# Margin Parameter Variation for an Adaptive Observer to a Class of Systems

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**Abstract**

In this paper, adaptive observer robustness is studies. The adaptive observer is proposed to a nonlinear system linearized by output injected for variable structure systems. Despites, the adaptive observer is suggesting to supply the parameter variation but it isn't usually verified. Unfortunately, the parameter uncertainty impact  state convergence. For that a margin parameter variation is determined which the convergence of the state is guaranty. Simulation results show that the adaptive observer is robust only in the definite parameter margin variation.

**Keywords:** Nonlinear Systems, Adaptive Observer, Margin Parameter Variation, Robustness.

## 1. INTRODUCTION

During the running, it is not usually possible for some systems to know about the exact value of the system parameters. These kinds of systems have two proposed methods to be studied. The first is to augment the state vector which will include the state and the unknown parameters [15]. The second is to build an adaptation law depending only on measured and estimated state variables. Indeed, in literature ([1], [2], [3], [4]), adaptive observers using parameter adaptation law to estimate state variables are considered.

In [5] the authors have compared these two methods and they conclude that these two techniques have the same results and the adaptive observer is more robust for nominal systems. In [6] an adaptive observer is defined as a recursive algorithm which allows state and parameter estimation. These parameters can be assumed as unknown inputs [7].

To determine the adaptation law for some class systems, the following techniques are proposed:

- Linearization by output injected for variable structure systems [6],
- Linearization by feed forward [7],
- Systems transformed into a canonical form in which the error becomes linear at the new coordinate ([8], [9], [10], [11]),
- Lipschitz nonlinear systems [12],

The studied adaptive observer made to a constant unknown parameter of system. But, really, the parameter varied in time. Recently, the authors in ([12], [13], [14]) studied the robustness of an adaptive observer. For that a set of parameter are determined which the state estimation are improved.

The aims contribution of this paper is to determine the parameter variation margin which the adaptive observer build to systems linearization by output injected for variable structure systems ([6]) remains still robust.

## 2. ADAPTIVE OBSERVER FOR SYSTEMS TRANSFORMED INTO A CANONICAL FORM ([8], [9], [10], [11])

### 2.1. Observer architecture:

Consider the nonlinear system:

$$\begin{cases} \dot{x} = f(x) + q_0(x,u) + \sum_{i=1}^{P} \theta_i q_i(x,u) \\ \dot{x} = f(x) + q_0(x,u) + \theta^T Q(x,u) \\ y = h(x) \end{cases} \tag{1}$$

Where:

$x \in R^n$; $u \in R^m$; $q_i \in R^n * R^m \to R^n$ $\quad for\ all\ 1 \le i \le p;\ Q(x,u) \in R^{p*n}$;

$\theta \in [\theta_1\ \theta_2 \dots \theta_p]^T \in R^p$; $y \in R$

In ([8], [9], [10], and [11]) the authors present the different steps to change the system (1) into the form (2) in which the error becomes linear. The system is transformed in a canonical form if there exists a global state space diffeomorphism and the parameters become independent as such:

$$\xi = T(x)\,,\ T(x_0) = 0$$

$$\begin{cases} \dot{\xi} = A_c\, \xi + \psi_0(x,u) + \sum_{i=1}^{P} \theta_i \psi_i(x,u) \\ \dot{\xi} = A_c\, \xi + \psi_0(x,u) + \Psi(x,u)\theta \\ y = C_c \xi \end{cases} \tag{2}$$

With:

$$A_c = \begin{bmatrix} 0 & 1 & 0 & \cdots\cdots\cdots 0 \\ 0 & 0 & 1 & \cdots\cdots\cdots 0 \\ \vdots & \vdots & 0 & \ddots \quad 0 \\ \vdots & \vdots & \vdots & \quad 1 \\ 0 & 0 & 0 & 0 \quad 0 \end{bmatrix}; C_c = \begin{bmatrix} 1 & 0 \cdots\cdots 0 \end{bmatrix}$$

The parameter adaptation algorithm designed for the systems can be in this form:

$$\begin{cases} \dot{z} = A_c\, z + \gamma(y,u) + b \sum_{i=1}^{P} \beta_i(t)\theta_i \\ \dot{z} = A_c\, z + \gamma(y,u) + b\beta^T(t)\theta \qquad ; z \in R^n \\ y = C_c\, z \end{cases} \tag{3}$$

Where:

$$z = \xi - M(t)\theta \quad ; M = \begin{bmatrix} 0 \\ N \end{bmatrix} \tag{4}$$

With N functioning as the solution of the matrix equation (5):

$$\dot{N} = \begin{bmatrix} -b_2 & 1 & 0 & \cdots & \cdots & 0 \\ \vdots & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -b_{n-1} & \cdots & \cdots & \cdots & \cdots & 1 \\ -b_n & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix} N + \begin{bmatrix} -b_2 & 1 & 0 & \cdots & \cdots & 0 \\ \vdots & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -b_{n-1} & \cdots & \cdots & \cdots & \cdots & 1 \\ -b_n & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix} \Psi(y,u) \tag{5}$$

The functions $\Psi(y,u)$ and $N$ are bounded, $b_i$ are the coefficients of a Hurwitz polynomial: $s^{n-1} + b_2 s^{n-2} + \cdots + b_n$ .

$$\beta^T(t) = \left[\beta_1(t) \quad \beta_2(t) \cdots\cdots \beta_p(t)\right] = C_c A_c M + C_c \Psi(y,u)$$

The adaptive observer is revealed by the following equations [8]:

$$\begin{aligned}
&\dot{\hat{z}} = (A_c + kC_c)\hat{z} + \gamma(y,u) + b\beta^T(t)\hat{\theta} - ky \\
&\dot{\hat{z}} = A\hat{z} + \gamma(y,u) + b\beta^T(t)\hat{\theta} - ky \\
&\dot{\xi}_i = A\xi_i + b\beta_i(t) \quad ; \xi_i \in R^n , \xi_i(0) = \xi_{i0} \\
&\dot{\xi}_0 = A\xi_0 + b\beta^T(t)\hat{\theta} \quad ; \xi_0 \in R^n , \xi_0(0) = \xi_{00} \\
&\eta^T = C_c\left[\xi_1 \, \xi_2 \cdots\cdots \xi_p\right] \\
&\dot{W} = -\alpha W + \eta\eta^T , W \in R^p * R^p , W(0) = W_0 \\
&\dot{w} = -\alpha w - \eta(C_c\xi_0 + y - C_c\hat{z}) , w \in R^p , w(0) = w_0 \\
&\dot{\hat{\theta}} = -\Gamma(W\hat{\theta} + w) , \hat{\theta}(0) = \hat{\theta}_0
\end{aligned} \tag{6}$$

Where:

The gain k is determined so that the matrix $A = A_c + kC_c$ is Hurwitz.

The vector $\beta(t)$ is bounded.

$\alpha$ is a positive reel.

$W_0$ is symmetric and semi defined positive.

$\eta(t)$ satisfy the existence condition if there exists $k > 0$ and $\delta > 0$ such as:

$$\int_t^{\delta+t} \eta(\tau)\eta^T(\tau)d\tau \geq k\tau$$

This type of observer is characterized by simplicity to implement it even though it is limited to a class of systems [11] and the error converges exponentially to an arbiter rate [8].

This observer is very robust when the parameters are constant. The efficiency of this architecture will be studied with a linear parameter variation in time. The maximal margin variation of the parameter is determined so that the convergence of the desired state is guaranteed.

### 2.2. Parameter Margin Variation

During the running the parameters of systems vary, to overcome this parameter variation an adaptive observer is used to estimate joint parameter and unmeasured state. In [16], the author's proved that the adaptation law isn't usually robust to estimate the state for all parameter variation.

In [13] the author's suppose that the unknown parameter is bounded for that the adaptive observer gives a good performance. In this section, a maximal margin of variation is defined for which the adaptation law is still robust, we will study the adaptive observer presented in ([8], [9], [10], [11]) in the case of varied parameter when the following assumptions are satisfied.

***Assumptions:***

The matrix $A = A_c + kC_c$ is invertible and stable.

$b\beta^T(t)$ is stable.

$e^T b\beta^T(t)\hat{\theta} \geq 0$ and $\tilde{\theta}^T \Gamma^{-1}\dot{\hat{\theta}} > 0$

$|\theta_i| > 1 \;\; for\ all\ i = 1 \cdots\cdots p$

If the vector of parameters varies in time, the nonlinear system transformed will have the form (7) however the observer has the same architecture as (6):

$$\dot{z} = A_c\, z + \gamma(y,u) + b\beta^T(t)\theta^{'} \qquad (7)$$

With

$$\theta^{'} = \theta + \Delta\theta \,; \Delta\theta = n\theta \;\; ; n \in R$$

When $\Delta\theta$ is the parameter variation, it is proportionally into the nominal parameters.

The new vector of unknown parameters can be written as such $\theta^{'} = (n+1)\theta$. We express the theorem:

***Theorem:***

If the assumptions are satisfied, the observers (6) still robust to estimate the state of systems (1) for all values of  satisfying this relation:

$$n \leq \frac{\|y\|}{\|\max(\theta_i)\|}$$

***Proof:***

Defining:

The error parameter $\tilde{\theta}^{'} = \theta^{'} - \hat{\theta} \Rightarrow \dot{\tilde{\theta}}^{'} = -\dot{\hat{\theta}}$

The error state $e = z - \hat{z}$

The error dynamics is:

$$\begin{aligned} \dot{e} &= A_c(z-\hat{z}) + b\beta^T(t)(\theta^{'} - \hat{\theta}) + ky - kC_c\hat{z} \\ \dot{e} &= (A_c + kC_c)e + b\beta^T(t)(\theta^{'} - \hat{\theta}) \\ \dot{e} &= (A_c + kC_c)e + b\beta^T(t)\tilde{\theta}^{'} \end{aligned} \qquad (8)$$

The stability of the observers (6) with varying parameters is developed with a Lyapunov function defined as:

$$V = \frac{1}{2}ee^T + \frac{1}{2}\tilde{\theta}^{'T}\Gamma^{-1}\tilde{\theta}^{'} \qquad (9)$$

The derived function of V is:

$$\dot{V} = \dot{e}e^T - \tilde{\theta}^{'T}\Gamma^{-1}\dot{\hat{\theta}}$$

$$\dot{V} = (A_c + kC_c)ee^T + b\beta^T(t)\tilde{\theta}^{'}e^T - \tilde{\theta}^{'T}\Gamma^{-1}\dot{\hat{\theta}}$$

$$\dot{V} = Aee^T + b\beta^T(t)(\theta + \Delta\theta - \hat{\theta})e^T - (\theta + \Delta\theta - \hat{\theta})^T\Gamma^{-1}\dot{\hat{\theta}}$$

As defined in assumptions $e^T b\beta^T(t)\hat{\theta} \geq 0$ and $\tilde{\theta}^T \Gamma^{-1}\dot{\hat{\theta}} > 0$ we have:

$$\dot{V} \leq Aee^T + b\beta^T(t)\theta e^T + b\beta^T(t)\Delta\theta e^T - \Delta\theta^T\Gamma^{-1}\dot{\hat{\theta}} + \hat{\theta}^T\Gamma^{-1}\dot{\hat{\theta}} \leq 0$$

Since

$$\Delta\theta^T\Gamma^{-1}\dot{\hat{\theta}} = (\Gamma^{-1}\dot{\hat{\theta}})^T\Delta\theta$$

Then:

$$\Delta\theta(b\beta^T(t)e^T - (\Gamma^{-1}\dot{\hat{\theta}})^T) \leq -(Aee^T + b\beta^T(t)\theta e^T + \hat{\theta}^T\Gamma^{-1}\dot{\hat{\theta}})$$

$$\Delta\theta \leq \frac{-(Aee^T + b\beta^T(t)\theta e^T + \hat{\theta}^T\Gamma^{-1}\dot{\hat{\theta}}}{(b\beta^T(t)e^T - (\Gamma^{-1}\dot{\hat{\theta}})^T)}) \leq \frac{1}{b\beta^T(t)e^T - (\Gamma^{-1}\dot{\hat{\theta}})^T}$$

$$\Delta\theta \leq \frac{1}{(\Gamma^{-1}\dot{\hat{\theta}})^T}$$

By including the equation (6) in this inequality we obtain:

$$\dot{\hat{\theta}} = -\Gamma(W\hat{\theta} + w)$$

$$\Delta\theta \leq \frac{1}{(-\Gamma^{-1}\Gamma(W\hat{\theta} + w))^T} \leq w^T$$

$$\Delta\theta \leq w^T \tag{10}$$

$$\dot{w} = -\alpha w - \eta(C_c\xi_0 + y - C_c\hat{z})$$

This implies

$$w = \frac{1}{\alpha}(-\dot{w} + \eta(C_c\xi_0 + y - C_c\hat{z}))$$

$$w \leq \eta(C_c\xi_0 + y - C_c\hat{z})$$

Where $t \to \infty \quad e_y \to 0$

Then:

$$w^T \leq \xi_0^{\ T} C_c^{\ T}\eta^T \tag{11}$$

Or $\quad \eta^T = C_c\left[\xi_1\,\xi_2 \cdots\cdots \xi_p\right]$

Therefore (11) becomes:

$$w^T \leq \xi_0^{\ T} C_c^{\ T} C_c\left[\xi_1\,\xi_2 \cdots\cdots \xi_p\right] \tag{12}$$

If A is invertible and stable then:

$$\dot{\xi}_i = A\xi_i + b\beta_i(t)$$

$$\xi_i = A^{-1}(\dot{\xi}_i - b\beta_i(t)) \leq \lambda_{\max}(A^{-1})(\dot{\xi}_i - b\beta_i(t))$$

With $\lambda_{\max}(A^{-1}) \leq 0$

This implies

$$\xi_i \leq \lambda_{\max}(A^{-1})b\beta_i(t)$$

$$\left[\xi_1\,\xi_2 \cdots\cdots \xi_p\right] \leq \lambda_{\max}(A^{-1})b\beta_i(t) \tag{13}$$

$$\dot{\xi}_0 = A\xi_0 + b\beta^T(t)\hat{\theta}$$

$$\xi_0 = A^{-1}(\dot{\xi}_0 - b\beta_i^{\ T}(t)\hat{\theta})$$

$$\xi^T{}_0 \leq \lambda_{\max}(A^{-1})\hat{\theta}^T\beta(t)b^T \tag{14}$$

With:

$$\dot{\hat{z}} = (A_c + kC_c)\hat{z} + \gamma(y,u) + b\beta^T(t)\hat{\theta} - ky$$

This implies

$$\hat{\theta}^T \beta(t) = (\dot{\hat{z}} - (A_c + kC_c)\hat{z} - \gamma(y,u) + ky)(b^{-1})^T$$

$$\hat{\theta}^T \beta(t) \le (\dot{\hat{z}} + ky)^T (b^{-1})^T \tag{15}$$

But:

$$\dot{\hat{z}} = C_c^{-1}\dot{y}$$

According to (13), (14) and (15) the equation (12) becomes:

$$w^T \le \lambda_{\max}(A^{-1})(\dot{\hat{z}} + ky)^T (b^{-1})^T b^T C_c{}^T C_c \lambda_{\max}(A^{-1}) b\beta(t)^T$$

Then:

$$w^T \le \left|\lambda_{\max}(A^{-1})\right|^2 (\dot{\hat{z}} + ky)^T C_c{}^T C_c b\beta(t)^T \tag{16}$$

The matrix $b\beta(t)^T$ is stable; this implies that $\lambda_{\max}(b\beta(t)^T) \le 0$ , the equation (16) becomes:

$$w^T \le \left|\lambda_{\max}(A^{-1})\right|^2 (\dot{\hat{z}} + ky)^T C_c{}^T C_c \lambda_{\max}(b\beta(t)^T)$$

$$w^T \le \left|\lambda_{\max}(A^{-1})\right|^2 (ky)^T C_c{}^T C_c \lambda_{\max}(b\beta(t)^T) \tag{17}$$

Since

$y = z_1$ And $C_c{}^T C_c = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & \cdots & 0 \end{bmatrix}$

$$w^T \le \left|\lambda_{\max}(A^{-1})\right|^2 k_1 z_1 \lambda_{\max}(b\beta(t)^T) \le z_1$$

The equation (10) will have the new form $\Delta\theta \le \|z_1\|$. Or $z_1 = y$ due to

$$\Delta\theta \le \|y\| \tag{18}$$

From (18), it becomes clear that the margin parameter variation depends only on the output. With proportional to the nominal parameter $\theta$, $\Delta\theta = n\theta$. The number $n$ verifies the relation:

$$n \le \frac{\|y\|}{\|\max(\theta_i)\|} \tag{19}$$

## 3. EXAMPLE

To verify and to make valid the robustness of the proposed adaptive observer when the parameter varies, one considers the robot arm motion equation of the robot arm studied in [8].

$$I\ddot{q} + \frac{1}{2}mgl\sin(q) = u$$

In which q: angle, u: the input torque; I: the moment of inertia of the link, g: the gravity constant, m: mass of link, l: the length of the link.

It is clear that the system can be put directly in the form (7) when choosing:

$$z_1 = q; z_2 = \dot{q}; y = q; \rho_1 = \frac{mgl}{2I}; \rho_2 = \frac{1}{I}$$

The equations in the state space are:

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -\rho_1 \sin(y) + \rho_2 u \\ y = z_1 \end{cases}$$

The adaptive observer designed in the robot arm model is given by the following equations:

$$\begin{bmatrix} \dot{\hat{z}}_1 \\ \dot{\hat{z}}_2 \end{bmatrix} = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix} \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \end{bmatrix} + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} y + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (-\hat{\rho}_1 \sin(y) + \hat{\rho}_2 u)$$

$$\dot{\xi}_1 = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix} \xi_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (-\sin(y))$$

$$\dot{\xi}_2 = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix} \xi_2 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$\dot{\xi}_0 = \begin{bmatrix} -k_1 & 1 \\ -k_2 & 0 \end{bmatrix} \xi_0 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (-\hat{\rho}_1 \sin(y) + \hat{\rho}_2 u)$$

$$\dot{W} = -\alpha W + \begin{bmatrix} \xi_{11}^2 & \xi_{11}\xi_{21} \\ \xi_{11}\xi_{21} & \xi_{21}^2 \end{bmatrix}$$

$$\dot{w} = -\alpha w + \begin{bmatrix} \xi_{11} \\ \xi_{21} \end{bmatrix} (C_c \xi_0 + y - \hat{z}_1)$$

$$\begin{bmatrix} \dot{\hat{\rho}}_1 \\ \dot{\hat{\rho}}_2 \end{bmatrix} = -\Gamma(W \begin{bmatrix} \hat{\rho}_1 \\ \hat{\rho}_2 \end{bmatrix} + w)$$

In simulation, the nominal parameters and simulation parameters used are as follows [8]:

$$m = 1Kg : g = 9.8 \,; l = 1m \text{ and } I = 0.5$$

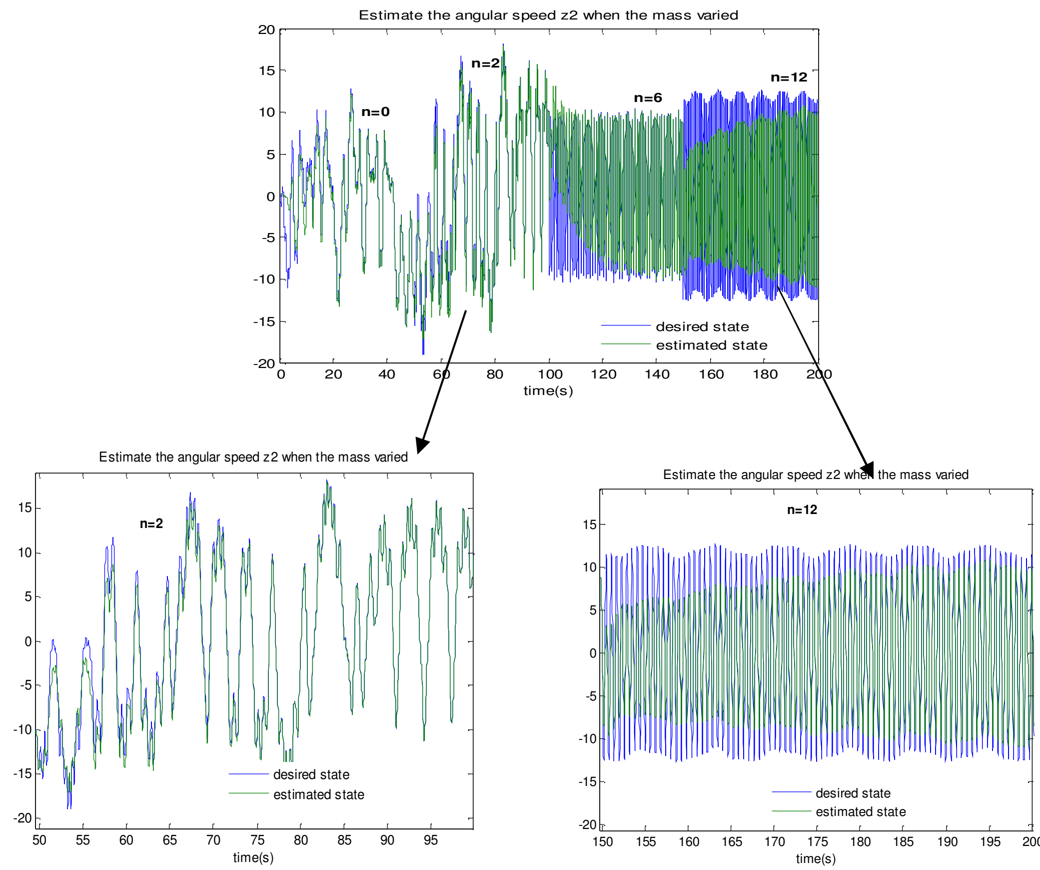
In literature the margin parameter variation is determined under a real application. In ([12], [14], [17]), the parameter variation is a percentage of the nominal parameter determined by the practical test or the constructer which the adaptive observer show a best performances. But the proposed bound of the external parameter margin variation can be enough or very large. To justify the robustness of the proposed margin parameter variation, the adaptive observer robustness (6) is tested through out the robot arm example under a variations on $\rho_1$ and $\rho_2$ are considered which the margin parameter variation taken is n<12.

The results obtained are reported as follows:

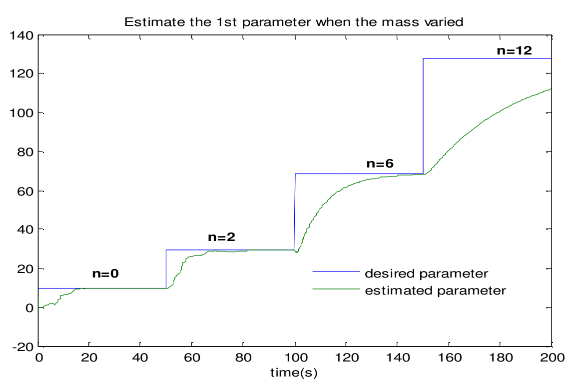
If the link mass varied such as m=1Kg (n=0) at t<50s, m=3Kg (n=2) at 50<t<100s, m=7Kg (n=6) at 100<t<150s and m=13Kg (n=12) at t>150s. The angular speed $z_2$ and the estimated parameter converge to the desired values at each variation after a time delay which is proportional to the margin variation applied (figure (1)). For a little variation of mass (n=2), the state and the parameter converge at the same time (tc=80s).But, if n is maximal (n=12) the estimated parameters and state take more time to converge to the reached value.

for that, if the inertia varied slowly, such as: I=0.5 at 0<t<70s, I=1 at 70s<t<140s and I=2 and t>140s, the state $z_2$ and the parameter $\rho_2$ converges rapidly to the reached values (figure (2.a), figure (2.c)) . Due to the inertia depend on $m$ the parameter $\rho_1$ did not usually converges which is requires more time than $\rho_2$ to converge (figure (2.b)).

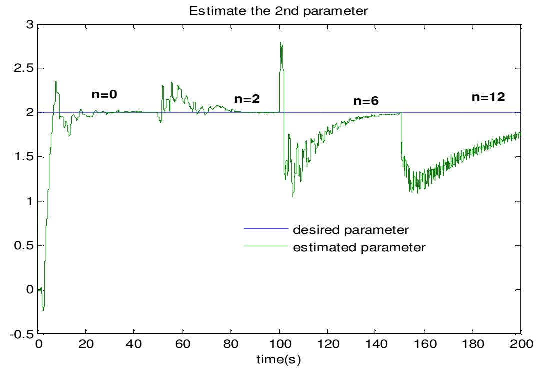
When the mass and the inertia vary simultaneously , to I=0.5 at 0<t<70s , I=1 at70s<t<140s, I=2 at t>140s and for the mass m=1Kg at 0<t<50s, m=3Kg at 50s<t<100s, m=7Kg at 100s<t<150s and m=13Kg at t>150s, we show that only the instance of the mass varying. In this case, it is clear that the state lasts longer time to converge to the desired values than the inertia (figure(3.a)).Thus, the parameter variation and the time converge become more efficient with the mass variation rather than with the inertia (figure(3.c)). This is also shown in the estimation of the parameter (figure (3.b)).




a. Estimation of the angular speed $z_2$ when the mass varied

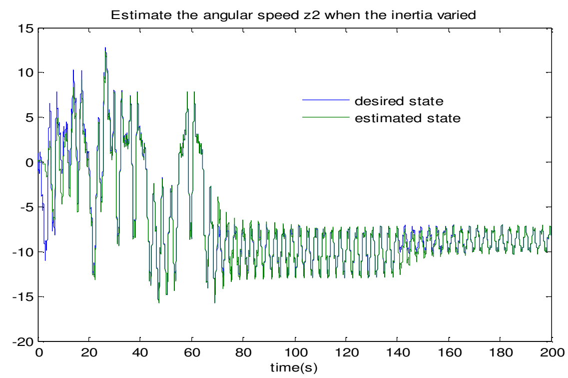



b. Estimation of the 1st parameter $\rho_1$

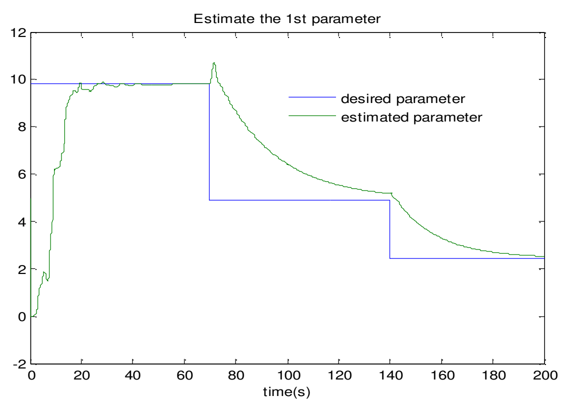



c. Estimation of the 2nd parameter $\rho_2$

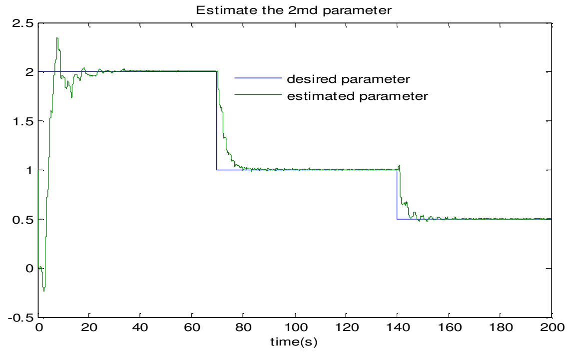
**FIGURE1:** Estimation of the angular speed z2 and the parameter when the mass varied




a. Estimation of the angular speed $z_2$ when the inertia varied

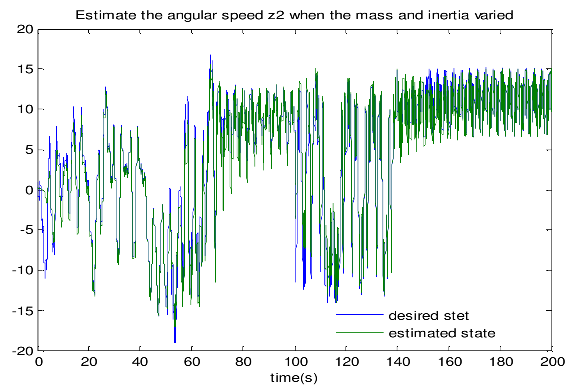



b. Estimate the 1st parameter $\rho_1$

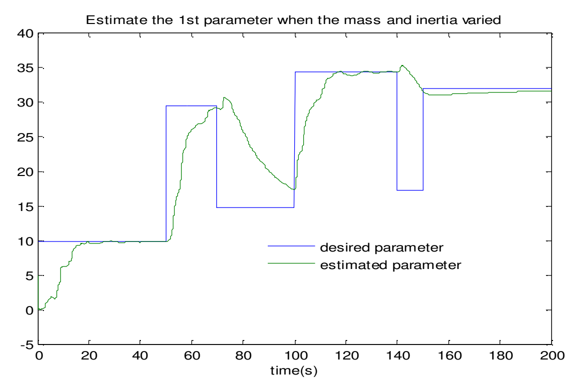



c. Estimate the 2nd parameter $\rho_2$

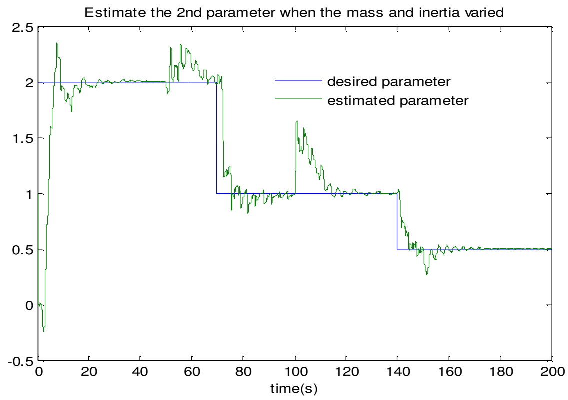
**FIGURE 2:** Estimation of the angular speed and the parameter when the inertia I varied




a. Estimation of the angular speed z2 when the inertia and mass varied




b. Estimate the 1st parameter $\rho_1$




c. Estimate the 2nd parameter $\rho_2$

**FIGURE 3:** Estimation of the angular speed $z_2$ and the parameter when the mass and inertia varied

We find that the studied adaptive observer is robust to estimate the state and the parameter only if parameter variations verify the inequality (18) and (19). The rapidity of convergence depends only on the varied parameter

## 4. CONSLUSION

In this paper, we have studied the robustness of the adaptive observer proposed in ([8], [9], [10], [11]) in the case of linear parameter variation of the system. A parameter variation margin is determined which the state convergence is guarantees In fact, the architecture of this adaptive observer shows good performance in the estimation of the state and of the unknown varied parameter.

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