Optimizing PID Tuning Parameters Using Grey Prediction Algorithm

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ABSTRACT

This paper considered a new way to tune the PID controller parameters using the optimization method and grey prediction algorithm. The grey prediction algorithm has the ability to predict the output or the error of the system depending on a small amount of data. In this paper the grey prediction algorithm is used to predict and estimate the errors of the system for a defined period of time and then the average of the estimated error is calculated. A mat lab program is developed using simulink to find the average of the estimated error for the system whose process is modeled in first order lag plus dead time (FOLPD) form. In the other hand optimization method with mat lab software program was used to find the optimum value for the PID controller gain (K_c (opt)) which minimizes specific performance criteria (ITAE performance criteria). The main goal of the optimization method is to achieve most of the systems requirements such as reducing the overshoot, maintaining a high system response, achieving a good load disturbances rejection and maintaining robustness. Those two parameters (the average of the estimated error and the PID controller gain (K_c (opt))) were used to calculate the PID controller parameters (gain of the controller (K_c), integral time (T_i) and the derivative time (T_d)). Simulations for the proposed algorithm had been done for different process models. A comparison between the proposed tuning rule and a well performance tuning rule is done through the Matlab software to show the efficiency of the new tuning rule.

Keywords: ITAE criteria; Grey prediction; AMIGO tuning rule; PID

1. INTRODUCTION

Since it was appear for the first time in 1922, the PID controller shows the ability of to compensate most practical industrial processes which have led to their wide acceptance in industrial applications. It has been stated, for example, that 98% of control loops in the pulp and paper industries are controlled by PI controllers (Bialkowski, 1996) and that, in more general process control applications, more than 95% of the controllers are of PID type (Åström and Hägglund, 1995). In order for the PID controller to work probably it has to be tuned which mean a selection of the PID controller parameters has to be made [2, 7]. The requirement to choose either two or three controller parameters has meant that the use of tuning rules to determine these parameters is popular. There are many tuning rules for the PID controller as it has been noted that 219 such tuning rules in the literature to specify the PI controller terms, with 381 tuning rules defined to specify the PID controller parameters (O'Dwyer, 2003), Though the use of tuning rules is practically important [9]. Even though, recent surveys indicate, 30 % of installed controllers operate in manual, 30 % of loops increase variability, 25 % of loops use default settings and 30 % of loops have equipment problems [1]. Most PID tuning rules are based on first-order plus time delay assumption of the plant hence cannot ensure the best control performance. Using modern optimization techniques, it is possible to tune a PID controller based on the actual transfer function of the plant to optimize the closed-loop performance. In this paper optimization method is being used to obtain only one of the PID controller parameter which is controller gain (K_c (opt)). A search of one parameter to be optimized lead to select the Integral of Time multiply by Absolute Error (ITAE) index performance criterion, since it can provide controllers with a high load disturbance rejection and minimize the system overshoot while maintain the robustness of the system. The Integral of Time multiply by Absolute Error (ITAE) index is a popular performance criterion used for control system design. The index was proposed by Graham and Lathrop (1953), who derived a set of normalized transfer function coefficients from 2nd-order to 8th-order to minimize the ITAE criterion for a step input [8]. The grey prediction algorithm had been used for several applications and it proves its ability to forecast the output accurately. In this paper the grey algorithm had been used to estimate the error of the system which will be used to obtain the PID controller parameters. This paper is organized as follows: - The grey prediction algorithm is discussed in section 2. An overview of the traditional and a best performance tuning rule is covered in section 3. The proposed tuning rule which derived from optimization method along with the grey prediction algorithm is outlined in section 4. In section 5 and section 6 graphical results showing the performance and robustness of FOLPD processes, compensated with the proposed PID tuning rule. The process is modeled as a first order lag plus time delay (FOLPD) model, and compensated by PID controllers whose parameters are specified using the proposed tuning rule. The results of the proposed tuning rule are plotted and are used to be compared in the face of the performance, robustness and load disturbance rejection against a well performance tuning rule. Conclusions of the work are drawn in Section 7.

2. GREY PREDICTION ALGORITHM

Grey theory was introduced in 1982. The Grey theory is able to deal with indeterminate and incomplete data to analyze and establish the systematic relations and a prediction model. Unlike conventional stochastic forecasting theory, Grey prediction simply requires as few as four lagged inputs to construct a Grey differential equation [5]. The Grey prediction has been widely used in studies of social sciences, agriculture, procreation, power consumption and management, as well as other fields. In Grey theory, two techniques are adopted to establish the model for applications. They are accumulated generating operation (AGO) and inverse accumulated generating operation (IAGO). The grey prediction model conducts a so called "accumulated generating operation" on the original sequence. The resultant new series is used to establish a difference equation whose coefficients are found via the least-squares method. The accumulated generating series prediction model value is then obtained. The estimated prediction value in the time-domain is calculated by means of an inverse accumulated generating operation. Only a few original sequence elements are needed, and one does not have to assume the distribution of the sequence. The generated value is then used to establish a set of Grey difference equation and Grey pseudo differential equation. The model is called the Grey Model. Generally, there are a few types used in the literature [5, 10]:

1) GM (1, 1): This represents first-order derivative, containing one input variable, generally used for prediction purposes.

2) GM (1, N): This represents first-order derivative, but containing N input variables, for multi-variable analysis.

3) GM (O, N): This represents zero-order derivative, containing N input variables, for prediction purposes.

In this thesis, the GM (1, 1) model is adopted to perform the prediction of system output response. The standard procedure is as follows [14]:

- Step 1: Collecting the original data sequence;

$$x^{(0)} = \{x^{(0)}(1), x^{(0)}(2) \dots x^{(0)}(n)\} \qquad n \geq 4 \dots \dots (1)$$

- Step 2: Conducting an accumulated generation operation, AGO, on the original data sequence in order to diminish the effect of data uncertainty;

$$x^{(1)}(k) = \text{AGO of } x^{(0)} = \sum_{i=1}^{k} x^{(0)}(i) \cdots (2) \qquad k = 1, 2, \dots n \quad \text{and } n \geq 4$$

- Step 3: Establishing Grey differential equation and then calculating its background values:-First we define z $^{(1)}$ as the sequence obtained by the MEAN operation to $x^{(0)}$ as follow:-

$$\begin{aligned} z^{(1)}(k) &= \text{MEAN of } x^{(1)} \\ &= 0.5 * [x^{(1)}(k) + x^{(1)}(k-1)] \qquad k = 2, 3, 4 \quad \dots \dots \dots (3) \end{aligned}$$

Secondly Grey differential equation can be obtained the as follow:-

$$x^{(0)}(k) + a * z^{(1)}(k) = u_q \dots\dots\dots\dots (4)$$

Where the parameters a, uq are called the development coefficient and the grey input, respectively. Equation (5) is called the whitening equation corresponding to the grey differential equation.

$$\frac{dx^{(1)}}{dt} + a * x^{(1)} = u_q \dots\dots\dots\dots (5)$$

- Step 4: decide the value of a, uq by means of the least –square method as follow:-

$$L = \begin{bmatrix} a \\ u_q \end{bmatrix} = (B^T * B)^{-1} * B^T * x^N \dots (6)$$

$$B = \begin{bmatrix} -z^{(1)}(2) \dots\dots\dots .1 \\ -z^{(1)}(3) \dots\dots\dots .1 \\ . \\ . \\ . \\ -z^{(1)}(n) \dots\dots\dots .1 \end{bmatrix} \dots\dots\dots ..(7)$$

$$x^{(N)} = \{x^{(0)}(2)\ x^{(0)}(3) \dots x^{(0)}(n)\}^T ..(8)$$

- Step 5: Deriving the solution to the Grey difference equation:-

$$\hat{x}^{(1)}(n + p) = \left[x^{(0)}(1) - \frac{u_q}{a}\right] * e^{-a(n+p-1)} + \frac{u_q}{a}$$

Where the parameter (p) is the forecasting step size and the up script "^" means the value $\hat{x}$ is a forecasting value of x.

- Step 6 conducting the inverse accumulated generation operation (IAGO) on $\hat{x}^{(1)}$ to obtain a prediction value as follow:-

$$x^{(0)}(n + p) = (1 - e^{a})\left[x^{(0)}(1) - \frac{u_q}{a}\right] * e^{-a(n+p-1)}$$

$$\hat{x}^{(0)}(k) = \text{IAGO of } x^{(1)} = x^{(1)}(k) - x^{(1)}(k - 1) \qquad k = 2, 3 \dots n \quad \dots (9)$$

$$n \geq 4 \text{ for all equations}$$

3. AMIGO TUNING RULE

The objective of AMIGO was to develop tuning rules for the PID controller in varying time-delay systems by analyzing different properties (performance, robustness etc.) of a process test batch. The AMIGO tuning rules are based on the KLT-process model obtained with a step response experiment. AMIOG tuning rule considered controller describe with the following equation:-

$$u(t) = k(by_{sp}(t) - y_f(t)) + k_i \int_0^t (y_{sp}(\tau) - y_f(\tau))\, d\tau + k_d \left(C * \frac{dy_{sp}(t)}{dt} + \frac{dy_f(t)}{dt}\right) \dots (10)$$

Where u is the control variable, ysp the set point, y the process output, and yf is the filtered process variable, i.e. $y_f(s) = G_f(s)y(s)$ The transfer function $G_f(s)$ is a first order filter with time constant T_f , or a second order filter if high frequency roll-off is desired.

$$G(s) = \frac{1}{(1 + sT_f)^2} \dots\dots\dots\dots(11)$$

Parameters b and c are called set-point weights. They have no influence on the response to disturbances but they have a significant influence on the response to set point changes. Neglecting the filter of the process output the feedback part of the controller has the transfer function.

$$C(s) = K_c\left(1 + \frac{1}{sT_i} + sT_d\right)\dots..(12)$$

The advantage of feeding the filtered process variable into the controller is that the filter dynamics can be combined with in the process dynamics and the controller can be designed as ideal controller. The AMIGO tuning rules are [3, 6]

$$K_c = \left(0.2 + 0.45 * \frac{T}{L}\right)\dots..(13)$$

$$T_i = \left(\frac{0.4L + 0.8T}{L + 0.1\,T}\right)L\dots..(14)$$

$$T_d = 0.\frac{5LT}{0.3L + T}\dots\dots\dots(15)$$

4. THE PROPOSED TUNING RULE

The proposed algorithm is depending on finding two important parameters which are the optimal PID controller gain and the average estimated error for the process. Using ITAE performance criteria as a factor to be optimized, Matlab program had been used to obtain the optimal value of the controller gain (K_c (opt)). A Matlab m-file is being used to calculate the ITAE index (the objective function) which is mathematically given by:-

$$IATE = \int_0^{\infty} [t\,|e(t)dt]\,| \dots\dots\dots..(16)$$

Where t is the time and e (t) is the error which is calculated as the difference between the set point and the output. A function of Matlab optimization toolbox (*f_{min}search*) is called to calculate the minimum of the objective function. Like most optimization problems, the control performance optimization function is needed to be initialized and a local minimum is required. To do so, the initial controller parameters are set to be determined by one of existing tuning rules. In this way, the controller derived is at least better than that determined by the tuning method. The stability margin based Ziegler-Nichols is used for initial controller parameters. On each evaluation of the objective function, the process model develop in the simulink is executed and the IATE performance index is calculated using multiple application Simpson's 1/3 rule. The simulation repeated with different values of the process parameters (K_P; T; L) and the values are recorded as shown in table (1). For each process parameters value (K_P, L and T) the grey prediction algorithm is then used to obtain the average of the estimated error (E). The average estimated error is computed as follow: - Initial value of the PID controller parameters is used. The selection of the initial values is made depending on the behavior of the system. In this paper the initial value of the PID parameters is K_c = 0.2, K_i = 0.003 and K_d = 0 so as to keep the system behavior under damp. This setting can be used to control different system as long as the system behavior with the initial values of the PI controller will be under damp and the ratio (L/T) is equal or less than 2. The estimated error is computed during the period from the start of the simulation until the steady state reached and then average value is taken from the computed values. The simulation step size is fixed to 0.1 sec so as to make it real time simulation since the suitable sampling time in the real time process can be equal to 0.1 sec. The average of the estimated error produced by grey prediction algorithm and the optimal values of the PID controller gain (K_c (opt)) produced by optimization method is recorded as shown in the table (1). These two parameters are then used to adjust the three parameters of the controller.

5. RSEULTS

Using Matlab simulation tools several processes with different parameters were taken under test. A record of the controller gain (K_c (opt)) that minimize ITAE performance criteria was observed along with the average of the estimated error (E) as shown in table below. The processes under test were first order plus dead time (FOPDT) process.

$$P_{KTL}(S) = \frac{K_P}{Ts + 1} * e^{-Ls} \dots\dots\dots (17)$$

Table1: Controller parameters for different Process parameters

E	K_p	L	T	L/T	K_c (opt) Matlab
0.38	0.5	0.1	1	0.1	11.7157
0.381	0.5	0.5	1	0.5	2.1989
0.387	0.5	2	1	2	0.0112
0.3872	0.5	1	2	0.5	2.2008
0.392	0.5	2	2	1	0.009
0.401	0.5	4	2	2	0.0058
0.3881	0.5	0.1	3	0	39.4482
0.3921	0.5	1	3	0.3	0.0115
0.416	0.5	6	3	2	0.0039
0.3942	0.5	0.5	4	0.1	8.8396
0.3963	0.5	1	4	0.3	4.2533
0.4111	0.5	4	4	1	0.0047
0.422	0.5	6	4	1.5	0.0036

0.433	0.5	8	4	2	0.003
0.363	1	1	1	1	0.009
0.37	1	2	1	2	0.0059
0.3701	1	1	2	0.5	0.0069
0.377	1	2	2	1	0.0047
0.3943	1	4	2	2	0.0029
0.378	1	1	3	0.3	0.0057
0.3863	1	2	3	0.7	0.8715
0.4044	1	4	3	1.3	0.0026
0.424	1	6	3	2	0.002
0.3821	1	0.5	4	0.1	4.0914
0.3864	1	1	4	0.3	2.1267
0.3912	1	1.6	4	0.4	1.3568
0.3952	1	2	4	0.5	1.1018
0.437	1	6	4	1.5	0.0018
0.464	1	8	4	2	0.0015
0.344	1.5	1	1	1	0.006
0.348	1.5	0.5	2	0.3	1.3646
0.3531	1.5	1	2	0.5	0.0044

0.3551	1.5	0.1	3	0	9.4915
0.3742	1.5	2	3	0.7	0.0027
0.3703	1.5	0.5	4	0.1	0.0046
0.3751	1.5	1	4	0.3	0.0033
0.45	1.5	6	4	1.5	0.0012
0.497	1.5	8	4	2	0.00099
0.318	2	0.1	1	0.1	0.0174
0.337	2	2	1	2	0.0029
0.3281	2	0.1	2	0.1	4.7457
0.3371	2	1	2	0.5	0.0033
0.3491	2	2	2	1	0.0024
0.3752	2	4	2	2	0.0015
0.3501	2	1	3	0.3	0.0027
0.3621	2	2	3	0.7	0.002
0.3532	2	0.1	4	0	9.4914
0.3642	2	1	4	0.3	1.0633
0.3792	2	2	4	0.5	0.0018
0.4132	2	4	4	1	0.0012

Using carve fitting techniques the tuning rule are found as shown below.
First a parameter (M_T) which describes the relation between the optimum value of the controller gain (K_c (opt)) and the average of the estimated error is found as follow:-

$$M_T = (0.25502 * EXP((0.0019648/(K_c * K_P)) - 0.32209 * K_c * K_P))/E \;.....(18)$$

Second a use of the parameter M_T to obtain the controller parameters is made as follow:-

$$K_c = \frac{0.2 + \frac{0.45}{M_T}}{K_p} \;......(19)$$

$$T_i = \frac{\sqrt{\frac{E - 0.305}{0.00328827}}}{4} \;......(20)$$

$$T_d = \frac{\left(\left(\frac{1}{E}\right) + 2.103\right)}{20.414} \dots\dots(21)$$

Where $K_i = \dfrac{K_c}{T_i}$ and $K_d = K_c * T_d$

If the value of $K_c \geq 50)$ then scale down the parameters value by factor equal to ($K_1(d$)) . The new controller parameters will be equal to:-

$$K_c = \frac{K_c}{K_d}) \quad \text{And} \quad K_i = \frac{K_i}{K_d}) \quad \text{and} \quad (K_1(d\ ) = 1)$$

6. MATLAB SIMULATION RESULTS

Several processes were taken under test to simulate the efficiency of the proposed tuning rules. All processes were Fist order Plus Dead Time. A reduction procedure is used to modulate the higher order models in the FOPDT model
The processes which are used in the simulation are:-

$$G_1(s) = \frac{1}{(s^2 + 1.4s + 1)} = \frac{0.99 * e^{-0.73s}}{0.885s + 1} \dots\dots(22)$$

$$G_2(s) = \frac{10 * e^{-S}}{(s+1)(s+2)(s+3)(s+4)} = \frac{0.4169 * e^{-2.16s}}{1.1696s + 1} \dots(23)$$

$$G_3(s) = \frac{10}{(s+1)(s+2)(s+3)(s+4)} = \frac{2.941 * e^{-0.76s}}{1.96s + 1} \dots(24)$$

$$G_4(s) = \frac{0.5 * e^{-0.5s}}{2s + 1} \dots\dots(25)$$

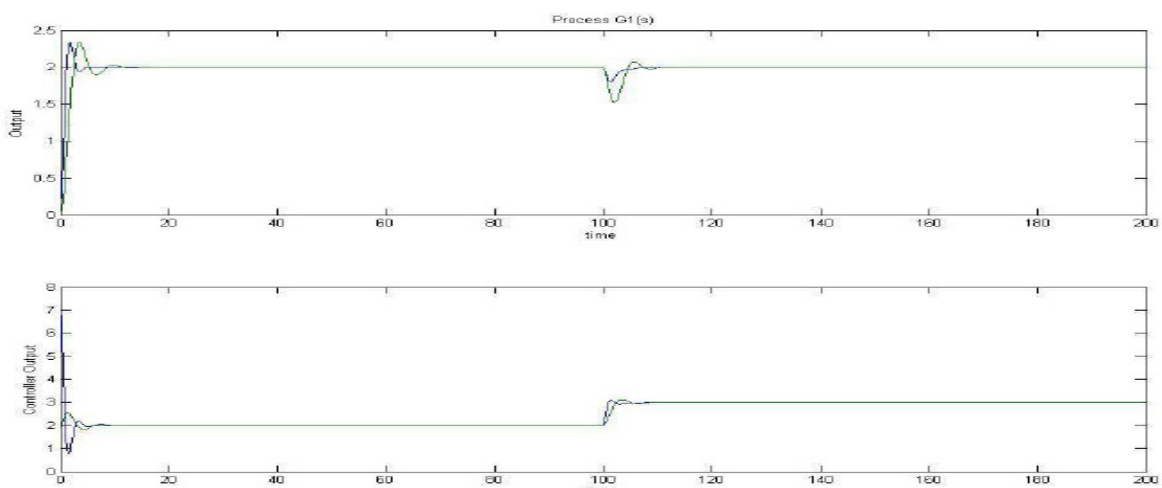

Figure 1: Second order process Step response

Table 2: The response parameters values of AMIGO and Proposed tuning rule for the process $G_1(s)$.

Algorithm	Rise time (s)	Settling Time (s)	Set point overshoot	IAE (disturbance)
AMIGO	2.3	5.00	15.08%	11.8479
Proposed tuning rule	1.1	2.5	16.82%	4.9772

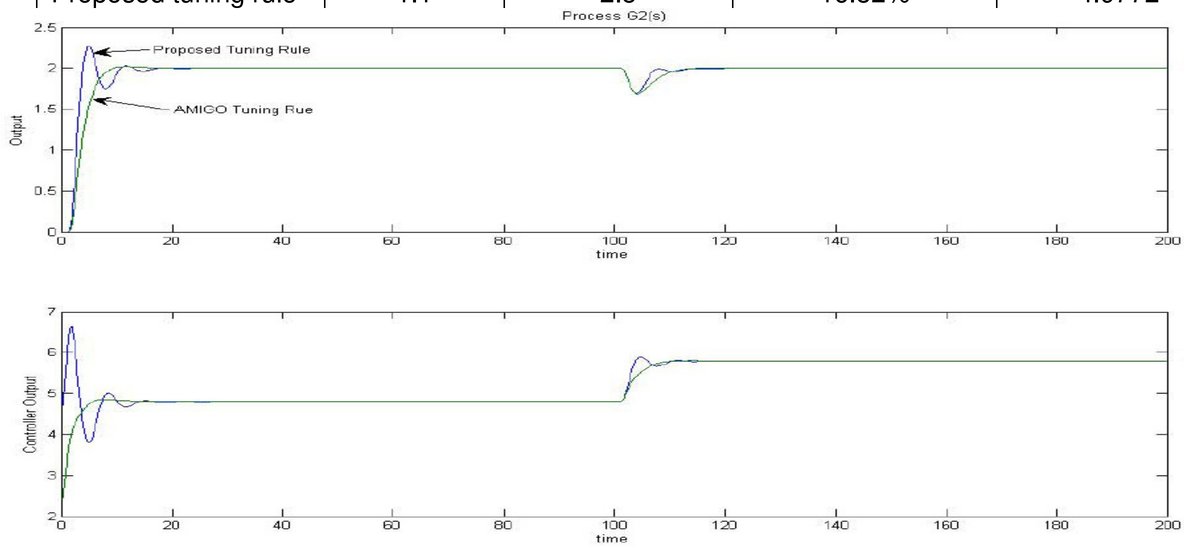

Figure 2: High order process with delay Step response

Table 3: The response parameters values of AMIGO and Proposed tuning rule for the process $G_2(s)$.

Algorithm	Rise time (s)	Settling Time (s)	Set point overshoot	IAE (disturbance)
AMIGO	6.2	7.3	0.9%	11.95
Proposed tuning rule	3.6	9.8	13.6%	16.03

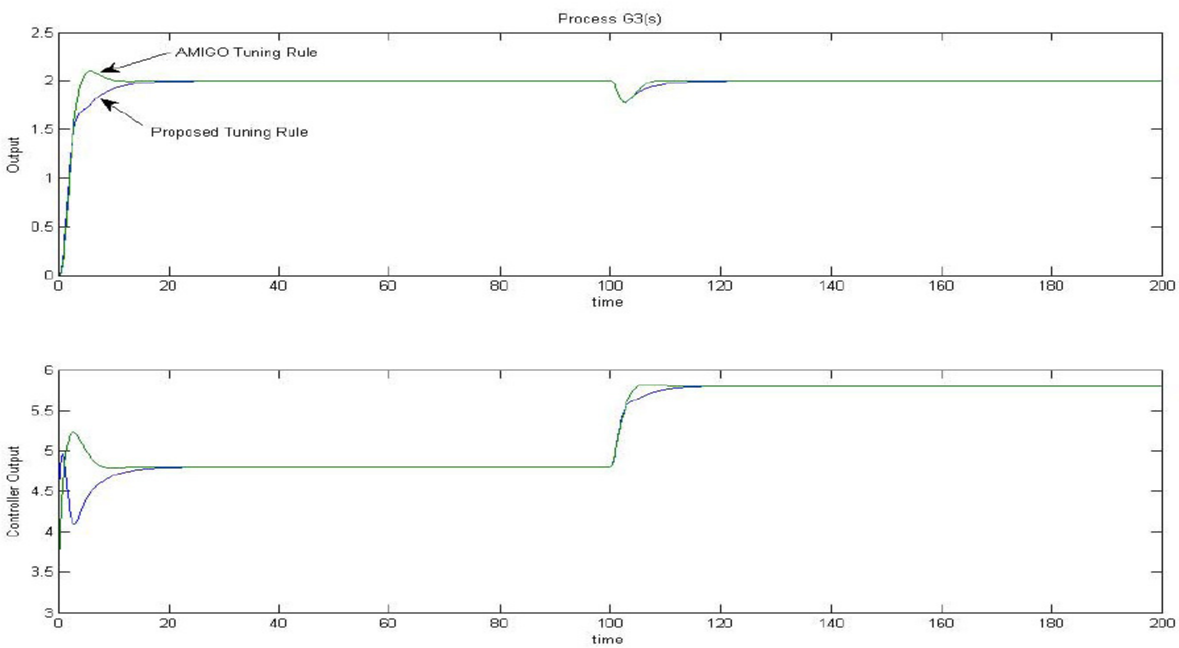

Figure 3: High order process without delay Step response

Table 4: The response parameters values of AMIGO and Proposed tuning rule for the process $G_3(s)$.

Algorithm	Rise time (s)	Settling Time (s)	Set point overshoot	IAE (disturbance)
AMIGO	6.2	3.8	4.9%	7.93
Proposed tuning rule	3.4	9.2	0.0%	11.74

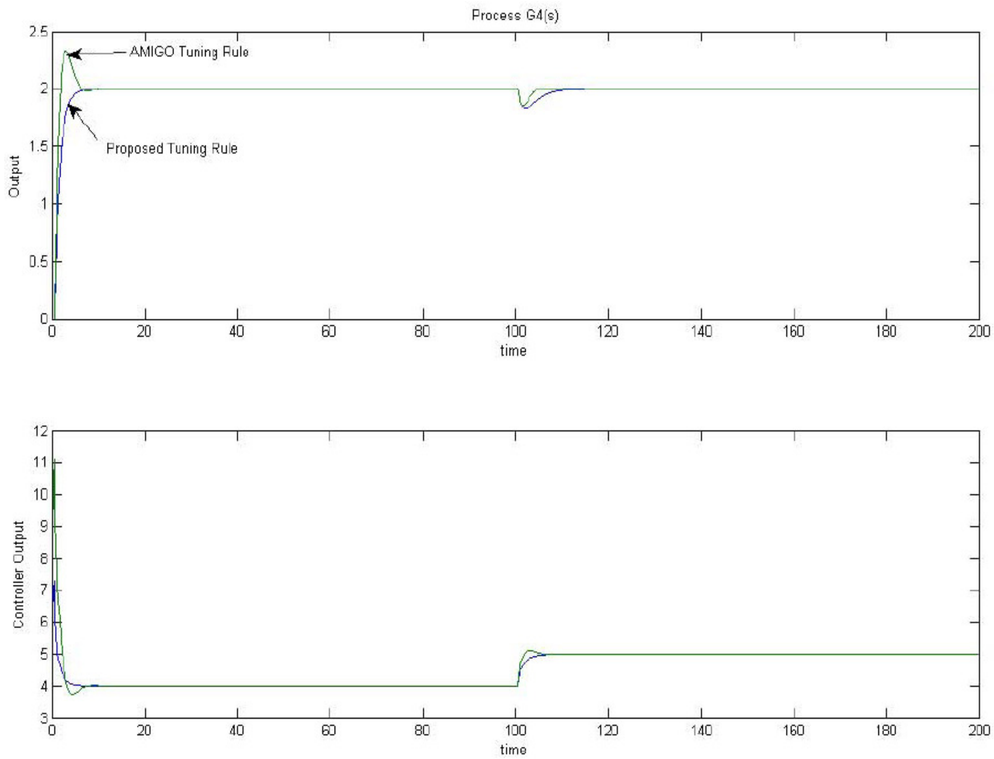

Figure 4: First order process with delay (FOPDT) Step response

Table 5: The response parameters values of AMIGO and Proposed tuning rule for the process $G_4(s)$.

Algorithm	Rise time (s)	Settling Time (s)	Set point overshoot	IAE (disturbance)
AMIGO	1.6	5	16.76%	3.21
Proposed tuning rule	3.0	4	0.0%	8.16

7. CONCLUSION

Based on the results of the simulations for the process models chosen for this analysis, conclusion can be drawn out that the proposed tuning rule may give slow but highly robust and stable response and no overshoot for most processes while it may give a fast response but in the expense of high overshoot and long settling time. The proposed tuning rule is the easiest to apply to find the necessary settings, and is applicable to a wide range of process types. Also, over a range of modified parameters, the proposed tuning rules can be improved slightly, but only by sacrificing stability and robustness. The most important advantage of this design is in the use of the IATE performance criteria index along with the grey prediction algorithm to find the new tuning rule since it can provide the controller with a good performance and also it eliminates the need of knowing the process parameters since we only need to know the average of the estimated error and the process gain K_c. As it appears from the simulation, the proposed tuning rule is able to deal with the possible variation of system parameters. It is so obvious that the proposed tuning rule has the same or better performance than AMIGO tuning rule. The observation from those results shows that a high overshoot appears in the output of the system for some cases of processes. This overshoot appears as expense of achieving a high response and a better load disturbance rejection. The proposed tuning rule is also able to deal with the process parameters change since the approximation of FOPDT is not accurate. The limitation of this proposed tuning rule is concentrate in the initial value of the PID controller because it must lead the system to behave under damp behavior. The concluded important contributions in this paper regarding the use of the proposed tuning rule are that it proves the ability of the proposed tuning rule in tuning the PID controller probably with only need of small information about the process parameters (only K_p is needed). Also it validates the flexibility of the proposed tuning rule to deal with different modeling systems with different parameters.

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