

Reduction of Ultimate Strength due to Corrosion - A Finite Element Computational Method

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Abstract

Bridge safety is of paramount importance in transportation engineering and maintenance management. Corrosion causes strength deterioration and weakening of aged steel structures. Therefore, it is a vital task to estimate the remaining strength of corroded steel structures in order to assure the public safety. Due to the economic constraints and increase of number of steel highway and railway bridge structures, it will be an exigent task to conduct tests for each and every aged bridge structure within their bridge budgets. Therefore, this paper proposes a method of evaluating the residual strength capacities by numerical approach and compares the non-linear FEM analyses results with their respective tensile coupon tests. Further, since it is not easy to measure several thousands of points, to accurately reproduce the corroded surface by numerical methods and to predict their yield and ultimate behaviors, a simple and reliable analytical model is proposed by measuring the maximum corroded depth ($t_{c,max}$), in order to estimate the remaining strength capacities of actual corroded members more precisely.

Keywords: Bridges, Corrosion, Maximum Corroded Depth, FEM Analysis, Remaining Strength.

1. INTRODUCTION

Corrosion of the members of a steel structure leads to impairment of its operation and progressive weakening of that structure. The consequences of corrosion are many and varied and effects of these on safe, reliable and efficient operation of structures are often considered than simply losing of a volume of metal. Various kinds of failures and the need of expensive replacements may occur even though the amount of metal destroyed is quite small. One of the major harmful effects of corrosion is the reduction of metal thickness leading to loss of mechanical strength and structural failure, causing severe disastrous and hazardous injuries to the people. Therefore, understanding of the influence of damage due to corrosion on the remaining load-carrying capacities is a vital task for the maintenance management of steel highway and railway infrastructures.

Though it's a maintenance issue, it can be addressed appropriately by specification of a proper corrosion system in the design phase. It has been proved that the corrosion played a significant role in the catastrophic collapse of both the Silver Bridge (Point Pleasant, WV) in 1967 and the Mianus River Bridge (Connecticut) in 1983, USA [1]. Those collapses indicated the paramount importance of attention to the condition of older bridges, leading to intensified inspection protocols and numerous eventual retrofits or replacements [2,3]. Therefore corrosion is not an issue to be taken lightly either in design phase or in maintenance stage. Detailed regular inspections are necessary in order to assure adequate safety and determine maintenance requirements, in bridge infrastructure management. But the number of steel bridge infrastructures in the world is steadily increasing as a result of building new steel structures and extending the life of older structures. So, there is a need of more brisk and accurate assessment method which can be used to make reliable decisions affecting the cost and safety.

During the past few decades, several experimental studies and detailed investigations of corroded surfaces were done by some researchers in order to introduce methods of estimating the remaining strength capacities of corroded steel plates [4-7]. But, to develop a more reliable strength estimation technique, only experimental approach is not enough as actual corroded surfaces are different from each other. Further, due to economic constraints, it is not possible to conduct tests for each and every aged bridge structure within their bridge budgets. Therefore, bridge engineers are faced with lack of experimental and field data. Therefore, nowadays, use of numerical analysis method could be considered to have a reliable estimation in bridge maintenance industry [8].

Sidharth *et al.* [9] stated the importance and reviewed the abreast development in the FE analysis technique used to study the corrosion effect on the plates. Ahmmad *et al.* [10] investigated the deformability of corroded steel plates under quasi-static uniaxial tension through both experimental and numerical analyses. They proposed empirical formulae to estimate the reduction in deformability and energy absorption capacity due to pitting corrosion and general corrosion under uniaxial tension. Therefore it can be seen that the finite element analysis method has now become the most common, powerful and flexible tool in rational structural analysis and makes it possible to predict the strength of complex structures more accurately than existing classical theoretical methods.

Even though, it is an exigent task to conduct detail investigations of all existing steel structures as the number of steel structures are steadily increasing in the world, it is necessary to assess those structures in regular basis to ensure their safety and determine the necessary maintenance. Therefore, developing a rapid and accurate methodology to estimate the remaining strength capacities of steel infrastructures is a vital task in maintenance engineering. Therefore, this paper proposes a simple, accurate and brisk analytical method which can be used to make reliable decisions on the maintenance management of existing steel infrastructures.

2. CORRODED TEST SPECIMENS

The test specimens were cut out from a steel girder of Ananai River in Kochi Prefecture on the shoreline of the Pacific Ocean, which had been used for about hundred years. This bridge had simply supported steel plate girders with six spans, with each of 13.5 m. All plate girders were constructed by rivet joints and were exposed to high airborne salt environment by strong sea wind for a long time. It was constructed as a railway bridge in 1900, and in 1975 changed to a pedestrian bridge, when the reinforced concrete slab was cast on main girders. The bridge was dismantled due to serious corrosion damage in year 2001. Many severe corrosion damages distributed all over the girder, especially, large corrosion pits or locally-corroded portions were observed on upper flanges and its cover plates. Then, 21 (F1-F21) and 5 (W1-W5) test specimens were cut out from the cover plate on upper flange and web plate respectively.

Before conducting the thickness measurements, all rusts over both surfaces were removed carefully by using the electric wire brushes and punches. Then, two new SM490A plates

($t=16\text{mm}$) were jointed to both sides of specimen by the butt full penetration welding for grip parts to loading machine, as shown in Figure 1. Here, the flange and web specimens have the widths ranged from 70-80 mm and 170-180 mm respectively. The test specimen configuration is shown in Figure 1. In addition, 4 corrosion-free specimens (JIS5 type) were made of each two from flange and web, and the tensile tests were carried out in order to clarify the material properties of test specimens. The material properties obtained from these tests are shown in Table 1.

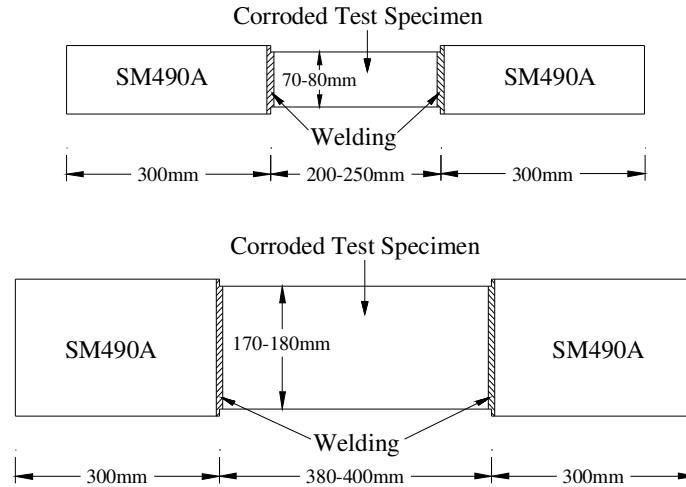


FIGURE 1: Dimensions of test specimens.

Specimen	Elastic modulus /(GPa)	Poisson's ratio	Yield stress /(MPa)	Tensile strength /(MPa)	Elongation at breaking /(%)
Corrosion-free plate (flange)	187.8	0.271	281.6	431.3	40.19
Corrosion-free plate (web)	195.4	0.281	307.8	463.5	32.87
SS400 JIS	200.0	0.300	245~	400~510	–

TABLE 1: Material properties

Accuracy, convenience, portability and lightness are highly demanded for choosing of a device for the on-site measurement of corroded surface irregularities. Therefore, the portable 3-dimensional scanning system, which can measure the 3-dimensional coordinate values at any arbitrary point on the corrosion surface directly and continuously, was used for the measurement of surface irregularities of the test specimens [11]. Here, the thickness of the corroded surface can be calculated easily from those measured coordinates. The measuring device has three arms and six rotational joints, and can measure the coordinates of a point on steel surface by using the non-contact scanning probe (laser line probe). The condition of thickness measurement is shown in Figure 2. Since this probe irradiates the steel surface with a laser beam, which has about 100mm width, the large number of 3-dimensional coordinate data can be obtained easily at a time. So, the thicknesses of all scratched specimens were measured by using this 3D laser scanning device and the coordinate data was obtained in a grid of 0.5mm intervals in both X and Y directions. Then, the remaining thicknesses of all grid points were calculated by using the difference of the coordinate values of both sides of those corroded specimens. Then, the statistical thickness parameters such as average thickness (t_{avg}), minimum thickness (t_{min}), standard deviation of thickness (σ_{st}) and coefficient of variability (CV) were calculated from the measurement results.



FIGURE 2: Condition of thickness measurement.

Even though, various types of corrosion conditions in actual steel structures can be seen as the corrosion damage can take place in many shapes and forms, it is necessary to categorize those different corrosion conditions to few general types for better understanding of their remaining strength capacities. So, in this study, all specimens were categorized into 3 typical corrosion types concerning their visual distinctiveness, amount of corrosion, expected mechanical and ultimate behaviors and minimum thickness ratio, μ (minimum thickness/ initial thickness) [7]. Here, the 3 different types of corrosion levels can be classified according to their severity of corrosion as follows:

- $\mu > 0.75$; Minor Corrosion
- $0.75 \geq \mu \geq 0.5$; Moderate Corrosion
- $\mu < 0.5$; Severe Corrosion

Three specimens F-14, F-13 and F-19 with minor, moderate and severe corrosion conditions respectively are shown in Figure 3.

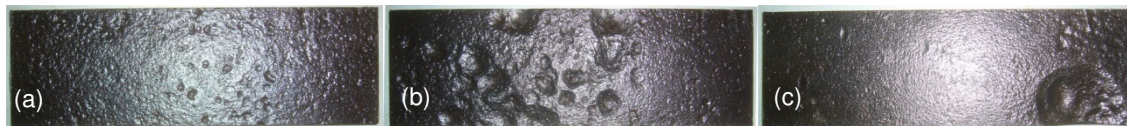


FIGURE 3: Plates with (a) minor [F-14], (b) moderate [F-13] and (c) severe [F-19] corrosion.

3. EXPERIMENTAL ANALYSIS AND RESULTS

Tensile loading tests were carried out at constant velocity under loading control by using a hydraulic loading test machine (maximum load: 2940KN) for all 26 specimens with different corrosion conditions. The loading velocity was set to 200N/sec for minor corroded specimens and 150N/sec for moderate and severe corroded specimens.

Figure 4 shows the load-elongation curves for three different corroded specimens with 3 corrosion types. The specimens F-14 and F-13 have comparatively larger minimum thickness ratio ($\mu = 0.783$ and 0.512 respectively) and the specimen F-19 in which the corrosion progression was more severe, the minimum thickness ratio is also diminutive ($\mu = 0.061$). Herein, the specimen (F-14) with minor corrosion has almost same mechanical properties (such as apparent yield strength and load-elongation behavior etc.) as the corrosion-free specimen. On the other hand, the moderate corroded specimen (F-13) and the severe corroded specimen (F-19) show obscure yield strength and the elongation of the specimen F-19 decreases notably. The

reason for this is believed to be that the local section with a small cross-sectional area yields at an early load stage because of the stress concentration due to irregularity of corroded steel plate. And this will lead moderate and severe corroded members elongate locally and reach to the breaking point.

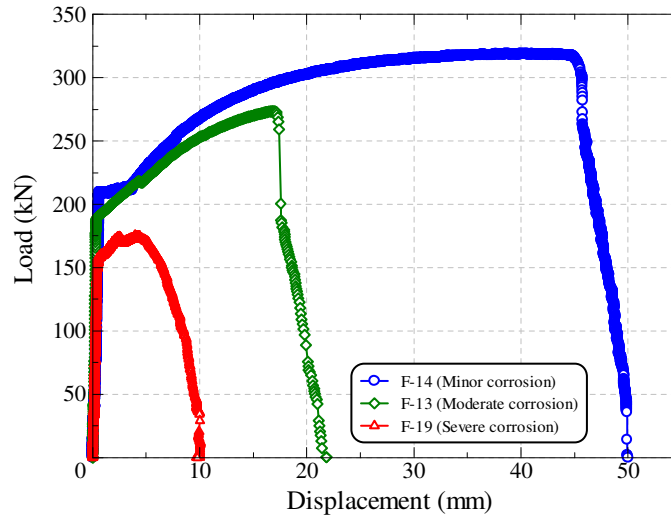


FIGURE 4: Load-displacement curves.

Even though the relation between the breaking section and the thickness distribution is not exactly clarified yet, it can be seen that most of minor corrosion members are failed in a section corresponds to a minimum thickness point (t_{min}) or a minimum average thickness point (t_{sa}) for the members with the thickness variation is very small. Also, it was noted that the breaking point of moderate and severe corrosion members are corresponds to a section of the minimum thickness point. Therefore, the local statistical parameters with the influence of stress concentration can be used for the yield and tensile strength estimations.

4. NUMERICAL INVESTIGATION

4.1 Analytical Model

The 3D isoparametric hexahedral solid element with eight nodal points (HX8M) and updated Lagrangian method based on incremental theory were adopted in these analyses. Non linear elastic-plastic material, Newton-Raphson flow rule and Von Mises yield criterion were assumed for material properties. Further, an automatic incremental-iterative solution procedure was performed until they reached to the pre-defined termination limit.

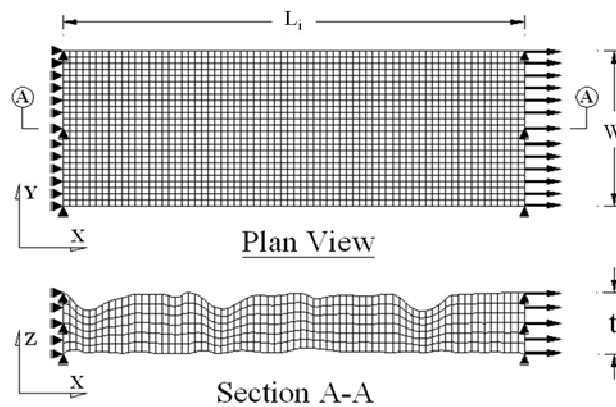


FIGURE 5: Analytical model of corroded member.

The analytical models with different length (L) and width (W) dimensions were modeled with their respective corrosion conditions as shown in Figure 5. One edge of the member's translation in X, Y and Z directions were fixed and only the Y and Z direction translations of the other edge (loading edge) were fixed to simulate with the actual experimental condition. Then the uniform incremental displacements were applied to the loading edge. Yield stress $\sigma_y = 294.7$ [MPa], Elastic modulus $E = 191.6$ [GPa], Poisson's ratio $\nu = 0.276$ were applied to all analytical models, respectively.

4.2 Ductile Fracture Criterion

Fracture is an important mode of failure in steel structures, and accurate assessment of fracture is necessary to understand the ultimate behaviors. The "Stress Modified Critical Strain Model (SMCS)" was proposed by Kavinde et al. (2006), to evaluate the initiation of ductile fracture as a function of multiaxial plastic strains and stresses [12]. This method was adopted in this analytical study. In SMCS criterion, the critical plastic strain ($\epsilon_p^{Critical}$) is determined by the following expression:

$$\epsilon_p^{Critical} = \alpha \cdot \text{Exp}\left(-1.5 \frac{\sigma_m}{\sigma_e}\right) \quad (1)$$

Where, α is toughness index and the stress traxiality $T = (\sigma_m/\sigma_e)$, a ratio of the mean or hydrostatic stress (σ_m) and the effective or von Mises stress (σ_e). The toughness index α is a fundamental material property and hence obtained from the tensile test conducted for the non corroded specimen and obtained as follows:

$$\alpha = \frac{\epsilon_p^{Critical}}{\text{Exp}\left(-1.5 \frac{\sigma_m}{\sigma_e}\right)} \quad (2)$$

The ultimate strength of each corroded specimen was calculated accordingly by using the SMCS criterion and compared with their experimental ultimate capacities to understand the feasibility of the numerical modeling approach for remaining strength estimation of corroded steel plates with different corrosion conditions.

4.3 Analytical Results and Discussion

The yield and ultimate strengths in analytical prediction were estimated and compared with that of the experimentally obtained values to evaluate the accuracy of the used analytical model. The percentage error in yield and tensile strength in analytical predictions are calculated as:

$$\% \text{ Error in } P_y = \left| \frac{P_{y[\text{Analytic al}]} - P_{y[\text{Experime ntal}]}}{P_{y[\text{Experime ntal}]}} \right| \cdot 100 \quad (3)$$

$$\% \text{ Error in } P_b = \left| \frac{P_{b[\text{Analytic al}]} - P_{b[\text{Experime ntal}]}}{P_{b[\text{Experime ntal}]}} \right| \cdot 100 \quad (4)$$

First, analytical modeling of the non-corroded specimen was done with above described modeling and analytical features to understand the accuracy of the adopted procedure. It was found that the analytical model results were almost same as the experimental results with having a negligible percentage error of 0.03% and 0.02% in yield and tensile strengths respectively. Then, all other experimentally successful specimens were modeled accordingly and their yield and ultimate strengths were compared with the experimentally obtained values.

(a) Minor Corrosion Members

The above described analytical procedure was used to model the members with minor corrosion to understand their yield and ultimate behaviors and to obtain their failure surfaces. Comparison of experimental and analytical load-displacement curves of specimen F-14 is shown in Figure 6. Here, the percentage errors in yield and tensile strength predictions of member F-14 are 0.53%

and 0.03% respectively. So, it is revealed that this numerical modeling technique can be used to predict the remaining strength capacities of minor corroded members accurately.

The failure surface was obtained from the most narrowed location of the deformed shape of the analytical model. Figure 7 shows that both experimental and analytical failure surfaces of the specimen F-14 are in good agreement.

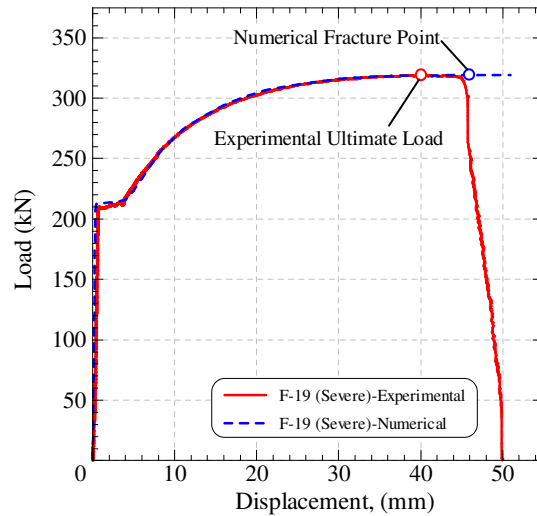


FIGURE 6: Comparison of load-displacement curves of minor corrosion member [F-14].

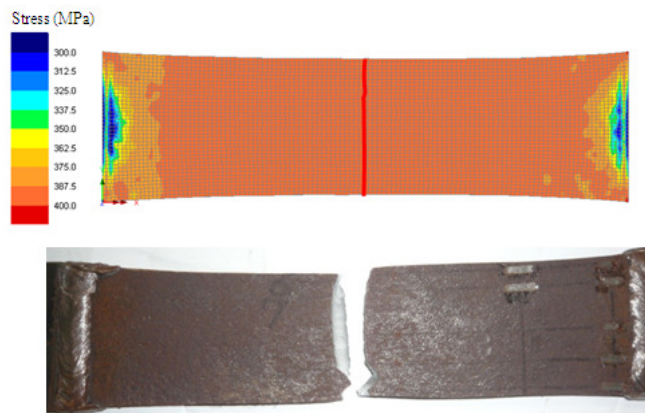


FIGURE 7: Stress distribution at ultimate load of minor corrosion member [F-14].

(b) Moderate Corrosion Members

Then, the members with moderate corrosion were also analyzed and compared with their respective experimental results. The Figure 8 shows the comparison of load-displacement behavior of moderately corroded member, F-13. It shows a very good comparison of experimental and analytical load-displacement behaviors in members with moderate corrosion as well and the percentage errors in yield and tensile strength predictions of member F-13 are 2.96% and 0.70% respectively.

Further, the Figure 9 shows that a similar failure surface could be obtained as in experimental analysis and this indicate the adaptability of this analytical modeling method and SMCS fracture criterion for moderately corroded members too, in order to predict their remaining strength capacities.

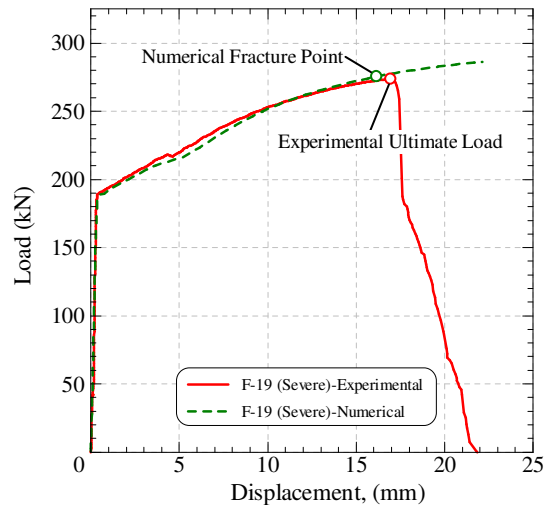


FIGURE 8: Comparison of load-displacement curves of moderate corrosion member [F-13].

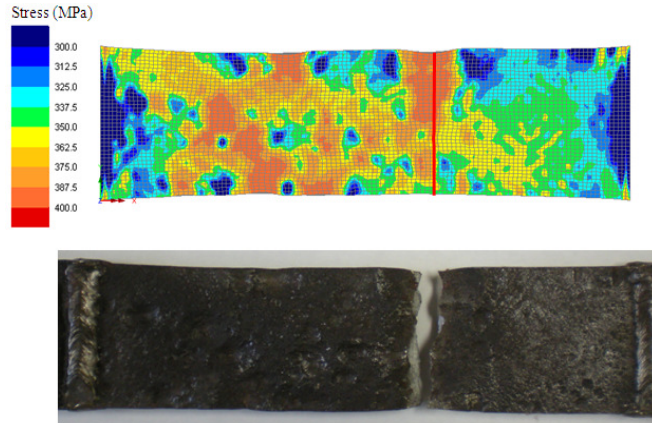


FIGURE 9: Stress distribution at ultimate load of moderate corrosion member [F-13].

(c) Severe Corrosion Members

Finally, the severely corroded members too analyzed and observed their yield and ultimate behaviors and their failure surfaces. The Figure 10 shows the comparison of load-displacement behavior of severely corroded specimen F-19. It shows that an adequate comparison of experimental and analytical load-displacement behaviors can be obtained in members with severe corrosion as well.

The percentage errors in yield and tensile strength predictions of member F-19 shown in Figure 10 are 3.20% and 5.53% respectively. Though the ultimate behavior shows a slight difference, the failure surface shown in Figure 11 is almost same as in the experimental analysis.

Although the results of numerical predictions for yield and ultimate strengths of minor and moderate corroded members show a very good comparison with the experimental results, severe corrosion members show a little bit deviation in their ultimate strength predictions. The reason could be that, some microscopic cracks could also build-up with the development of corrosion which they could eventually results a loss of strength. Hence, there we can see a loss of tensile strength in experimental analyses than expected or predicted by analytical models in severely corroded specimens. So, careful microscopic observations for severe corroded surfaces and smoothing them neatly could reduce such errors with analytical prediction.

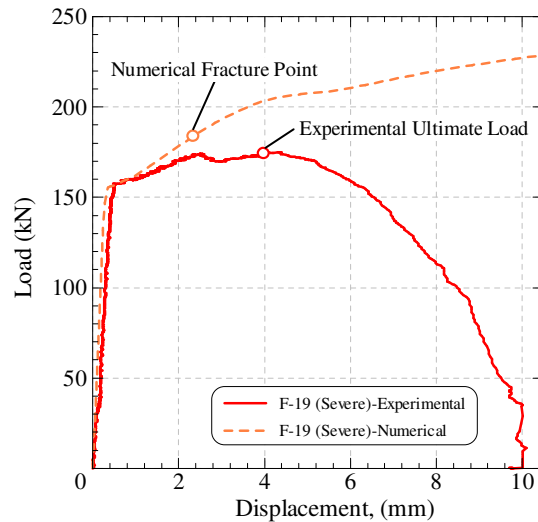


FIGURE 10: Comparison of load-displacement curves of severe corrosion member [F-19].

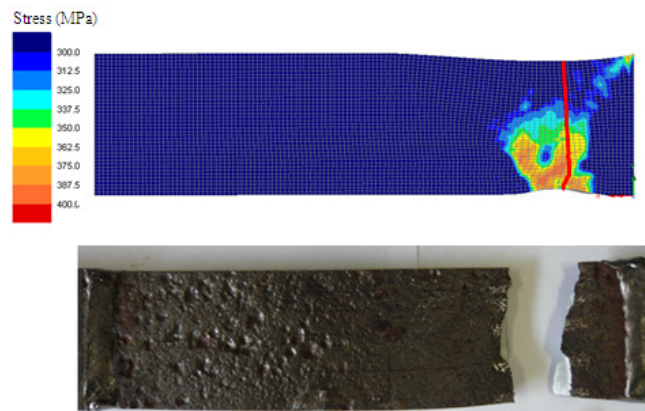


FIGURE 11: Stress distribution at ultimate load of severe corrosion member [F-19].

(d) Discussion

All experimentally successful flange and web specimens were modeled accordingly and their yield and ultimate strengths and failure surfaces were compared with the experimental behaviors. A good comparison of the experimental and analytical load-elongation behaviors for all three classified corrosion types was obtained. Figure 12 shows the comparison of ultimate load capacities of experimental and numerical analyses with 2mm models. Having a coefficient of correlation of $R^2 = 0.994$ indicates the accuracy and the possibility of numerical investigation method to predict the tensile strength of actual corroded specimens.

Failure surfaces obtained from the analytical method and the experimental analysis indicated a very good comparison for all three corrosion categories and hence this fact too signifies the accuracy of the adopted numerical modeling method. Further, it can be seen that the elongation of members with minor corrosion have comparatively same elongation as corrosion free members, where as the moderately corroded members show lesser elongation than the minor corrosion members. Also the members with severe corrosion show significant reduction in elongation as well. The reason for this could be the local section with a small cross-sectional area yields at an early load stage because of the stress concentration due to irregularity of corroded steel plate which elongates locally and reach to the breaking point.

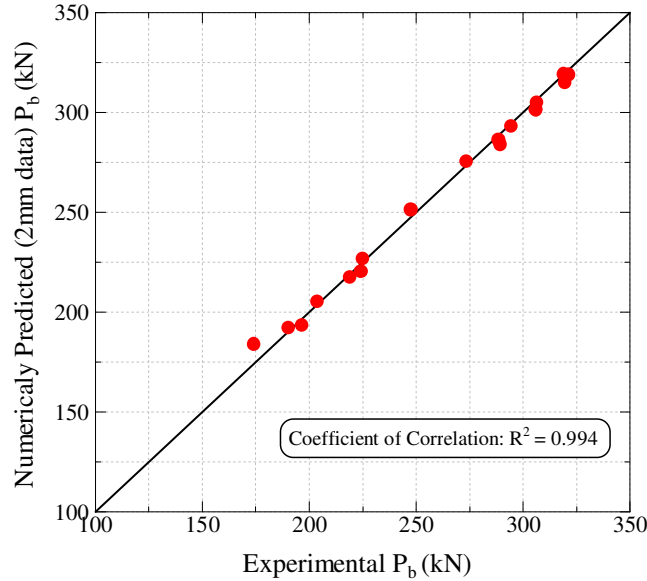


FIGURE 12: Comparison of experimental and analytical ultimate load capacities.

5. DEVELOPMENT OF BRISK ANALYTICAL METHOD

5.1 Corrosion Condition Modeling (CCM) Parameters

Since it is an exigent task to conduct detail corroded surface measurements for all aged steel infrastructures as the number of steel structures are steadily increasing in the world, a simple, accurate and brisk method is deemed necessary to model different corroded surfaces numerically and predict their yield and ultimate behaviors. Therefore two parameters were defined to model the corroded surface considering the stress concentration effect and to obtain the yield and ultimate behaviors more accurately. The following Figure 13(a) shows the variation of diameter of the maximum corroded pit vs. maximum corroded depth ($t_{c,max}$) and Figure 13(b) shows the normalized average thickness (t_{avg}/t_0) vs. normalized maximum corroded depth ($t_{c,max}/t_0$). Both Figures show a very good linear relationship and hence these parameters were used to develop an analytical model which can be used to predict the yield and ultimate behaviors of corroded steel plates with different corrosion conditions.

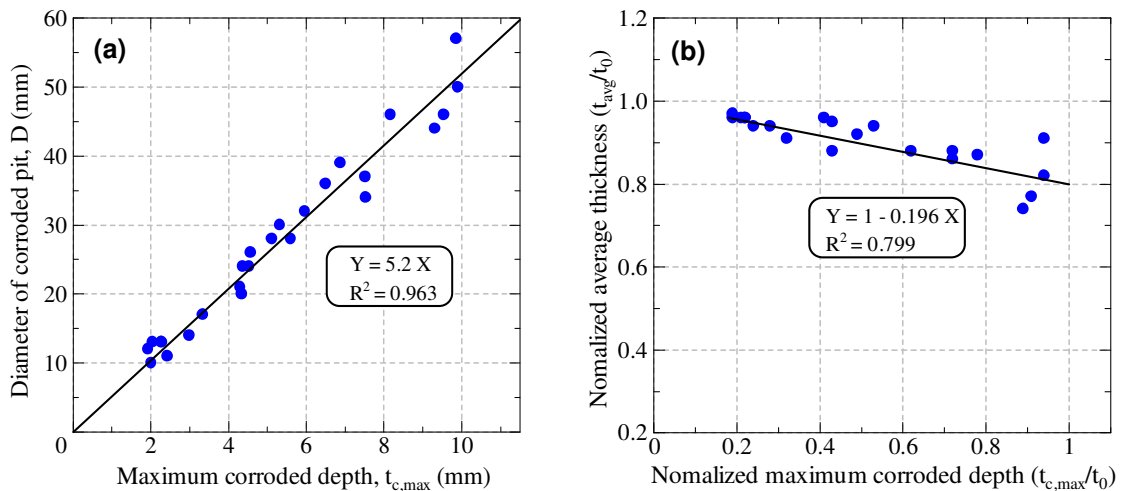


FIGURE 13: Relationship of (a) D vs. $t_{c,max}$ and (b) normalized t_{avg} vs. $t_{c,max}$.

Therefore, by considering the Figures 13(a) and (b), the two equations for the corrosion condition modeling (CCM) parameters can be defined as:

$$D^* = 5.2 t_{c,max} \quad (5)$$

$$t^*_{avg} = t_0 - 0.2 t_{c,max} \quad (6)$$

where D^* and t^*_{avg} are the representative diameter of maximum corroded pit and representative average thickness respectively.

5.2 Analytical Model

An analytical model is developed with the above CCM parameters for each corroded specimen with different corrosion conditions as shown in Figure 14. Here, the maximum corroded pit was modeled by using the representative diameter (D^*) which could account the stress concentration effect and the material loss due to corrosion was considered by using the representative average thickness parameter (t^*_{avg}). The same modeling features and analytical procedure as described in section 4 were adopted for the analyses. Then the results of this model were compared with the experimental results to understand the effectiveness of the proposed model.

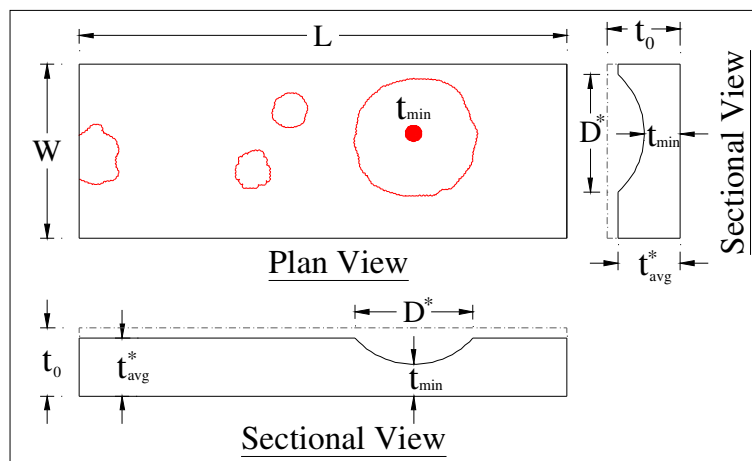


FIGURE 14: Analytical model with CCM parameters.

5.3 Analytical Results and Discussion

The load-elongation behavior of 3 members F-14, F-13 and F-19 with minor, moderate and severe corrosion conditions respectively are shown in Figure 15 below. It was revealed that a very good comparison of the load-elongation behavior can be seen for the all three classified corrosion types. Here, the percentage errors in yield and tensile strength predictions of the proposed analytical model for the three corrosion types are 0.13% and 0.83% in F-14, 0.38% and 1.01% in F-13 and 3.51% and 2.69% in F-19 respectively. Further it was noted that there is no significance in % errors even though they tend to increase with the severity of corrosion.

Then, all experimentally successful specimens were modeled accordingly with the proposed analytical model and their yield and ultimate strengths and failure surfaces were compared with the experimental results. The Figure 16 shows the comparison of ultimate load capacities of proposed model and experimental results. A very good comparison with a coefficient of correlation of $R^2 = 0.992$ was attained. This fact divulges the accuracy of the proposed model and the possibility of the use of proposed analytical model instead of the model with detailed corroded surface measurements. Therefore it can be comprehended that the adopted method and the proposed analytical model can be used to predict the yield and ultimate behaviors precisely. Further it was noticed that a good agreement of the failure surfaces of experimental and proposed model can be obtained too.

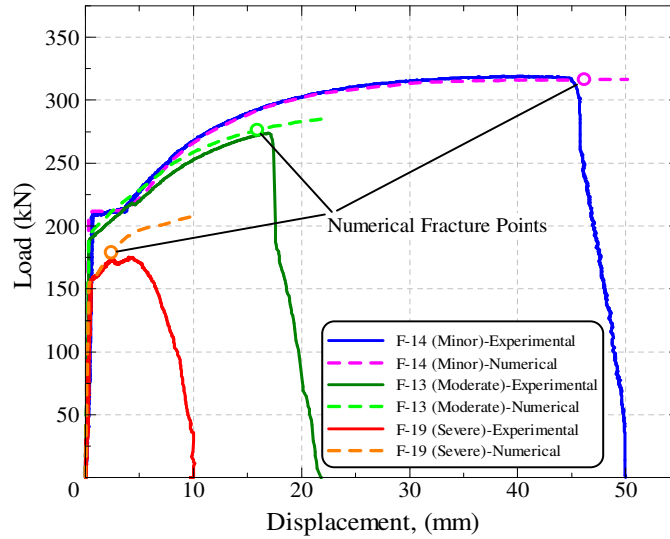


FIGURE 15: Comparison of load-elongation curves of proposed analytical model.

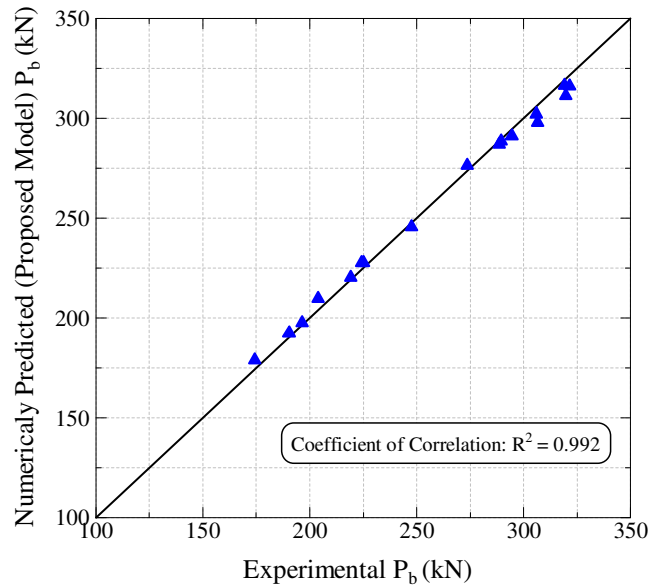


FIGURE 16: Comparison of experimental and proposed model's ultimate load capacities.

Further, it was noted that the available numerical methods and empirical equations developed by various authors require conducting detail corroded surface measurements and the measurement of average thickness and/or the minimum average thickness, in order to develop the analytical model and predict the yield and ultimate behaviors of that corroded members. But, the proposed model require only the measurement of maximum corroded depth ($t_{c,max}$), which can be easily identified through a careful visual inspection of the corroded surface, in order to develop the analytical model considering the stress concentration effect and the material loss due to corrosion. Therefore, this method can be used as a simple, reliable and brisk analytical method for the maintenance management of steel infrastructures.

A further detailed study comprises with experimental and numerical analysis of more specimens with different corrosion conditions is deemed necessary to understand the significance of the proposed method and verify this for different corroded levels and environmental conditions.

6. CONCLUSIONS

The 26 specimens taken out of the scrapped plate girder which had been used for about 100 years with severe corrosion, was used to perform the tensile tests to estimate the mechanical properties of corroded plates and understand the relationship of strength reduction and their level of corrosion. A non-linear FEM analysis was carried out to understand the mechanical behavior, stress distribution, ultimate behavior etc. for members with different corroded conditions. The main conclusions are as follows:

1. The corrosion causes strength reduction of steel plates and minimum thickness ratio (μ) can be used as a measure of the level of corrosion and their strength degradation.
2. A very good agreement between the experimental and non linear FEM results can be seen for all three classified corrosion types. Further, failure surfaces of those specimens too showed a very good comparison with the experimental results. So, the adopted numerical modeling technique can be used to predict the remaining strength capacities of actual corroded members accurately.
3. Proposed analytical model with CCM parameters showed a very good agreement with the experimental results for all three classified corrosion types. Further, the proposed method is simple and gives more accurate remaining strength estimation of corroded steel plates. Therefore this analytical model, developed by the measurement of maximum corroded depth only, can be used as a reliable and brisk method for the maintenance management of corroded steel infrastructures.

7. ACKNOWLEDGEMENTS

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