Radiation, Chemical Reaction, Double Dispersion effects on Heat and mass transfer in Non-Newtonian fluids

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Abstract

Radiation and chemical reaction effects on heat and mass transfer in non-Darcy non-Newtonian fluid over a vertical surface is considered. In this article we have maintained the constant temperature. A mathematical model is developed taking into the account the new elements introduced. Numerical solutions for the governing nonlinear momemtum, energy and concentration are obtained.The governing boundary layer equations and boundary conditions are simplified by using similarity transformations. The governing equations are solved numerically by means of Fourth-order Runge-Kutta method coupled with double-shooting technique. The influence of viscosity index n, thermal and solute dispersion, velocity, temperature, concentration, Heat and mass transfer rates are discussed.

Keywords: Radiation, Chemical reaction, double dispersion, free convection, heat and mass transfer.

1. INTRODUCTION

Transport processes in porous media play significant roles in various applications such as geothermal engineering, thermal insulation, energy conservation, petroleum industries solid matrix heat exchangers, chemical catalytic reactor, underground disposal of nuclear waste materials and many others. Convective heat transfer from impermeable surfaces embedded in porous media has numerous thermal engineering applications such as geothermal systems, crude oil extraction, thermal insulating and ground water pollution.Nield and Bejan [1] and Pop and Ingham [2] have made comprehensive reviews of the studies of heat transfer in relation to the above applications. Radiation effects on convection can be quite important in the context of many industrial applications involving high temperatures such as nuclear power plant, gas turbines and various propulsions engines for aircraft technology.Ali et.al [3] studied the natural convection-radiation interaction in boundary layer flow over a horizontal surfaces.Hossain and Pop [4] considered the effect of radiation on free convection of an optically dense viscous incompressible fluid along a heated inclined flat surface maintained at uniform temperature placed in a saturated porous medium .Yih [5] investigated the radiation effect on the mixed convection flow of an optically dense viscous fluid adjacent to an isothermal cone embedded in a saturated porous medium. Bakier [6] analyzed the effect of radiation on mixed convection from a vertical plate in a saturated medium.Cheng and Minkowycz [7] presented similarity solutions for free convective heat and mass transfer from a vertical plate in a fluid saturated porous medium. Chen and Chen [8] studied free convection from a vertical wall in a non-Newtonian fluid saturated porous medium. Mehta and Rao [9] studied the free convection heat transfer in a porous medium past a vertical flat plate with non-uniform surface heat flux at the wall. Nakayama and Koyama [10] analyzed the more general case of free convection over a non-isothermal body of arbitrary shape embedded in a porous medium. Anjalidevi and Kandasamy [11] studied the effects caused by the chemical-diffusion mechanisms and the inclusion of a general chemical reaction of order n on the combined forced and natural convection flows over a semi-infinite vertical plate immersed in an ambient fluid. They stated that the presence of pure air or water is impossible in nature and some foreign mass may present either naturally or mixed with air or water.El-Amin [12] studied the effects of chemical reaction and double dispersion on non-Darcy free convective heat and mass transfer in porous medium. The effects of double dispersion on natural convection heat and mass transfer in a non-Newtonian fluid saturated non-Darcy porous medium has been investigated by Murthy et.al [13]. Murti and Kameswaran [14] analyzed the effects of Radiation, chemical reaction and double dispersion on heat and mass transfer in non-Darcy free convective flow. The study by Chamka [15] of MHD flow over a uniformly stretched vertical permeable surface used similar assumptions for the chemical reaction.

The objective of this paper is to study the effects of double dispersion on natural convection heat and mass transfer in non-Darcy, non-Newtonian fluid with Radiation and chemical reaction along the vertical surface.

2. NOMENCLATURE

C Concentration
d Pore diameter
D Mass diffusivity
f Dimensionless stream function
g Gravitational acceleration
Gc Modified Grashof number
k Molecular thermal conductivity
K Permeability of the porous medium
K_0 Chemical reaction parameter
k_d Dispersion thermal conductivity
k_e Effective thermal conductivity
Le Lewis number
m Order of reaction
N Buoyancy ratio
Nu_x Local Nusselt number

$\Pr$ Prandtl number
q Heat transfer rate
Ra Ralyleigh number
Re_x Local Reynolds number
Sc Schmidt number
w Surface conditions
μ^* Fluid consistency of the in elastic non-Newtonian power law fluid
Gr^* Non-Darcy parameter
ε Porosity of the saturated porous medium
m Order of reaction
j_w Local mass flux
T Temperature
u, v Velocity components in the x and y directions
u_r Reference velocity
x , y Axes along and normal to the plate
α Molecular thermal diffusivity
σ Electrical conductivity. mho
α_d Dispersion diffusivity
α_x, α_y Components of the thermal diffusivity in x and y directions
β Thermal expansion coefficient
β^* Solutal expansion coefficient
χ Non-dimensional chemical reaction-porous media parameter
ϕ Dimensionless concentration
γ Mechanical thermal-dispersion coefficient
η Similarity space variable
R Radiation parameter $\dfrac{16av\sigma_R T_\infty^2}{\rho C_p U_\infty^2}$
λ Non-dimensional chemical reaction parameter
μ Fluid dynamic viscosity
υ Fluid kinematic viscosity
θ Dimension less temperature
ρ Fluid density
ψ Stream function
ζ Mechanical solutal-dispersion coefficient
∞ Conditions away form the surface
k^* Intrinsic permeability of the porous medium for flow of power law fluid
b Coefficient of Forchheimer term
σ_0 Stefan-Boltzman constant
k_1^* Mean absorption coefficient

3. MATHEMATICAL FORMULATION

The chemical reaction of order m on natural convective heat and mass transfer in a non-Darcian porous medium saturated with a non-Newtonian fluid adjacent to a vertical surface is considered. The co-ordinate system $x \to y$ is attached to the vertical surface as shown in Fig.1.

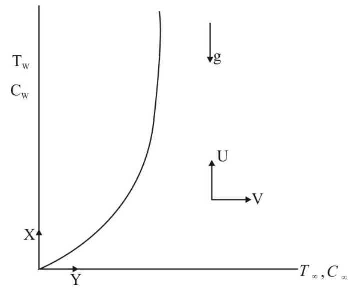

Figure 1. Schematic diagram of the problem

The x axis is taken along the plate and y axis is normal to it. The wall is maintained at constant temperature T_w and concentration C_w respectively. Taking into account the effects of thermal dispersion, the governing equations for steady non-Darcy flow in a saturated porous medium can be written as follows.

Continuity Equation:

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

Momentum Equation:

$$\frac{\partial u^n}{\partial y}+\rho_\infty\frac{bk^*}{\mu^*}\frac{\partial u^2}{\partial y}=\frac{\rho_\infty g k^*}{\mu^*}\left(\beta_T\frac{\partial T}{\partial y}+\beta_C\frac{\partial C}{\partial y}\right) \tag{2}$$

Energy Equation :

$$u.\frac{\partial T}{\partial x}+v.\frac{\partial T}{\partial y}=\frac{\partial}{\partial y}\left(\alpha_y\frac{\partial T}{\partial y}\right)-\frac{1}{\rho C_p}\frac{\partial q}{\partial y} \tag{3}$$

where $\frac{\partial q}{\partial y}=-16a\sigma R T_\infty^3\left(T_\infty-T\right)$

Concentration Equation:

$$u.\frac{\partial C}{\partial x}+v.\frac{\partial C}{\partial y}=\frac{\partial}{\partial y}\left(D_y\frac{\partial C}{\partial y}\right)-K_0\left(C-C_\infty\right)^m \tag{4}$$

$$\rho=\rho_\infty\left[1-\beta\left(T-T_\infty\right)-\beta^*\left(C-C_\infty\right)\right] \tag{5}$$

where u and v are velocities in the $x-y$ directions respectively, T is the temperature in the thermal boundary layer, K is the permeability constant. β_T and β_C are the coefficients of thermal and solutal expansions, ρ_∞ is the density at some reference point, g is the acceleration due to gravity. The energy equation includes radiation heat transfer effect with Jowl heating. The radiative heat flux term q is written using the Rosseland approximation as $q=-\frac{4\sigma_0}{3k_1^*}\frac{\partial T^4}{\partial y}$, where σ_0 and k_1^* are Stefan-Boltzman constant and the mean absorption coefficient respectively. The chemical reaction effect is added as the last-term in the right hand side of equation (4) where the power m is the order of reaction. It is assumed that the normal component of the velocity near the boundary is small compared with the other components of the velocity and the derivatives of any quantity in the normal direction are large compared with derivatives of the quantity in the direction of the wall.

The necessary boundary conditions for this problem are

$$\left.\begin{aligned} & y=0\ ;\ v=0,\ T=T_w, C=C_w \\ & y\to\infty ; u=u_\infty, T=T_\infty, C=C_\infty \end{aligned}\right\} \tag{6}$$

The quantities of α_y and D_y are variables defined as $\alpha_y=\alpha+\gamma d\left|\mathbf{v}\right|$ and $D_y=D+\zeta\, d\left|\mathbf{v}\right|$ represent thermal dispersion and solutal diffusivity, respectively (Telles and Trevisan [16]).The value of these quantities lies between $\frac{1}{7}$ and $\frac{1}{3}$, Also b is the empirical constant associated with the Forchheimer porous inertia term and μ^* is the consistency index, the modified permeability of the flow k^* of the non-Newtonian power law fluid is defined as

$$k^*=\frac{1}{2c_t}\left(\frac{n\varepsilon}{3n+1}\right)^n\left(\frac{50K}{3\varepsilon}\right)^{\frac{n+1}{2}} \tag{7}$$

where

$$K=\frac{\varepsilon^3 d^2}{150\left(1-\varepsilon\right)^2} \quad \text{and}$$

$$c_t=\begin{cases} \dfrac{25}{12} \\ \dfrac{2}{3}\left(\dfrac{8n}{9n+3}\right)^n\left(\dfrac{10n-3}{6n+1}\right)\left(\dfrac{75}{16}\right)^{\frac{3(10n-3)}{10n+11}} \end{cases}$$

and for $n=1, c_t=\dfrac{25}{12}$

Introducing the stream function ψ such that $u=\dfrac{\partial\psi}{\partial y}$ and $v=-\dfrac{\partial\psi}{\partial x}$.

we introduce similarity variables as

$$\psi=f\left(\eta\right)\alpha\sqrt{Ra_x}\,,\eta=Ra_x^{\frac{1}{2}}.\frac{y}{x}\ ,\ \theta\left(\eta\right)=\frac{T-T_w}{T_w-T_\infty},\ \phi\left(\eta\right)=\frac{C-C_w}{C_w-C_\infty}$$

where $Ra_x=\dfrac{x}{\alpha}\left[\dfrac{\rho_\infty k^* g\beta_T\left(T_w-T_\infty\right)}{\mu^*}\right]^{\frac{1}{n}}$

The above transformation reduces the system of partial differential equations in to the system of ordinary differential equations

$$\left(n\,f^{'n-1}+2Gr^*f^{'}\right)f^{''}=\theta^{'}+N\phi^{'} \tag{8}$$

$$\theta^{''}+\frac{1}{2}f\theta^{'}+\gamma Ra_d\left(f^{'}\theta^{''}+f^{''}\theta^{'}\right)+\frac{4R}{3}\left[3\theta^{'2}\left(\theta+c_r\right)^2+\theta^{''}\left(\theta+c_r\right)^3\right]=0 \tag{9}$$

$$\phi^{''}+\frac{1}{2}Lef\phi^{'}+\zeta Ra_d\,Le\left(f^{'}\phi^{''}+f^{''}\phi^{'}\right)-Sc\lambda\frac{Gc}{\mathrm{Re}_x^2}\phi^n=0 \tag{10}$$

where the primes denote the differentiation with respect to the similarity variable η.

$Ra_d = \frac{d}{\alpha}\left(\frac{k^* \rho_\infty g \beta_T \theta_w}{\mu^*}\right)^{\frac{1}{n}}$ is the modified pore-diameter-dependent Ralyleigh number, and $N = \frac{\beta^*(C_w - C_\infty)}{\beta(T_w - T_\infty)}$ is the buoyancy ratio parameter, $R = \frac{4\sigma\theta_w^3}{kk_1^*}$ is the conduction radiation parameter. With analogy to Mulolani and Rahman [17]. Aissa and Mohammadein [18], we define Gc to be the modified Grashof number, Re_x is the local Reynolds number, Sc and λ are the Schmidt number and non-dimensional chemical reaction parameter as

$$Gc = \frac{\beta^* g (C_w - C_\infty)^2 x^3}{\upsilon^2},\ \mathrm{Re}_x = \frac{u_r x}{\upsilon},\ Sc = \frac{\upsilon}{D},\ \lambda = \frac{K_0 \alpha d (C_w - C_\infty)^{n-3}}{kg\beta^*}$$

$Gr^* = b\left(\frac{k^{*2}\rho_\infty^2 [g\beta_T\theta_w]^{2-n}}{\mu^{*2}}\right)^{\frac{1}{n}}$ is the non-Darcy Parameter or Grashof number based on permeability for power law fluid. where the diffusivity ratio Le is the ratio of Schmidt number and Prandtl number and $u_r = \sqrt{g\beta d (T_w - T_\infty)}$ is the reference velocity as defined by Elbashbeshy [19]. The equation (10) can be written as

$$\phi'' + \frac{1}{2} Lef\phi' + \zeta Ra_d\, Le\left(f'\phi'' + f''\phi'\right) - \chi\phi^n = 0 \tag{11}$$

With analogy to Prasad et. al [20], Aissa and Mohammadein [18], the non-dimensional chemical reaction parameter χ is defined as $\chi = \frac{Sc\lambda Gc}{\mathrm{Re}_x^2}$.

and the boundary conditions become

$$\left.\begin{array}{l} f(0) = 0,\ \theta(0) = \phi(0) = 1 \\ f'(\infty) = 1, \theta(\infty) = \phi(\infty) = 0 \end{array}\right\} \tag{12}$$

It is noted that $Gr^* = 0$ corresponds to the Darcian free convection, thermal dispersion effect $\gamma = 0$ and the solute-dispersion effects $\zeta = 0$ are neglected. In equation (8) $N > 0$ indicates the aiding buoyancy and $N < 0$ indicates the opposing buoyancy. On the other hand from the definition of stream function, the velocity components become $u = \frac{\alpha Ra_x}{x} f'$ and $v = -\frac{\alpha Ra_x^{\frac{1}{2}}}{2x}\left[f - \eta f'\right]$.

The local heat transfer rate from the surface of the plate is given by

$$q_w = -k_e\left[\frac{\partial T}{\partial y}\right]_{y=0}. \tag{13}$$

The local Nusselt number

$$Nu_x = \frac{q_w x}{(T_w - T_\infty) k_e} \tag{14}$$

where k_e is the effective thermal conductivity of the porous medium which is the sum of the molecular and thermal conductivity k and the dispersion thermal conductivity k_d .Substituting $\theta(\eta)$ and equation (13) in equation (14) the modified Nusselt number is obtained as

$\frac{Nu_x}{(Ra_x)^{\frac{1}{2}}} = -\left[1+\gamma Ra_d f^{'}(0)\right]\theta^{'}(0)$ .Also the local mass flux at the vertical wall is given by $j_w = -D_y\left(\frac{\partial C}{\partial y}\right)_{y=0}$ defines dimensionless variable and the local Sherwood number is $\frac{Sh_x}{(Ra_x)^{\frac{1}{2}}} = -\left[1+\zeta\, Ra_d f^{'}(0)\right]\phi^{'}(0)$.

4. SOLUTION PROCEDURE

The dimensionless equations (8) (9) and (11) together with the boundary conditions (12) are solved numerically by means of the fourth order Runge-Kutta method coupled with double shooting technique. By giving appropriate hypothetical values for $f^{'}(0), \theta^{'}(0)$ and $\phi^{'}(0)$ we get the corresponding boundary conditions at $f^{'}(\infty)$, $\theta(\infty)$, $\phi(\infty)$ respectively. In addition, the boundary condition $\eta \to \infty$ is approximated by $\eta_{\max} = 4$ which is found sufficiently large for the velocity and temperature to approach the relevant free stream properties. This choice of $\eta_{\max}$ helps in comparison of the present results with those of earlier works.

5. RESULTS & DISCUSSION

The chemical reaction Natural convective problem with radiation effect from a vertical plate with constant wall temperature is analyzed considering both buoyancy aiding and opposing flows. The results obtained are prescribed in graphical form for selected non-dimensional groups having chemical reaction effect, radiation and double dispersion through Figures 2-9. Variation of temperature and velocity within the boundary along with heat and mass transfer characteristics are prescribed in sequential order. The heat transfer coefficient values obtained here for various values of power law index with $\gamma = 0, \varsigma = 0, N = 0, Gr^* = 0, \chi = 0, R = 0$ match well with those tabulated values in Chen and Chen [8] .When $R = 0, \chi = 0$ the problem reduces to effect of double dispersion on natural convection heat and mass transfer in Non-Newtonian fluid saturated non-Darcy porous medium. Results obtained here are good in agreement with Chen and Chen [8] and Murthy [13]. The following values for parameters are considered for discussion. $0.5 \le n \le 1.5$, $0 \le Ra_d \le 3$, $\gamma = 0, 0.3$, $\varsigma = 0, 0.3$, $0 \le Le \le 4$, , $0 \le Gr^* \le 0.3$ $0 \le \chi \le 0.08$.

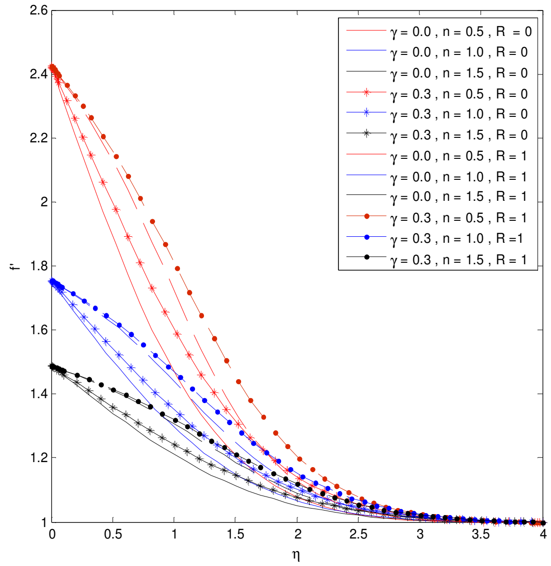

Fig. 2. Variation of velocity with similarity space variable η (opposing)

$$Gr^{*} = 0.07, N = -0.1, Le = 0.5, Ra_{d} = 0.7, \xi = 0, \chi = 0.02$$

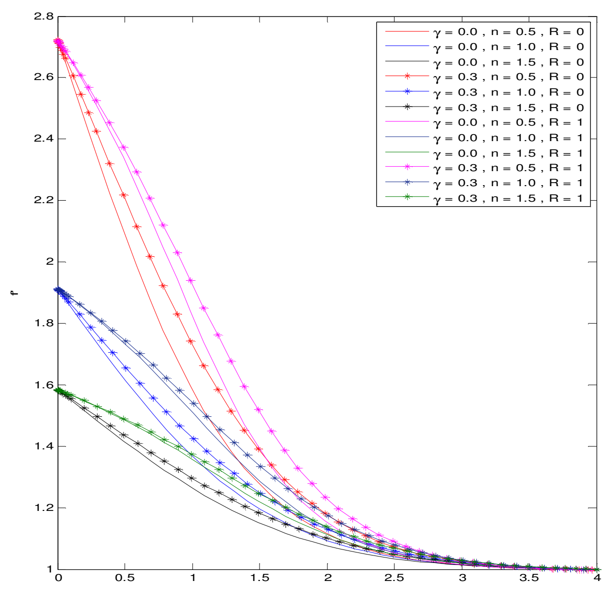

Fig. 3. Variation of velocity with similarity space variable η (aiding)

$$Gr^{*} = 0.07, N = 0.1, Le = 0.5, Ra_{d} = 0.7, \xi = 0, \chi = 0.02$$

Figures 2 and 3 describe the velocity variation in the boundary layer with applied chemical reaction and radiation effects. It is found that for a given chemical reaction parameter, with increase in radiation the velocity decreases. However with increase in radiation parameter, the velocity decreases with in the boundary at a given location, velocity decreases with increase in thermal dispersion in presence of radiation. But this decrement is less in presence of Radiation than absence. The same trend is observed in opposing flow also.

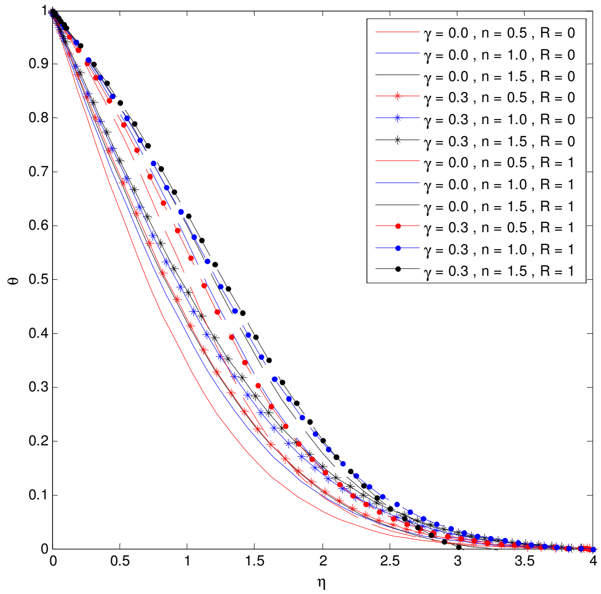

Fig. 4. Effect of temperature with similarity space variable η (opposing)

$$Gr^{*} = 0.07, N = -0.1, Le = 0.5, Ra_{d} = 0.7, \xi = 0, \chi = 0.02$$

Figure 4. shows that the effect of non-dimensional temperature profiles for various values of power law index n and γ for fixed values of other parameters in opposing case. It is clearly seen that in the aiding case increasing in power law index tend to increase the boundary layer thickness. With increase in radiation a rise in temperature distribution in the boundary layer is seen.

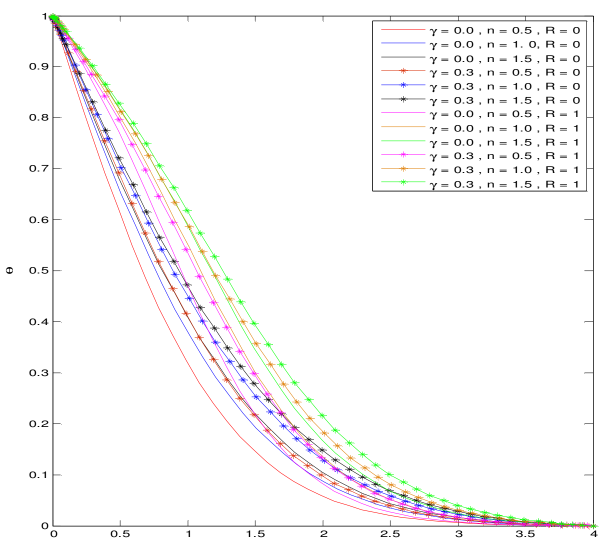

Fig. 5. Effect of temperature with similarity space variable η (aiding)

$$Gr^{*} = 0.07, N = 0.1, Le = 0.5, Ra_{d} = 0.7, \xi = 0, \chi = 0.02$$

Figure 5. shows that the effect of non-dimensional temperature profiles for various values of power law index n and γ for fixed values of other parameters in aiding case. With increase in

radiation parameter temperature within the boundary increases. However for a fixed value of radiation parameter temperature increases at a particular location.

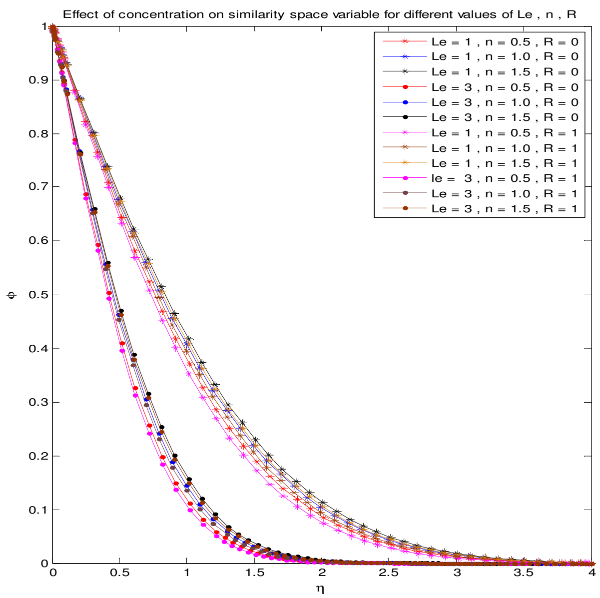

Fig. 6. Effect of concentration with similarity space variable η

$$Gr^{*} = 0.21, N = -0.1, \gamma = 0, , \xi = 0, \chi = 0.02, Ra_{d} = 0.7$$

Figure 6.shows that the effect of radiation and natural convection on concentration profile. Concentration with in the boundary decreases with increase in radiation parameter. Further with increase in power law index also the temperature increases. Further it indicates that the effect of radiation parameter on concentration profile is considerable only at the middle of the boundary and negligible at the end of the plate and at the edge of the boundary.

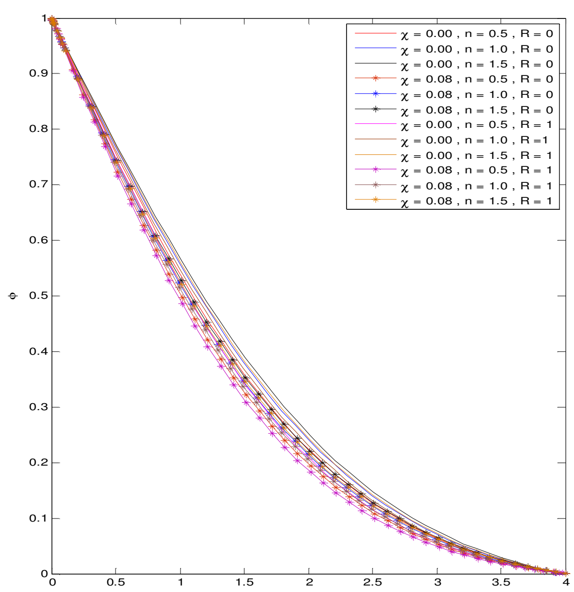

Fig. 7. Effect of concentration with similarity space variable η

$$Gr^{*} = 0.21, N = -0.1, \gamma = 0, \xi = 0, Le = 0.5, Ra_{d} = 0.7$$

From Figure 7. we observe that concentration increases with increase in power law index in presence of radiation r. For a fixed value of chemical reaction parameter concentration decreases with increase in radiation parameter.

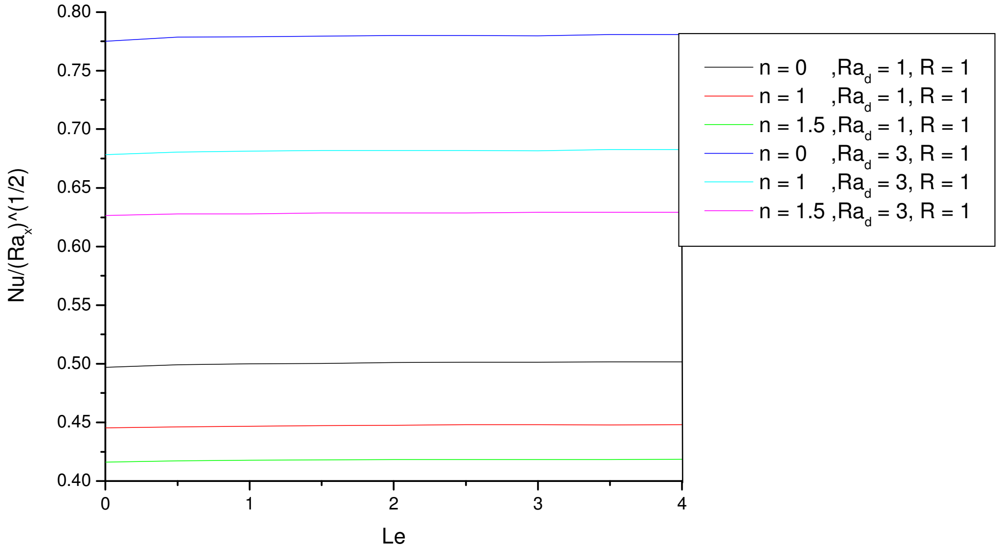

Fig. 8. Effect of Lewis number on Nusselt number

$$Gr^* = 0.21, N = -0.1, \gamma = 0.3, \xi = 0.3, Le = 0.5, \chi = 0.02$$

From Figure 8. we observe that for a fixed value of Ra_d nusselt number decreases with increase in power law index. Increase in Ra_d nusselt number decreases. Hence the heat transfer coefficient decreases with increase in power law index.

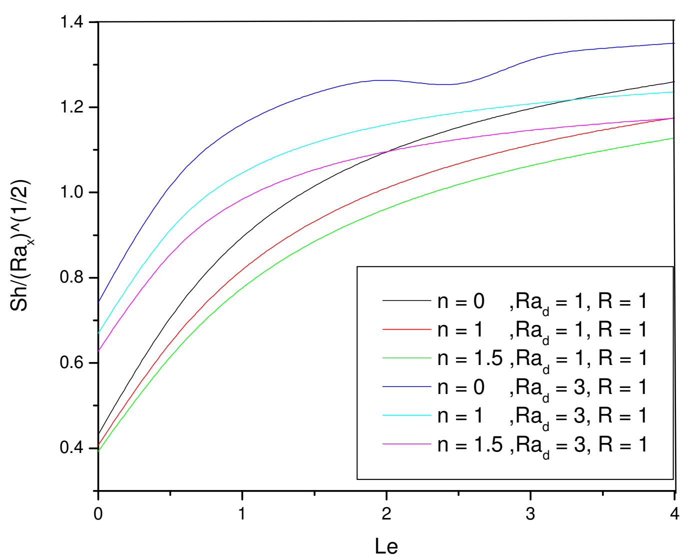

Fig. 9. Effect of Lewis number on Sherwood number

$$Gr^* = 0.21, N = -0.1, \gamma = 0.3, \xi = 0.3, Le = 0.5, \chi = 0.02$$

From Figure 9. we observe that for a fixed value of Ra_d Sherwood number decreases with increase in power law index. Increase in Ra_d Sherwood number decreases. Mass transfer coefficient increases with increase in power law index.

6. CONCLUSIONS

Increase in radiation parameter, the velocity decreases with in the boundary. Increase in radiation a rise in temperature distribution in the boundary layer. Concentration increases with increase in power law index in presence of radiation parameter.Nusselt number decreases with increase in Ra_d parameter. Mass transfer coefficient increases and the heat transfer coefficient decreases with increase in power law index.

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