CFD and Artificial Neural Networks Analysis of Plane Sudden Expansion Flows

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Abstract

It has been clearly established that the reattachment length for laminar flow depends on two non-dimensional parameters, the Reynolds number and the expansion ratio, therefore in this work, an ANN model that predict reattachment positions for the expansion ratios of 2, 3 and 5 based on the above two parameters has been developed. The R^2 values of the testing set output Xr1, Xr2, Xr3, and Xr4 were 0.9383, 0.8577, 0.997 and 0.999 respectively. These results indicate that the network model produced reattachment positions that were in close agreement with the actual values. When considering the reattachment length of plane sudden-expansions the judicious combination of CFD calculated solutions with ANN will result in a considerable saving in computing and turnaround time. Thus CFD can be used in the first instance to obtain reattachment lengths for a limited choice of Reynolds numbers and ANN will be used subsequently to predict the reattachment lengths for other intermediate Reynolds number values. The CFD calculations concern unsteady laminar flow through a plane sudden expansion and are performed using a commercial CFD code STAR-CD while the training process of the corresponding ANN model was performed using the *NeuroShell*^{TM} simulator.

Keywords: Sudden Expansion, ANN, CFD, Reattachment, Laminar flow.

1. INTRODUCTION

When a fluid is flowing through a plane sudden-expansion duct, the boundary layers separate at the expansion plane because of a singularity in the wall geometry. The separating shear layer will reattach at a downstream location that varies with the Reynolds number defined as $\mathrm{Re} = Ud/\nu$ and the expansion ratio equal to D/d, where D and d refer to the downstream and upstream duct respectively, see Figure 1. At reattachment, part of the shear layer is deflected upstream into the recirculation zone and the other part proceeds downstream as a recovering boundary layer. The behavior of such flows and especially the reattachment lengths downstream

of the expansion plane are important topics of consideration for design of fluidic devices, heat-exchangers and mixing systems.
It is known that laminar flow downstream of a sudden expansion remains symmetric with two equal separation zones on either side of the centerline of the duct for low Reynolds numbers. However, beyond a critical value of the Reynolds number the flow becomes asymmetric, producing two or three unequal separation zones. The exact underlying phenomenon is still not well understood but is thought to be due to a Coanda like phenomenon and the appearance of asymmetric flow is thought to coincide with a bifurcation of the Navier-Stokes equations. It has been found both experimentally and numerically that the value of the critical Reynolds number beyond which the flow behavior becomes asymmetric is dependent on the expansion ratio. Of particular interest in these flows, is the variation of reattachment lengths with Reynolds number and expansion ratio.

Plane sudden expansion flows have been investigated experimentally by Durst et al [1], Cherdron et al [2] and Fearn et al. [3]. These investigations included flow visualisation and fluid velocity measurements by laser Doppler anemometry. Numerical investigations were also conducted by Durst et al [1], Fearn et al [3] and Battaglia et al [4]. The techniques used were generally based on time integration by finite difference/element techniques of the incompressible form of the Navier-Stokes equations. The calculations of these flows with mesh sizes necessary to resolve the flow field, provide grid-independent solutions and to obtain the salient features of the flow for every Reynolds number can be very time consuming. For example, using a Computational Fluid Dynamic (CFD) commercial code, around 27 hours of computing time on a SUN-20 workstation, are necessary to obtain a converged solution for a single Reynolds number.

ANNs are computer models that are trained in order to recognize both linear and non-linear relationships among the input and the output variables in a given data set. In general, ANN applications in engineering have received wide acceptance. The popularity and acceptance of this technique stems from its features, which are particularly attractive for data analysis. These features include handling of fragmented and noisy data, speed inherent to parallel distributed architectures, generalization capability over new data, ability to effectively incorporate a large number of input parameters, and its capability of modeling non-linear systems. In general, ANN models consists of three basic elements: an organized architecture of interconnected processing elements, a method for encoding information during training, and a method for recalling information during testing. Simpson (1990) provides a coherent description of these elements and presents comparative analyses, applications, and implementation of 27 different ANN paradigms. This tool can be used by mechanical engineers in conjunction with other analytical and graphical techniques for data analysis, optimisation and model evaluation.

When considering the reattachment length of sudden-expansions the judicious combination of CFD calculated solutions with ANN will result in a considerable saving in computing and turnaround time. Thus CFD can be used in the first instance to obtain reattachment lengths for a limited choice of Reynolds numbers and ANN will be used subsequently to predict the reattachment lengths for other intermediate Reynolds number values.

The purpose of this work is twofold. First present a CFD analysis of flows through plane sudden expansions. Second, the results of this analysis will be used in conjunction with ANN to demonstrate that the hybrid combination of ANN and CFD modelling allows a considerable time saving in the prediction of reattachment lengths for plane sudden expansions.

The next section of this paper will present the theoretical model used to obtain finite-volume solutions to the flow through plane sudden expansions chosen from previously published work. Section three presents the ANN model, the results are presented in section 4 followed by conclusions.

2. NUMERICAL SIMULATION IN PLANE SYMMETRIC SUDDEN EXPANSIONS

2.1 Model Equations and Numerical Method

The flow is assumed to be laminar, two-dimensional and unsteady, the fluid viscous and incompressible. Under these assumptions, the continuity and momentum equations are therefore written in their conservative form and for Cartesian systems of co-ordinates as:

$$\frac{\partial U}{\partial x}+\frac{\partial V}{\partial y}=0 \tag{1}$$

$$\frac{\partial(U)}{\partial t}+\frac{\partial(U^2)}{\partial x}+\frac{\partial(UV)}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial(V)}{\partial t}+\frac{\partial(UV)}{\partial x}+\frac{\partial(V^2)}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}\right) \tag{3}$$

where, (U, V) are the fluid velocity vector Cartesian components, P represents the pressure and ρ and μ the density and viscosity of the fluid respectively.

The finite-volume technique is used to solve the above equations. The second order QUICK scheme of Leonard [5] was employed for the discretization convection fluxes and a second-order centred difference scheme was adopted for diffusive fluxes. The pressure field P is solved with the PISO algorithm, see Issa [6]. Temporal integration is achieved through a fully implicit formulation with a time step of 10^{-5}. Convergence was assumed when the global rates of change of the variables were between 10^{-10} and 10^{-11} for laminar flow and 10^{-7} for turbulent flow and monitoring of variables at relevant positions in the flow field. All the calculations were carried on a SUN-20 workstation, to 64-bits precision.

2.2 Initial-boundary conditions and meshes

The geometrical details of the configuration are shown on figure 1. The computational grid was rectangular and extended from the exit plane of the expansion to a downstream position giving a length of up to 70 step-heights.

The calculations proceeded by impulsively starting the flow from rest. At the inlet boundary, a fully developed parabolic profile was prescribed.

At the outlet plane, a zero gradient condition is enforced for all variables and this boundary was located at a downstream distance of 50 and in some instances 70 step heights. The axial length of the domain of integration was assumed to be long enough to capture the flow details and remove upstream influence of the outlet boundary face value. At solid boundaries, the usual law of the wall was used.

The mesh was rectangular with uniform distributions along the stream wise and cross-stream directions. Three grid sizes were used 150×55 and 200×93 and 250×110 to generate grid independent solutions as judged by the profiles of the axial velocity. There were no significant differences between the results of the last two grids and therefore the intermediate one was adopted in all the computations.

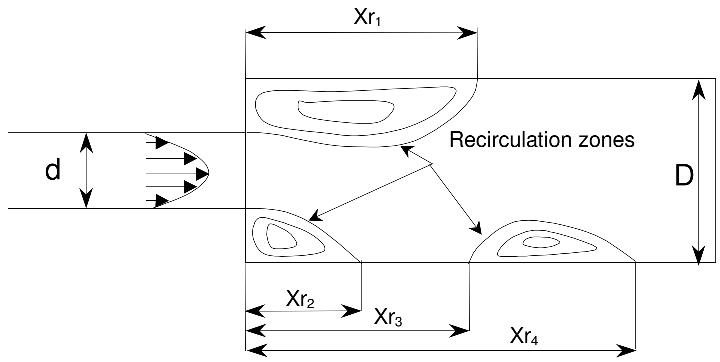


FIGURE 1: Geometrical configuration of the plane sudden-expansion.

3. ARTIFICIAL NEURAL NETWORKS MODEL

3.1 Artificial Neural Networks

An artificial neural network (ANN) method is a computational mechanism able to acquire, represent, and compute a mapping from one multivariate space of information to another, given a set of data representing that mapping. ANNs have the ability to mimic the human brain as well as their ability to learn and respond [7]. This technique found wide acceptance in various engineering applications since they have proven to be effective in performing complex operations, process and functions in a variety of fields.

An ANN can be considered as a collection of numerous simple processors organised in layers called neurons or nodes. These nodes are the basic organizational units of a neural network that are arranged in a series of layers to create the ANN by unidirectional communication channels (connections) that carry numerical data. Nodes are classified as input, output, or hidden layer nodes depending on their location and function within the network see Stern [8]. Data is received from sources external to the neural network through the input layer nodes, while data is transmitted out of the neural network through the output layer nodes. Hidden layer neurons act as the computational nodes in the neural network, communicating between input nodes and other hidden layer or output nodes. The number of nodes in the input layer is equal to the number of independent variables entered into the network which represent the input parameters whereas the number of output nodes corresponds to the number of variables to be predicted. The number of hidden layers and nodes used within the hidden layer vary according to the complexity of the task the network must perform see DeTienne et al.[9].

3.2 Model Development

It has been clearly established that the reattachment length for laminar flow depends on two non-dimensional parameters, the Reynolds number and the expansion ratio (E.R.=D/d). This dimensional analysis relationship forms the basis of the Artificial Neural Network model which is developed below.

Thus, an ANN model was developed for predicting reattachment positions for the expansion ratios of 2, 3 and 5. The ANN Architecture for this model is shown in Figure 2. The model consists of three layers, an input layer, a hidden layer and an output layer. The neurones in the input layer receive two input representing the Reynolds number (Re) and the Expansion ratios (E.R.); hence, two neurones were used for input in the ANN architecture. The output layer, on the other hand, consists of four neurones representing the reattachment positions (Xr1, Xr2, Xr3, and Xr4 of figure 1). The single hidden layer used consisted of 5 neurones. The initial number of nodes used in the single hidden layer was calculated using the equation developed by Carpenter and Hoffman [10]:

$$N = \eta[H(I+1) + n(H+1)] \tag{4}$$

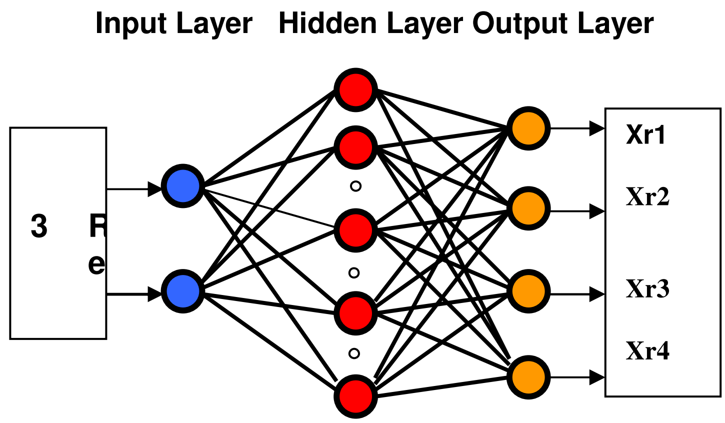


FIGURE 2: The Architecture for the Developed ANN Model.

Where, η is a constant greater than 1.0 (i.e. η =1.25 would give a 25% over-determined approximation, *N* the number of training pairs available, *H* the number of hidden nodes to be used in the network with one hidden Layer, *I* and n the number of input and output nodes respectively.

The result obtained using equation (4) indicated that four hidden nodes could be used in the hidden layer to develop the ANN architecture. However, the best result for the model was found with eight nodes in the hidden layer. Research in this area by Lapeds and Forber [11] and Hecht-Nielsen [12] proved that one or two hidden layers with an adequate number of neurones is sufficient to model any solution surface of practical interest

4. RESULTS

4.1 CFD results

The results of the CFD simulations are presented first. A detailed comparison with measured velocity data is conducted so as to provide the necessary confidence in other predicted flow parameters such as reattachment lengths for other values of Reynolds number.

The experimental results of Fearn et al. [3] are used in the study of the velocity profiles predictions. The heights of the upstream and downstream ducts were 4 and 12 mm respectively and the area and aspect ratios 1:3 and 8:1 respectively. Below a Reynolds number of 60, the flow was found to be symmetric.

Figure 3 illustrates detailed comparison of the calculated axial velocity profiles and their experimental counter parts, as measured by Fearn et al. [3], for several downstream positions for a Reynolds number equal to 187. The agreement between the calculated and measured profiles is excellent. The profile inside the third recirculation zone is also accurately predicted.

Figure 4 shows the predicted reattachment lengths with Reynolds number for the three expansion ratios of 2, 3 and 5. They correspond to the experimental studies and configurations of Cherdron et al [2], Fearn et al [3] and Ouwa et al [13]. The reattachment location is determined as the position where the stream wise velocity is zero at the first grid point from the wall. The trend of all the three curves reveals a symmetric flow at low values of the Reynolds number and transition to asymmetric flow occurs at a critical value of the latter. Initially, only two recirculation zones are present with a third one appearing later. The part of the curves that corresponds to symmetric flow extrapolates to a finite recirculation length at zero Reynolds number. Ouwa et al. [13] reported variations of the two first reattachment lengths (Xr1 and Xr2) with Reynolds number and their results agree remarkably well with the present calculations. The inferred values of the critical Reynolds number for the three ratios of 2, 3 and 5 are 180, 60 and 45 respectively and compare well with the experimental values of 185, 54 and 45. The source of the discrepancies is difficult to trace but can be explained in terms of difficulty in obtaining an exact value for the transitional Reynolds number.

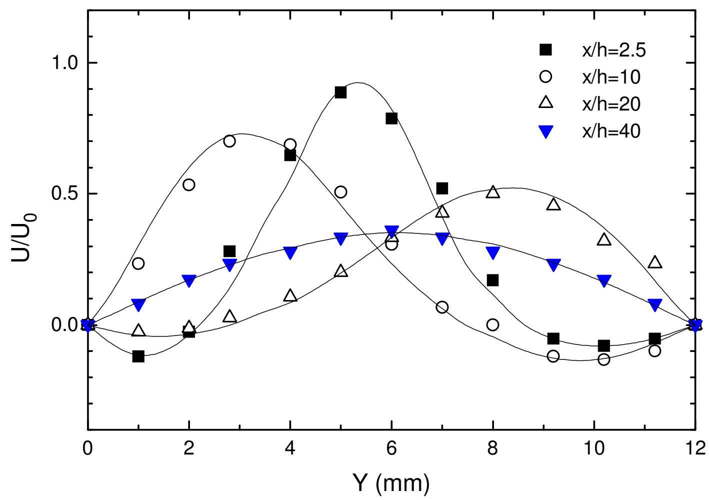
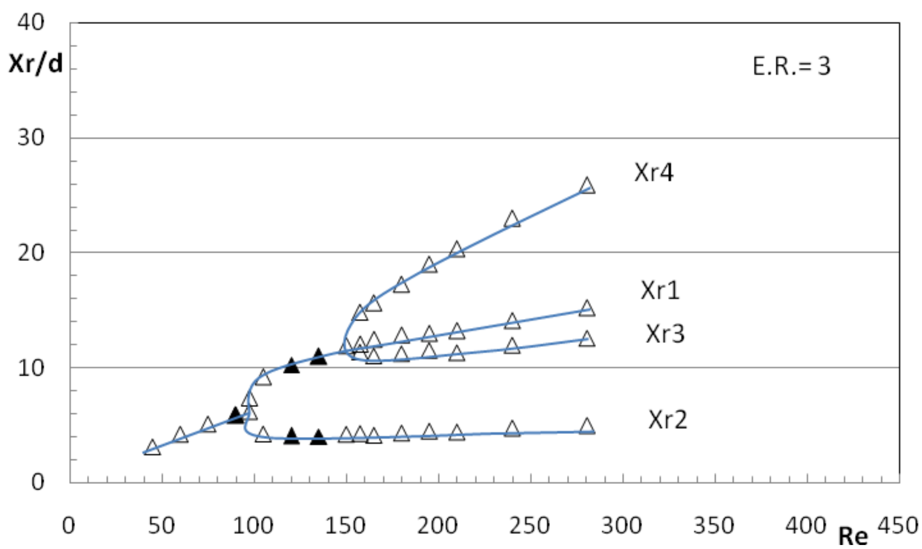
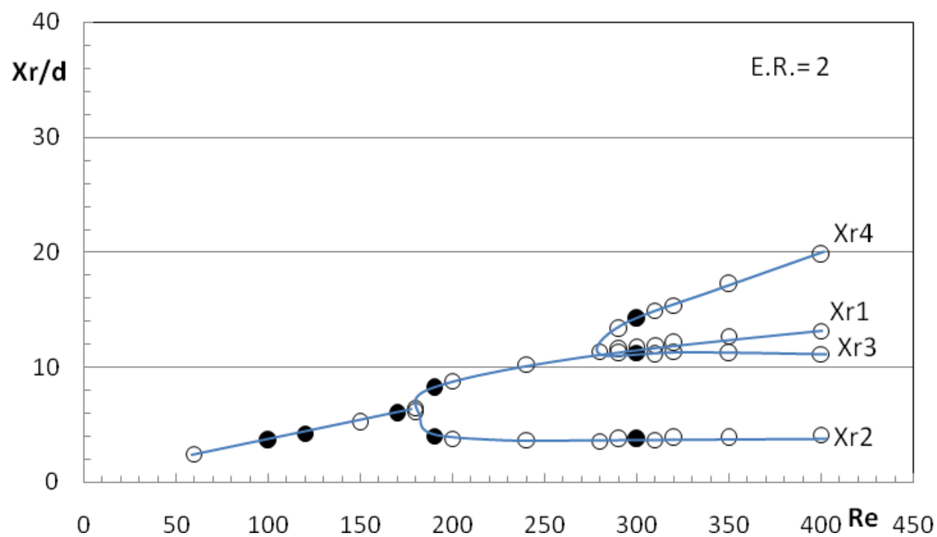


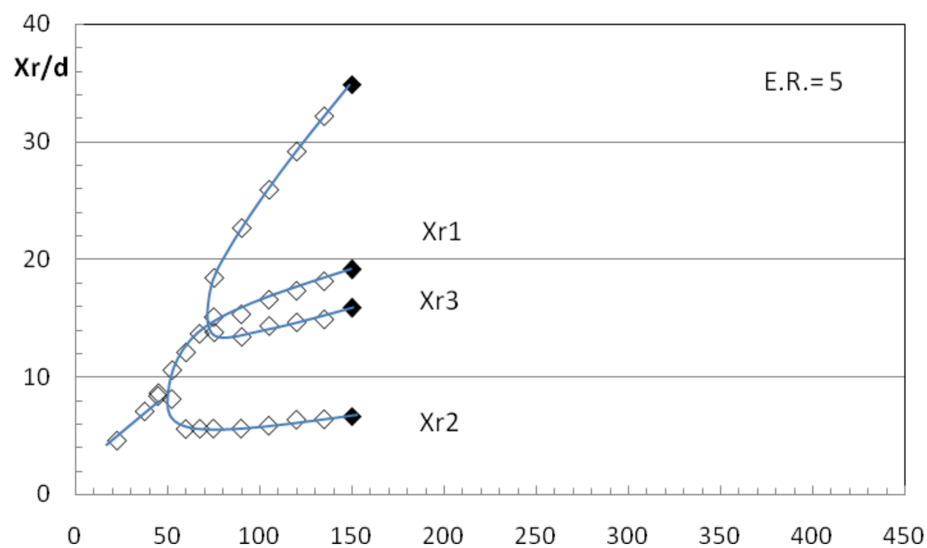
FIGURE 3: Axial Mean velocity (Fearn et al), Re=187.



(a)



(b)



(c)

FIGURE 4: Reattachment lengths with Reynolds number for E.R=2, 3 and 5, lines are just for visual help, filled dots represent the 9 randomly chosen cases by the simulator for testing.

4.2 ANN data preparation, network training, validation, testing and results

Before the ANN model can be used to provide the desired output, the model needs to be trained to recognise the relationships between the input (Re and E.R.) and the desired attachment positions (Xr1, Xr2, Xr3, and Xr4). These relationships will be stored as connection weights between the different neurones. The process of determining the weights is called the training or the learning process. Prior to conducting the training process, a set of input-output patterns are first prepared. The data set used in the development of this model was taken from the results of the extensive CFD calculations for the expansion ratios of 2, 3, and 5 respectively of figure 4. The data set consisted of 44 cases or data sets. 35 of these cases were used for model development and training while the remaining 9 cases, shown as filled dots on figure 4 (20%) of the data set were used to evaluate the developed model performance. These nine cases were selected randomly from the data set, but were chosen to be representative of the span of the

Reynolds numbers at hand and the three expansion ratios. The multi-layer feed-forward network used in this work was trained using the Back-propagation (BP) paradigm developed by Rumelhart and McClelland [14]. Simpson [15] gives a different equation that provides a generalised description of how the learning and recall process is performed by the BP algorithm.

The training process of this ANN model was performed using the *NeuroShell*^{TM} simulator. After completing 180429 epoch, the network converged to a threshold of 10^{-5}. The network model goodness of fit, R^2 (R^2 is defined as the coefficient of determination), demonstrated that the developed ANN model produced attachment positions that were in close agreement with the actual values. The R^2 values of the outputs Xr1, Xr2, Xr3, and Xr4 were 0.8617, 0.8325, 0.9397 and 0.9682 respectively. Having trained the network successfully, the next step is to test the network's generalisation capability using a different data set in order to judge its performance.

Using the 9 cases that were randomly selected from the data set, the developed model was tested to assess its generalisation capabilities. The R^2 values for the Xr1, Xr2, Xr3, and Xr4 were 0.9383, 0.8577, 0.997 and 0.999 respectively. Figures 5(a, b, c, d) and 6 (a, b, c, d) illustrate the relationship between actual and predicted attachment positions of Xr1, Xr2, Xr3, and Xr4 for the training data and the testing data sets respectively.

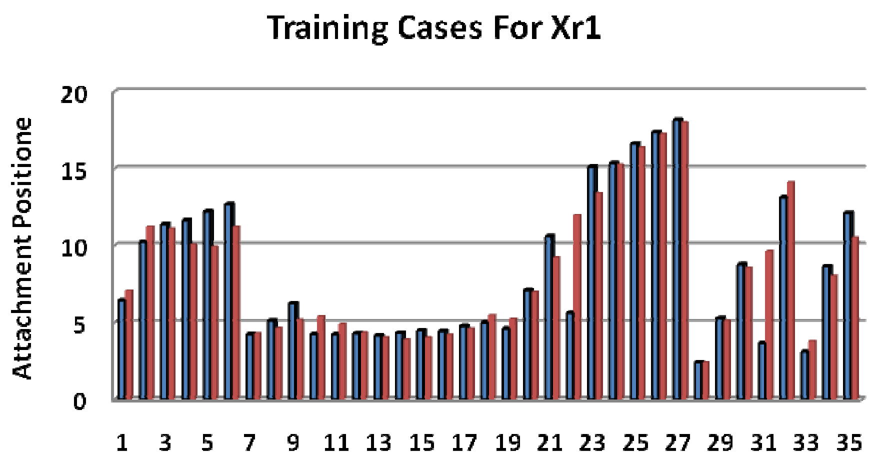


FIGURE 5a: Actual vs. Predicted Results for the output Xr1 in the Developed ANN Model.

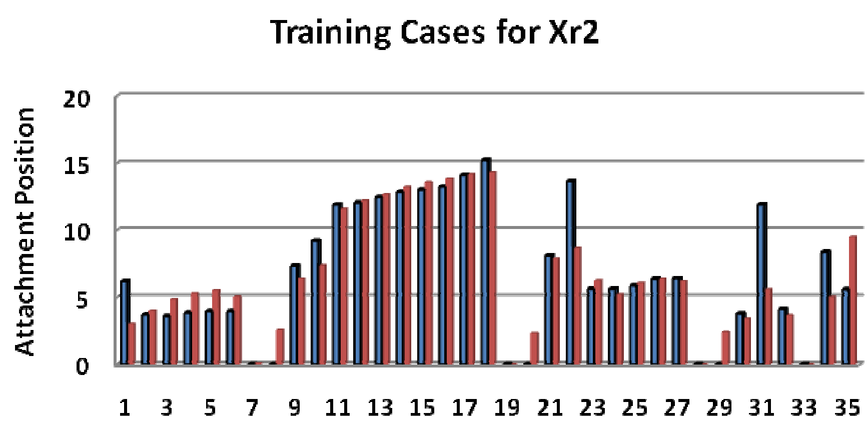


FIGURE 5b: Actual vs. Predicted Results for the output Xr2 in the Developed ANN Model.

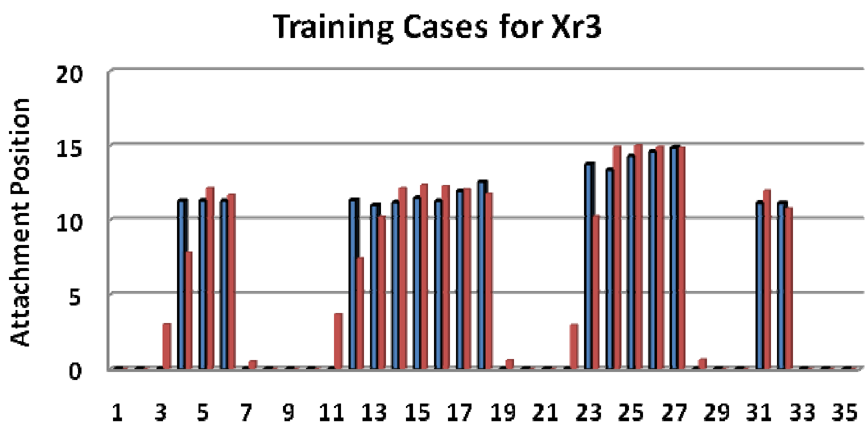


FIGURE 5c: Actual vs. Predicted Results for the output Xr3 in the Developed ANN Model.

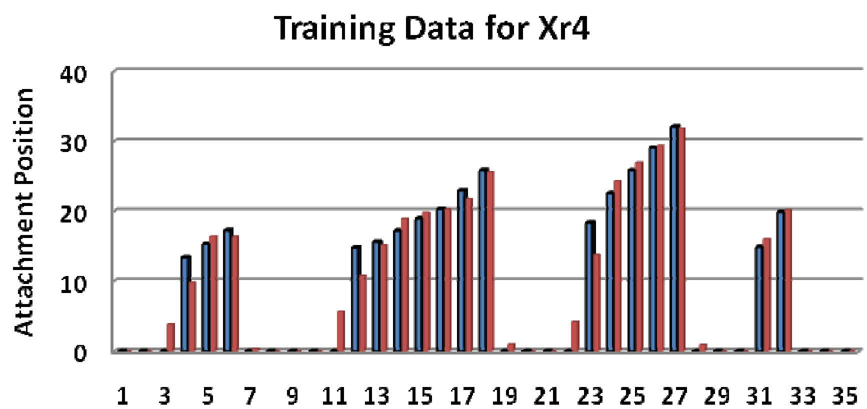


FIGURE 5d: Actual vs. Predicted Results for the output Xr4 in the Developed ANN Model.

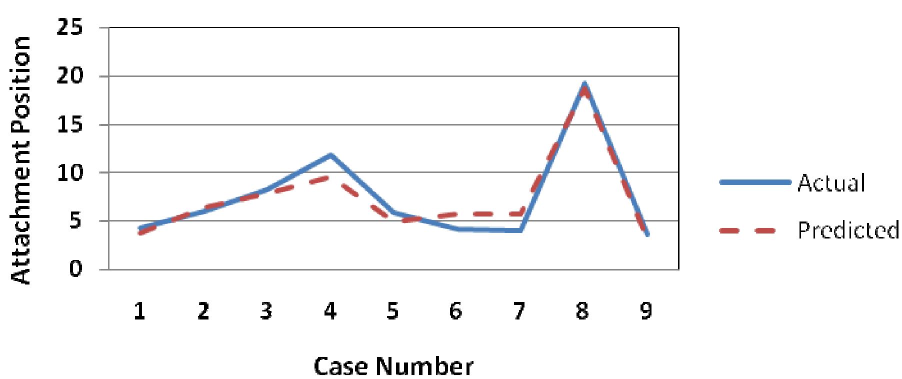


FIGURE 6a: Actual vs. Predicted attachment position for Xr1 in the Testing Cases.

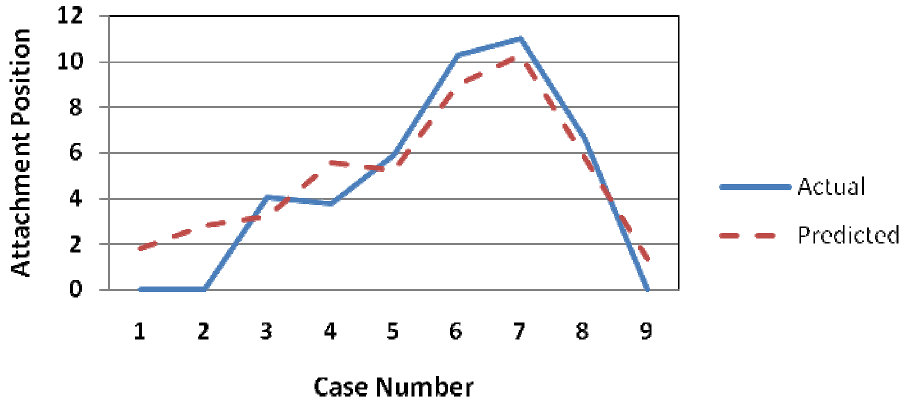


FIGURE 6b: Actual vs. Predicted attachment position for Xr2 in the Testing Cases.

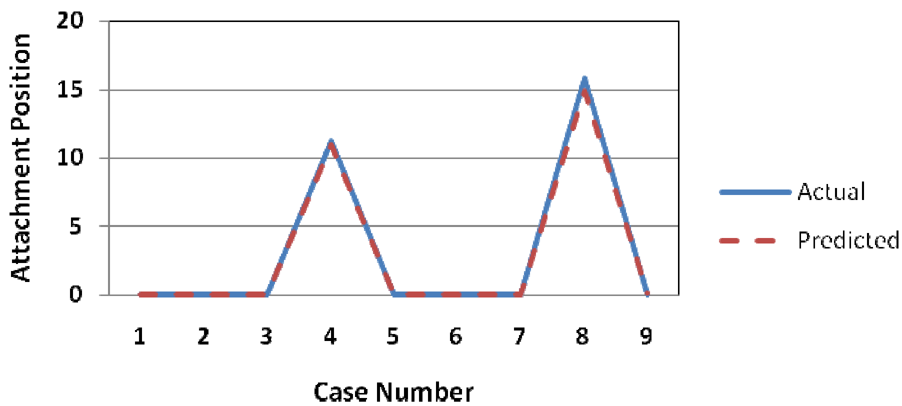


FIGURE 6c: Actual vs. Predicted attachment position for Xr3 in the Testing Cases.

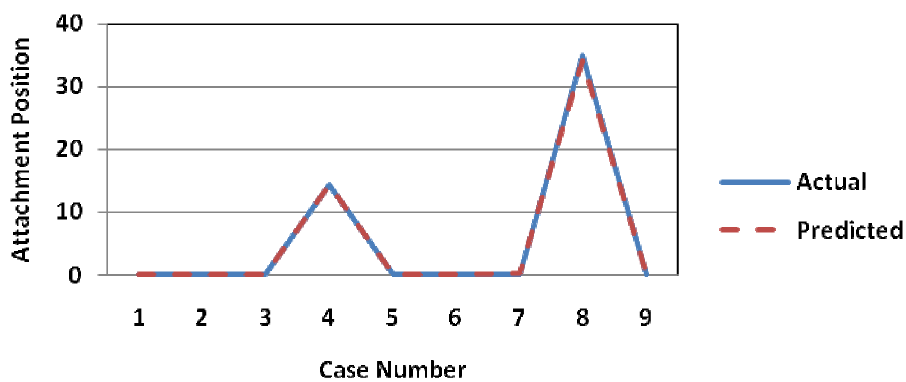


FIGURE 6d: Actual vs. Predicted attachment position for Xr4 in the Testing Cases.

Examination of the results obtained from the testing set of ANN model, suggests that there are very close agreement between actual and predicted values for the attachment positions Xr2, Xr3, and Xr4 and a moderate agreement for Xr1. As shown in figure 6a the predicted attachment position for Xr2 differ slightly from the actual attachment position, the predicted value in case 1 was 1.81 while the actual was 0.0, similarly there was small difference in the predicted value of the second testing case the actual was 0.0 while the ANN model predicted 2.82. All the other seven cases predictions are in close agreement with the actual attachment position the R^2 value for Xr2 was 0.8577. For Xr1 the results are shown in figure 6a. The testing results indicates that Xr1 was predicted with higher accuracy the R^2 value is 0.9383 which indicate that 93.83 percent of the variation in the data provided can be explained by the two input given and the developed model. Figures 6c and 6d outlined the highly accurate prediction of attachment position for Xr3 and Xr4 when compared with actual results. The R^2 value was 0.997 and 0.999 for Xr3 and Xr4 respectively.

Intensive double precision (because of small magnitude of numbers) computations are necessary to obtain the CFD solutions. For a single Reynolds number as long as 27 hours of computing time are necessary to reach a converged solution on a SUN-20 workstation. Hence tremendous effort was necessary to produce the results of figure 3. In addition to being computer intensive, CFD solution provide a large amount of information as part of the solution that has limited usefulness if one is only interested in reattachment lengths. In engineering design where parametric studies are common, time management becomes of primary importance. The present study has shown that in the case of laminar flows through plane sudden expansions, it is only necessary to obtain CFD solutions for a limited number of Reynolds numbers. An ANN model can then be trained to reproduce with good accuracy the variation of reattachment length with Reynolds number. These results indicate that reattachment positions can be predicted instantly with good accuracy for any E.R between 2 and 5 and Re value between 0.0 and 400 using the developed ANN model.

5. CONCLUSIONS

Computations were performed using both CFD and ANN. In engineering design where parametric studies are common time management becomes of primary importance. The present study has shown that in the case of laminar flows through plane sudden expansions, it is only necessary to obtain CFD solutions for a limited number of Reynolds numbers and expansion ratios. An ANN model can then be trained to reproduce with good accuracy the variation of reattachment length with Reynolds number and expansion ratio. The time needed to develop and test the ANN model is very short; if the data are available, and the person is experienced and familiar with the ANN software, model development and testing can be accomplished in approximately one hour or two. If the model was already developed and only some new data to be tested or changes are needed to be performed on the model, then it is a matter of very few minute. Neural networks offer a number of advantages, including shorter modelling time, less formal statistical training requirements, ability to implicitly detect complex nonlinear relationships between dependent and independent variables, ability to detect all possible interactions between predictor variables, and the availability of multiple training algorithms. Artificial neural networks models also allow fast processing of large amounts of information. Their generalisation capability makes them more robust to noise.

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