understood and implemented in practice. Finding design methods that lead to the optimal operation of PID controllers is therefore of significant interest. It has been stated, for example, that 98% of control loops in the pulp and paper industries are controlled by PI controllers (Bialkowski, 1996) and that, in more general process control applications, more than 95% of the controllers are of PID type (Åström and Hägglund, 1995). In order for the PID controller to work probably it has to be tuned which mean a selection of the PID controller parameters has to be made [8]. The requirement to choose either two or three controller parameters has meant that the use of tuning rules to determine these parameters is popular. There are many tuning rules for the PID controller as it has been noted that 219 such tuning rules in the literature to specify the PI controller terms, with 381 tuning rules defined to specify the PID controller parameters (O'Dwyer, 2003), Though the use of tuning rules is practically important [3, 11]. Even though, recent surveys indicate, 30 % of installed controllers operate in manual, 30 % of loops increase variability, 25 % of loops use default settings and 30 % of loops have equipment problems [1, 10]. Most PID tuning rules are based on first-order plus time delay assumption of the plant hence cannot ensure the best control performance. Using modern optimization techniques, it is possible to tune a PID controller based on the actual transfer function of the plant to optimize the closed-loop performance. In this paper optimization method is being used to obtain PID controller parameters. A search of one parameter to be optimized lead to select the Integral of Time multiply by Absolute Error (ITAE) index performance criterion, since it can provide controllers with a high load disturbance rejection and minimize the system overshoot while maintain the robustness of the system.

The Integral of Time multiply by Absolute Error (ITAE) index is a popular performance criterion used for control system design. The index was proposed by Graham and Lathrop (1953), who derived a set of normalized transfer function coefficients from 2nd to 8th-order to minimize the ITAE criterion for a step input [10].

This paper is organized as follows: - an overview of the traditional and a best performance tuning rule is covered in section 2. The proposed tuning rule which derived from optimization method is outlined in section 3. Section 4 outlines the optimized PID parameters values that obtained from using the ITAE criteria performance index. In section 5 graphical results showing the performance and robustness of FOLPD processes, compensated with the proposed PID tuning rule. The process is modeled as a first order lag plus time delay (FOLPD) model, and compensated by PID controllers whose parameters are specified using the proposed tuning rule. The results of the proposed tuning rule are plotted and are used to be compared in the face of the performance, robustness and load disturbance rejection against the traditional tuning rule and more over against a well performance tuning rule. Conclusions of the work are drawn in Section 6.

2. CONTROLLER TUNING

Controller tuning methods provide the controller parameters in the form of formulae or algorithms. They ensure that the obtained control system would be stable and would meet given objectives. Also, great advances on optimal methods based on stabilizing PID solutions have been achieved. These methods require certain knowledge about the controlled process. This knowledge, which depends on the applied method, usually translates into a transfer function [9]. In fact, since Ziegler–Nichols proposed their first tuning rules [5], an intensive research has been done from modifications of the original tuning rules to a variety of new techniques: analytical tuning; optimization methods; gain and phase margin optimization, just to mention a few. Recently, tuning methods based on optimization approaches with the aim of ensuring good stability and robustness has received attention in the literature [2, 6]. In this section some of PID tuning algorithms is considered.

2.1 Ziegler-Nichols tuning rule

Ziegler-Nichols tuning rule was the first such effort to provide a practical approach to tune a PID controller. According to the rule, a PID controller is tuned by firstly setting it to the P-only mode but adjusting the gain to make the control system in continuous oscillation. The corresponding gain is referred to as the ultimate gain (K_u) and the oscillation period is termed as the ultimate period (P_u). Then, the PID controller parameters are determined from K_u and P_u using the Ziegler-Nichols tuning table.

Table 1:- Ziegler-Nichols tuning rule

controller	K_c	T_i	T_d
P	$K_u/2$		
PI	$K_u/2.2$	$P_u/1.2$	
PID	$K_u/1.7$	$P_u/2$	$P_u/8$

The key step of the Ziegler-Nichols tuning approach is to determine the ultimate gain and period [5]. However, to determine the ultimate gain and period experimentally is time consuming.

2.2. AMIGO tuning rules

AMIGO tuning rule consider a controller described by:-

$$u(t) = k(by_{sp}(t) - y_f(t)) + k_i \int_0^t (y_{sp}(\tau) - y_f(\tau)) \, d\tau + k_d \left(\frac{Cdy_{sp}(t)}{dt} - \frac{y_f(t)}{dt} \right) \ldots\ldots(1)$$

Where u is the control variable, ysp the set point, y the process output, and y_f is the filtered process variable, i.e. $Y_f(s) = G_f(s)Y(s)$ The transfer function $G_f(s)$ is a first order filter with time constant T_f , or a second order filter if high frequency roll-off is desired [7].

$$G(s) = \frac{1}{(1 + sT_f)^2} \ldots\ldots\ldots\ldots\ldots(2)$$

Parameters b and c are called set-point weights. They have no influence on the response to disturbances but they have a significant influence on the response to set point changes. Neglecting the filter of the process output the feedback part of the controller has the transfer function

$$C(s) = K\left[1 + \frac{1}{sT_i} + sT_d\right] \ldots\ldots(3)$$

The advantage by feeding the filtered process variable into the controller is that the filter dynamics can be combined with in the process dynamics and the controller can be designed designing an ideal controller for the process P(s) $G_f(s)$. The objective of AMIGO was to develop tuning rules for the PID controller in varying time-delay systems by analyzing different properties (performance, robustness etc.) of a process test batch. The AMIGO tuning rules are based on the KLT-process model obtained with a step response experiment. The AMIGO tuning rules are

$$K_c = \left(0.2 + 0.45 * \frac{T}{L}\right) \ldots\ldots(4)$$

$$T_i = \left(\frac{0.4L + 0.8T}{L + 0.1T}\right)L \ldots\ldots(5)$$

$$T_d = 0.\frac{5LT}{0.3L + T} \ldots\ldots\ldots\ldots(6)$$

In order to use the PID controller with filtering, the rules are extended as follows:

$$\begin{cases} k_c = K_c \\ K_i = \dfrac{K_c}{T_i} \\ k_d = K_c * T_d \end{cases} \qquad \begin{cases} b = \begin{cases} 0 \text{ if } \tau \le 0.5 \\ 1 \text{ if } \tau > 0.5 \end{cases} \\ c = 0 \\ T_f = \begin{cases} 0.\dfrac{05}{\omega_{gc}} \text{ if } \tau \le 0.2 \\ 0.1 * L \text{ if } \tau > 0.2 \end{cases} \end{cases} \ldots\ldots\ldots(7)$$

Where: - ω_{gc} is the gain crossover frequency and $\tau = L/(L + T)$ is the relative dead-time of the process, which has turned out to be an important process parameter for controller tuning [4, 7].

3. THE PROPOSED TUNING RULE

The proposed tuning rule is driven using several steps

- Step 1:- Find relations between the controller tuning parameters and process parameters as stated below:-

  $K_c = f_1(K_P; L; T)$; $T_i = f_2(K_P; L; T)$; $T_d = f_3(K_P; L; T)$

Function f_1, f_2 and f_3 should be determined such that the load disturbances response is minimized and the robustness constraint is satisfied.

- Step 2:-, Create dimension less expressions through diving and multiplying the factors of the process parameters with appropriate scale factors of each other such as L/T or ; T_i/L or T_i/T ; T_d/L or T_d/T ; K_c*K_P
- Step 3:- Select one factor of the above to find the relations between the tuning parameters and the process parameters. In this paper the factor (L/T) is being selected.
  $K_c * K_P = K_1$ (L/T) ; $T_i/L = K_2$ (L/T) ; $T_d/L = K_3$ (L/T)
- Step 4:- For a defined values of the factor L/T determine the optimal value of the tuning parameters K_c; T_i; T_d which minimize a specific performance criteria (ITAE). In order to take FOPDT processes with a very small, medium and fairly long value of dead time into account, the values of the dimensionless factor L/T are considered from 0.1 to 2.
- Step 5:- Find the values of $K_c * K_P$; T_i/L; T_d/L corresponding to the values of L/T.
- Step 6:- Drive the equations of K_1; K_2; K_3 using carve fitting techniques.

In step 4 a Matlab m-file is defined to calculate the ITAE index (the objective function) which is mathematically given by:-

$$\mathbf{IATE} = \int_0^{\infty} [\mathbf{t} \mid \mathbf{e}(\mathbf{t})\mathbf{dt}] \mid \quad \ldots\ldots\ldots..(8)$$

Where t is the time and e (t) is the error which is calculated as the difference between the set point and the output. A function of Matlab optimization toolbox (f*minsearch*) is called to calculate the minimum of the objective function. Like most optimization problems, the control performance optimization function is needed to be initializing and a local minimum is required. To do so, the initial controller parameters are set to be determined by one of existing tuning rules. In this way, the controller derived is at least better than that determined by the tuning method. The stability margin based Ziegler-Nichols is used for initial controller parameters and for performance comparison.

On each evaluation of the objective function, the process model develop in the simulink is executed and the IATE performance index is calculated using multiple application Simpson's 1/3 rule. The simulation s repeated with different values of the process parameters (T; L; K_P)

4. RSEULTS

Using Matlab simulation tools several processes with different parameters were taken under test. A record of the controller parameters (K_c, T_i and T_d) that minimize ITAE performance criteria was observed as shown in table (2). The processes under test were first order plus dead time (FOPDT) process.

$$\mathbf{P_{KTL}(S)} = \frac{\mathbf{K_P}}{\mathbf{Ts+1}} * \mathbf{e^{-Ls}} \ldots\ldots\ldots.(9)$$

Table 2:- Controller parameters for different Process parameters

Kp = 5   T = 4

L	L/T	KC	Ti	Td
0.1	0.025	2.2829	4	0.0336
1	0.25	0.4171	4.009	0.3333
2	0.5	0.2204	4.2592	0.699
3	0.75	0.1602	4.555	0.9955
4	1	0.131	4.8604	1.2591
5	1.25	0.1133	5.1709	1.5046

Kp = 2   T = 3

L	L/T	KC	Ti	Td
0.1	0.033	8.1627	3	0.0335
1	0.33	0.8009	3.0347	0.3626
2	0.667	0.4358	3.3332	0.672
3	1	0.3273	3.6418	0.944
4	1.33	0.2718	3.9496	1.1847
5	1.67	0.2404	4.2664	1.4205

Kp = 3   T = 3

L	L/T	KC	Ti	Td
0.1	0.03	4.8005	3	0.0335
1	0.33	0.5339	3.0347	0.3626
2	0.67	0.2906	3.3351	0.672
3	1	0.2182	3.6419	0.944
4	1.33	0.1812	3.9497	1.1847
5	1.667	0.1603	4.2665	1.4205

Kp = 2   T = 2

L	L/T	KC	Ti	Td
0.1	0.05	5.4418	2	0.0334
0.5	0.25	1.0633	2.005	0.1667
1	0.5	0.5509	2.1298	0.344
2	1	0.3273	2.4278	0.6227
3	1.5	0.2554	2.7456	0.8689
4	2	0.2198	3.0588	1.0861

Kp = 1   T = 3

L	L/T	KC	Ti	Td
0.1	0.033	17.574	3	0.0335
0.5	0.167	3.1872	3	0.1634
1	0.33	1.6018	3.0347	0.3626
2	0.67	0.8715	3.3331	0.672
3	1	0.6547	3.6418	0.944
4	1.33	0.5435	3.9497	1.1847
5	1.67	0.4808	4.2664	1.4205

Kp = 1   T = 2

L	L/T	KC	Ti	Td
0.1	0.05	11.1413	2	0.0334
1	0.5	1.1018	2.1298	0.3442
2	1	0.6547	2.4279	0.6227
3	1.5	0.5109	2.7458	0.8687
4	2	0.4397	3.0588	1.0861

Kp = 5   T = 1

L	L/T	KC	Ti	Td
0.1	0.1	0.9602	1	0.0328
0.5	0.5	0.2199	1.0615	0.1433
1	1	0.1312	1.217	0.3146
1.5	1.5	0.1022	1.373	0.4369
2	2	0.0871	1.5198	0.5423

Kp = 4   T = 1

L	L/T	KC	Ti	Td
0.1	0.1	1.4645	1	0.0328
0.5	0.5	0.2749	1.0616	0.1433
1	1	0.164	1.2171	0.3146
1.5	1.5	0.1277	1.373	0.4367
2	2	0.1109	1.5389	0.5424

Kp = 3   T = 1

L	L/T	KC	Ti	Td
0.1	0.1	1.8139	1	0.0355
0.5	0.5	0.3666	1.0616	0.1433
1	1	0.2186	1.2172	0.3146
1.5	1.5	0.1703	1.373	0.4429
2	2	0.1479	1.5389	0.5321

Kp = 2   T = 1

L	L/T	KC	Ti	Td
0.1	0.1	2.7668	2	0.0328
0.5	0.5	0.5499	2.005	0.1433
1	1	0.3279	2.1298	0.3146
1.5	1.5	0.2555	2.2712	0.4425
2	2	0.2219	2.4279	0.5184

Kp = 1   T = 1

L	L/T	KC	Ti	Td
0.1	0.1	5.8578	1	0.0328
0.5	0.5	1.1016	1.0647	0.1433
1	1	0.6559	1.2171	0.3146
1.5	1.5	0.5109	1.373	0.4425
2	2	0.4354	1.5199	0.5184

Using carve fitting techniques the tuning rule are found as shown below.

$$K_c = \left(0.3 + 0.38 * \frac{T}{L} + 0.007 * \left(\frac{T}{L}\right)^2\right) \ldots\ldots\ldots (10)$$

$$T_i = L * \left(0.5 + 0.5 * \frac{T}{L} + 0.01 * \left(\frac{T}{L}\right)^1 .5\right) \ldots\ldots (11)$$

$$T_d = \frac{1.4 * LT}{0.9 * L + 2.9 * T} \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

5. MATLAB SIMULATION RESULTS

Several process models were examined in this analysis representing different types of processes. After finding the controller settings for the different processes, the responses of the systems were plotted. All processes were Fist order Plus Dead Time. A reduction procedure is used to modulate the higher order models in the FOPDT model.

$$G_1(s) = \frac{1 * e^{-5s}}{(3s+1)(s+1)} = \frac{0.99 * e^{-5.64s}}{2.85s+1}$$

Table 3: Controller settings of AMIGO and proposed tuning rule for process $G_1(s)$

Algorithm	K_C	T_i	T_d
Z-N	0.9238	7.7556	1.9389
AMIGO	0.432	4.3178	1.7695
Proposed tuning rule	0.499	4.4020	1.6868

Table 4: The response parameters values of Z-N, AMIGO and Proposed tuning rule for the process $G_1(s)$.

Algorithm	Rise time (s)	Settling Time (s)	Set point overshoot %	IAE
Z-N	10.2	22.8	11.42	83.28
AMIGO	15.5	17.2	4.19	100.36
Proposed tuning rule	13.5	14.6	7	88.54

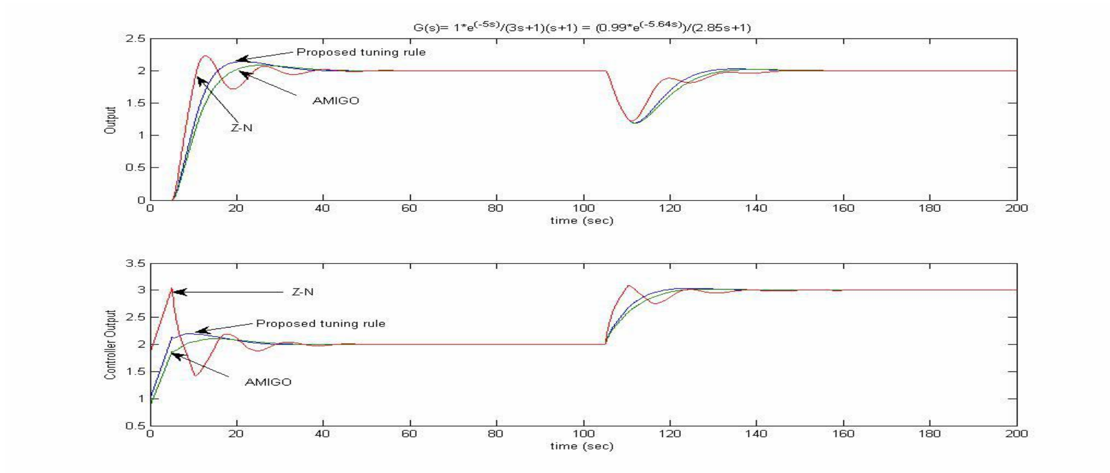

Figure 1:- Step response for the second order process with delay

$$G_2(s) = \frac{10 * e^{-s}}{(s+1)(s+2)(s+3)(s+4)} = \frac{0.419 * e^{-1.2s}}{(s^2 + 2s + 1)} = \frac{0.4167 * e^{-2.16s}}{1.1696s + 1}$$

Table 5: Controller settings of Z-N, AMIGO and proposed tuning rule for process $G_2(s)$

Algorithm	KC	Ti	Td
Z-N	2.7819	2.7641	0.6910
AMIGO	1.065	1.707236	0.694965
Proposed tuning rule	1.219	1.71901	0.662852

Table 6: The response parameters values of Z-N, AMIGO and Proposed tuning rule for the process $G_2(s)$.

Algorithm	Rise time (s)	Settling Time (s)	Set point overshoot %	IAE Yd
Z-N	3.2	6	18.13	9.94
AMIGO	6.2	7.3	0.9	16.03
Proposed tuning rule	5.3	9	2.36	14.10

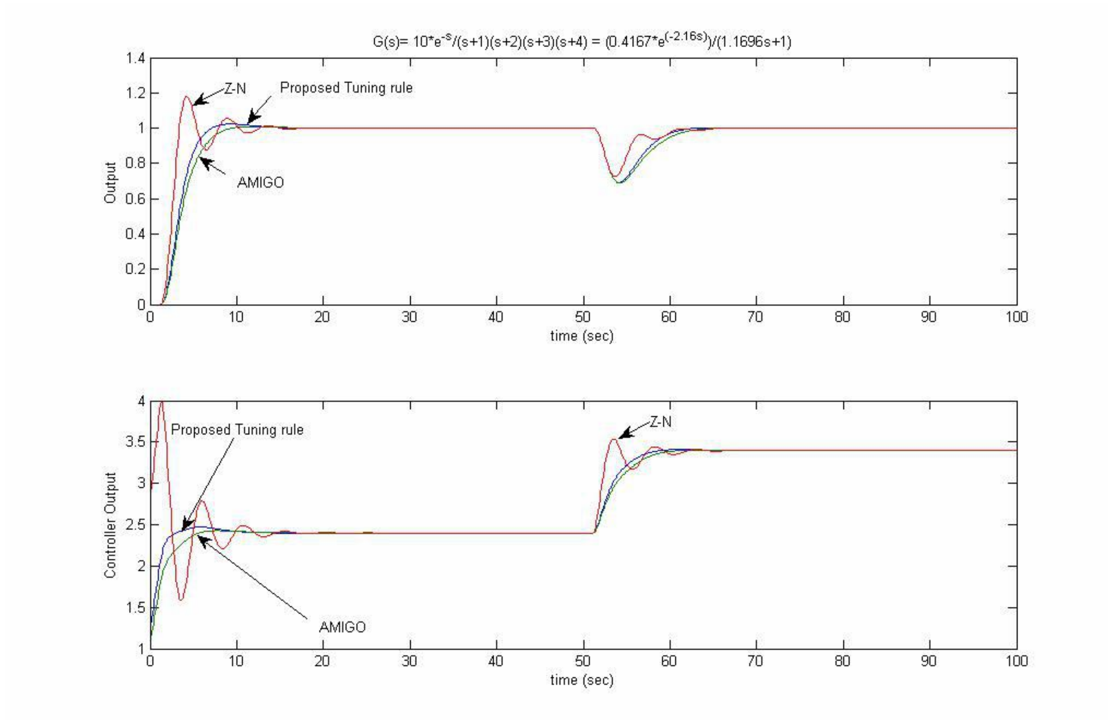

Figure 2:- Step response for the high order process with delay

$$G_3(s) = \frac{1 * e^{-3s}}{(2s+1)(s+1)^2} = \frac{0.9993 * e^{-4.69s}}{2.61s+1}$$

Table 7: Controller settings of AMIGO and proposed tuning rule for process $G_3(s)$

Algorithm	K_C	T_i	T_d
Z-N	1.0345	6.4481	1.6120
AMIGO	0.451	3.755031	1.523637
Proposed tuning rule	0.514	3.762972	1.354542

Table 8: The response parameters values of AMIGO and Proposed tuning rule for the process $G_3(s)$.

Algorithm	Rise time (s)	Settling Time (s)	Set point overshoot %	IAE Yd
Z-N	7.6	17.7	14.45	65.5
AMIGO	12.8	15	1.48	83.33
Proposed tuning rule	10.9	11.9	3.66	72.99

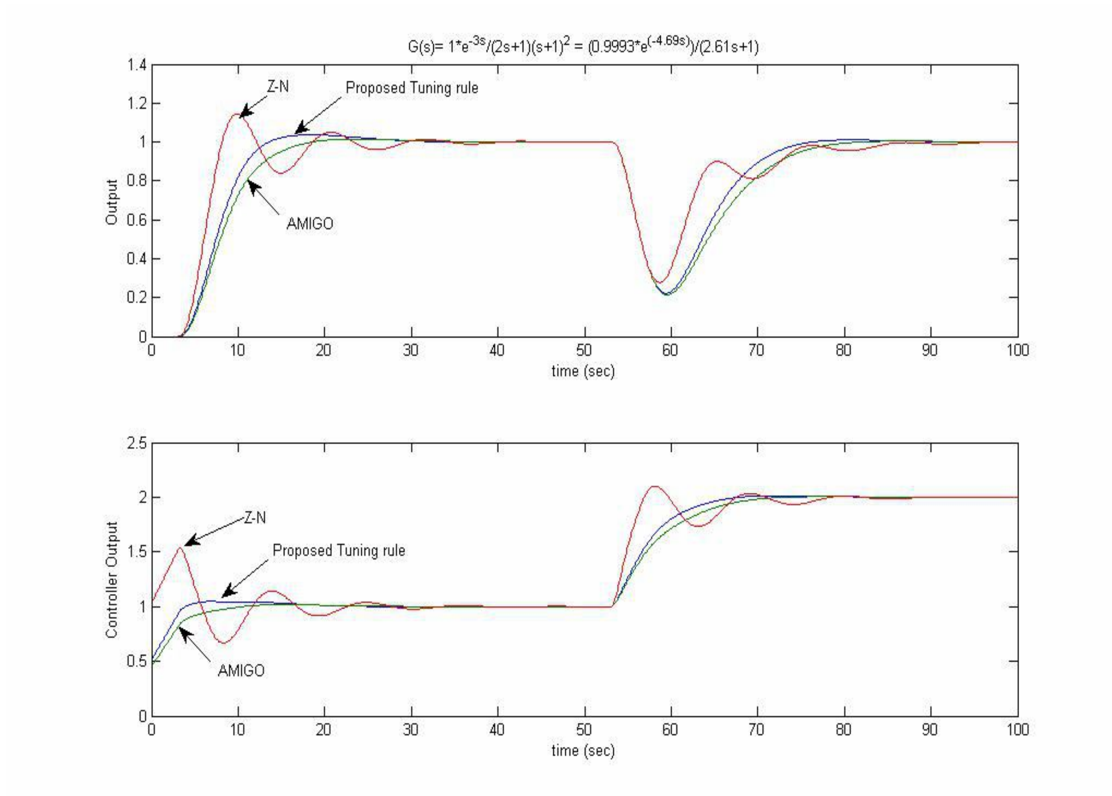

Figure 3:- Step response for the third order process with delay

$$G_4(s) = \frac{1 * e^{-4s}}{(2s+1)}$$

Table 9: Controller settings of AMIGO and proposed tuning rule for process $G_4(s)$

Algorithm	K_C	T_i	T_d
Z-N	0.9101	5.4901	1.3725
AMIGO	0.425	3.047619	1.25
Proposed tuning rule	0.492	3.113137	1.191489

Table 10: The response parameters values of AMIGO and Proposed tuning rule for the process $G_4(s)$.

Algorithm	Rise time (s)	Settling Time (s)	Set point overshoot %	IAE
Z-N	6.8	23	16.39	60.24
AMIGO	11.1	12.9	1.78	71.94
Proposed tuning rule	9.3	10.3	3.65	63.29

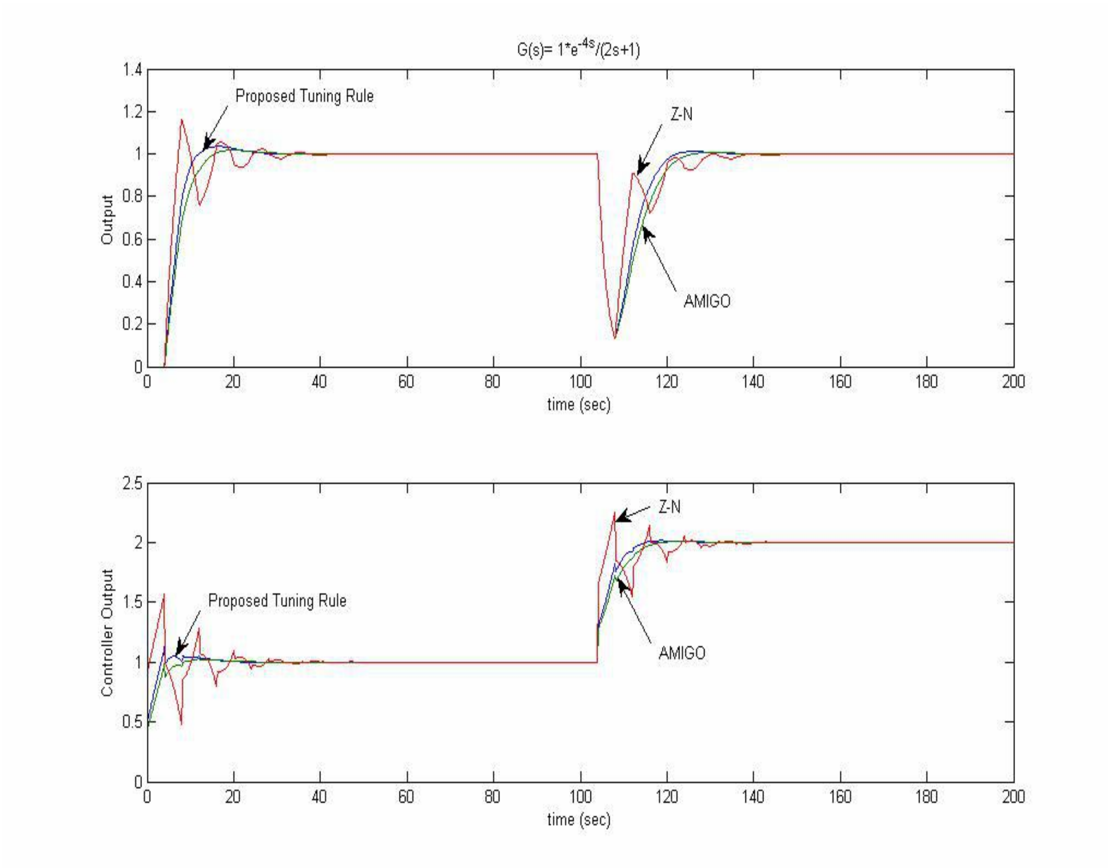

Figure 4:- Step response for FOPDT process

$$G_5(s) = \frac{10}{(s+1)(s+2)(s+3)(s+4)} = \frac{0.419 * e^{-0.3s}}{(s^2+2s+1)} = \frac{0.416 * e^{-1.14s}}{1.1696s+1}$$

Table 11: Controller settings of AMIGO and proposed tuning rule for process $G_5(s)$

Algorithm	K_C	T_i	T_d
Z-N	7.5449	1.4050	0.3512
AMIGO	1.591	1.262184	0.441037
Proposed tuning rule	1.676	1.16577	0.422533

Table 12: The response parameters values of AMIGO and Proposed tuning rule for the process $G_5(s)$.

Algorithm	Rise time (s)	Settling Time (s)	Set point overshoot %	IAE
Z-N	1.3	44.18	4.1	1.86
AMIGO	3.4	4.91	3.8	7.93
Proposed tuning rule	3.1	8.64	6.7	6.95

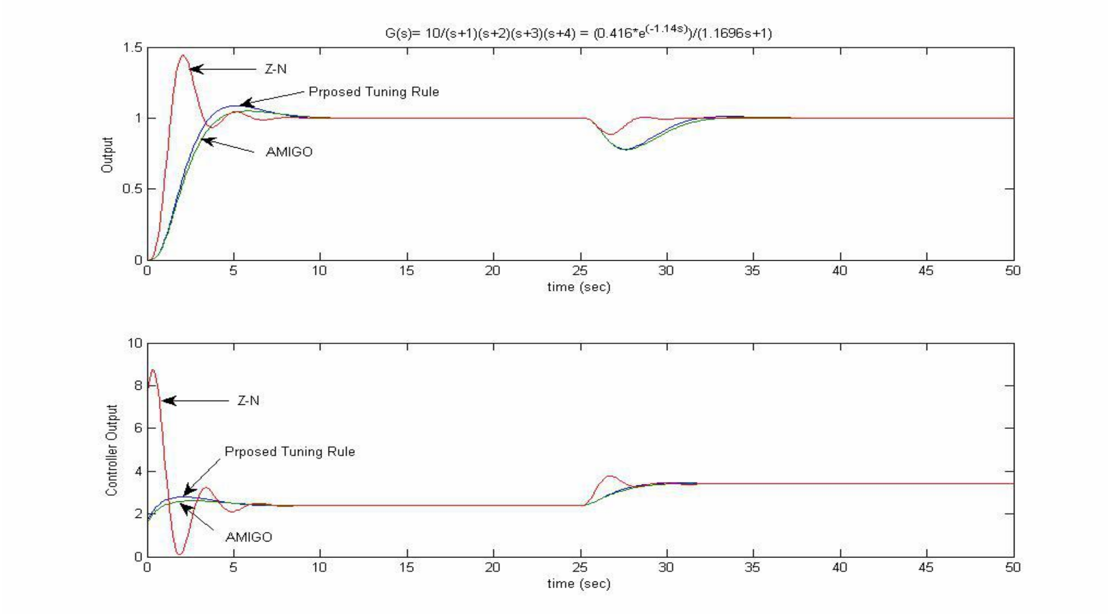

Figure 5:- Step response for high order process without delay

6. CONCLUSION

The analysis shows that the proposed tuning and AMIGO settings give the least oscillatory response than Z-N setting. It is also seen that the IAE (integral of absolute error) for the disturbance for the proposed tuning settings is less than the AMIGO setting but slightly higher than Z-N setting. The proposed tuning setting give a small rise time comparing to that of the AMIGO tuning, but slightly higher than ZN setting. In the other hand the proposed tuning gives a settling time faster than Z-N's. Test batch of different process had been used to simulate the proposed tuning. The most important advantage of this design is in the use of the IATE performance criteria index to find the new tuning rule since it can provide the controller with a good performance. As it appears from the simulation, the proposed tuning rule is able to deal with the possible variation of system parameters. It is so obvious that the proposed tuning rule has the same or better performance than AMIGO tuning rule and a much better performance than Z-N tuning rule. The observation from those results shows that a high overshoot appears in the output of the system for some cases of processes. This overshoot appears as expense of achieving a high response and a better load disturbance rejection. In the other hand the proposed tuning rule maintain robustness. The concluded important contributions in this paper regarding the use of the proposed tuning rule are that it proves the ability of the proposed tuning rule in tuning the PID controller probably. Also it validates the flexibility of the proposed tuning rule to deal with different modeling systems with different parameters. As a future work, the proposed tuning rule can be used in a practical experiment so as to prepare this proposed tuning rule to be used in the practical industrial applications.

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