Artificial Chattering Free on-line Fuzzy Sliding Mode Algorithm for Uncertain System: Applied in Robot Manipulator

Farzin Piltan *SSP.ROBOTIC@yahoo.com*
Department of Electrical and Electronic Engineering,
Faculty of Engineering, Universiti Putra Malaysia 43400
Serdang, Selangor, Malaysia

N. Sulaiman *nasri@eng.upm.edu.my*
Department of Electrical and Electronic Engineering,
Faculty of Engineering, Universiti Putra Malaysia 43400
Serdang, Selangor, Malaysia

Samira Soltani *SSP.ROBOTIC@yahoo.com*
Industrial Electrical and Electronic Engineering
SanatkadeheSabze Pasargad. CO (S.S.P. Co), NO:16 ,
PO.Code 71347-66773, Fourth floor Dena Apr , Seven Tir
Ave , Shiraz , Iran

Samaneh Roosta *SSP.ROBOTIC@yahoo.com*
Industrial Electrical and Electronic Engineering
SanatkadeheSabze Pasargad. CO (S.S.P. Co), NO:16 ,
PO.Code 71347-66773, Fourth floor Dena Apr , Seven Tir
Ave , Shiraz , Iran

Atefeh Gavahian *SSP.ROBOTIC@yahoo.com*
Industrial Electrical and Electronic Engineering
SanatkadeheSabze Pasargad. CO (S.S.P. Co), NO:16 ,
PO.Code 71347-66773, Fourth floor Dena Apr , Seven Tir
Ave , Shiraz , Iran

Abstract

In this research, an artificial chattering free adaptive fuzzy sliding mode control design and application to uncertain robotic manipulator has proposed in order to design high performance nonlinear controller in the presence of uncertainties. Regarding to the positive points in sliding mode controller, fuzzy logic controller and adaptive method, the output has improved. Each method by adding to the previous controller has covered negative points. The main target in this research is design of model free estimator on-line sliding mode fuzzy algorithm for robot manipulator to reach an acceptable performance. Robot manipulators are highly nonlinear, and a number of parameters are uncertain, therefore design model free controller using both analytical and empirical paradigms are the main goal. Although classical sliding mode methodology has acceptable performance with known dynamic parameters such as stability and robustness but there are two important disadvantages as below: chattering phenomenon and mathematical nonlinear dynamic equivalent controller part. To solve the chattering fuzzy logic inference applied instead of dead zone function. To solve the equivalent problems in classical sliding mode controller this paper focuses on applied fuzzy logic method in classical controller. This algorithm works very well in certain environment but in uncertain or various dynamic parameters, it has slight chattering phenomenon. The system performance in sliding mode controller and fuzzy sliding mode controller are sensitive to the sliding function. Therefore, compute the optimum value of sliding function for a system is the next challenge. This problem has solved by adjusting sliding function of the adaptive method continuously in real-time. In this way, the overall system performance has improved with respect to the classical sliding mode controller. This controller solved chattering phenomenon as well as mathematical nonlinear equivalent part by applied fuzzy supervisory method in sliding mode fuzzy controller and tuning the sliding function.

Keywords: chattering phenomenon, chattering free adaptive sliding mode fuzzy control, nonlinear controller, fuzzy logic controller, sliding mode controller, mathematical nonlinear dynamic equivalent controller part.

1. INTRODUCTION

A robot system without any controllers does not to have any benefits, because controller is the main part in this sophisticated system. The main objectives to control robot manipulators are stability and robustness. Many researchers work on designing the controller for robotic manipulators in order to have the best performance. Control of any systems divided in two main groups: linear and nonlinear controller [1].

However, one of the important challenge in control algorithms is to have linear controller behavior for easy implementation of nonlinear systems but these algorithms however have some limitation such as controller working area must to be near system operating point and this adjustment is very difficult especially when the system dynamic parameters have large variations and when the system has hard nonlinearities [1]. Most of robot manipulators which work in industry are usually controlled by linear PID controllers. But the robot manipulator dynamic functions are, nonlinear with strong coupling between joints (low gear ratio), structure and unstructured uncertainty and Multi-Inputs Multi-Outputs (MIMO) which, design linear controller is very difficult especially if the velocity and acceleration of robot manipulator be high and also when the ratio between joints gear be small [2]. To eliminate above problems in physical systems most of control researcher go toward to select nonlinear robust controller.

One of the most important powerful nonlinear robust controllers is Sliding Mode Controller (SMC). Sliding mode control methodology was first proposed in the 1950 [3, 4]. This controller has been analyzed by many researchers in recent years. Many papers about the main theory of SMC are proposed such as references [1, 5, 6]. This controller has been recently used in wide range of areas such as in robotics, process control, aerospace applications and in power converters. The main reason to opt for this controller is its acceptable control performance wide range and solves some main challenging topics in control such as resistivity to the external disturbance and uncertainty. However pure sliding mode controller has some disadvantages. First, chattering problem can caused the high frequency oscillation of the controllers output.

Secondly, sensitive where this controller is very sensitive to the noise when the input signals is very close to zero. Equivalent dynamic formulation is another disadvantage where calculation of equivalent control formulation is difficult since it is depending on the nonlinear dynamic equation [7]. Many papers were presented to solve these problems as reported in [8-11].

Since the invention of fuzzy logic theory in 1965 by Zadeh, it has been used in many areas. Fuzzy Logic Controller (FLC) is one of the most important applications of fuzzy logic theory [12]. This controller can be used to control nonlinear, uncertain and noisy systems. Fuzzy logic control systems, do not use complex mathematically models of plant for analysis. This method is free of some model-based techniques as in classical controllers. It must be noted here that the application of fuzzy logic is not limited only to modeling of nonlinear systems [13-17] but also this method can help engineers to design easier controller. However pure FLC works in many engineering applications but, it cannot guarantee two most important challenges in control, namely, stability and acceptable performance [18].

Some researchers had applied fuzzy logic methodology in sliding mode controllers (FSMC) in order to reduce the chattering and to solve the nonlinear dynamic equivalent problems in pure sliding mode controller (FSMC) [19-23, 63-68] and the other researchers applied sliding mode methodology in fuzzy logic controller (SMFC) as to improve the stability of the systems [24-28].

Adaptive control used in systems whose dynamic parameters are varying and need to be trained on line. In general states adaptive control can be classified into two main groups: traditional adaptive method and fuzzy adaptive method, where traditional adaptive method need to have some information about dynamic plant and some dynamic parameters must be known but fuzzy adaptive method can train the variation of parameters by expert knowledge. Adaptive fuzzy inference system provide a good knowledge tools to adjust a complex uncertain nonlinear system with changing dynamics to have an acceptable performance [29] Combined adaptive method to artificial sliding mode controllers can help the controllers to have a better performance by online tuning the nonlinear and time variant parameters [30-35, 61-68].

This paper is organized as follows: In section 2, main subject of proposed methodology is presented. Detail of fuzzy logic controllers and fuzzy rule base, the main subject of sliding mode controller and formulation, the main subject of designing fuzzy sliding mode controller and the design of sliding mode fuzzy artificial chatter free fuzzy sliding mode controller are presented which this method is used to reduce the chattering and estimate the equivalent part. In section 3, modeling robot manipulator and PUMA robot manipulator formulation are presented. This section covers the following details, introducing the dynamic formulation of robot manipulator and calculates the dynamic equation of PUMA robot manipulator. the simulation result is presented in section 4 and finally in section 5, the conclusion is presented.

2. PROPOSED METHODOLOGY

Sliding Mode Controller: This section provides a review of classical sliding mode control and the problem of formulation based on [4]; [37-39, 61-68]. Basically formulation of a nonlinear single input dynamic system is:

$$x^{(n)} = f(\vec{x}) + b(\vec{x})u \quad \textbf{(1)}$$

Where u is the vector of control input, $x^{(n)}$ is the n^{th} derivation of x, $x = [x, \dot{x}, \ddot{x}, \dots, x^{(n-1)}]^T$ is the state vector, $f(x)$ is unknown or uncertainty, and $b(x)$ is of known *sign* function. The control input has the following form:

$$\tau_{(q,t)} = \begin{cases} \tau_i^{+}(q,t) & if\ S_i > 0 \\ \tau_i^{-}(q,t) & if\ S_i < 0 \end{cases} \quad \textbf{(2)}$$

The control problem is truck to the desired state it means that $x_d = [x_d, \dot{x}_d, \ddot{x}_d, \dots, x_d^{(n-1)}]^T$, and have an acceptable error which is given by:

$$\tilde{x} = x - x_d = [\tilde{x}, \dots, \tilde{x}^{(n-1)}]^T \quad \textbf{(3)}$$

A time-varying sliding surface $s(x,t)$ is given by the following equation [61-68]:

$$s(x,t) = (\frac{d}{dt} + \lambda)^{n-1}\, \tilde{x} = 0 \quad \textbf{(4)}$$

where λ is the constant and it is positive. The main derivative of S is

$$\dot{S} = \ddot{q}_d + \lambda \dot{e} \quad \textbf{(5)}$$

The Lyapunov function V is defined as:

$$V = \frac{1}{2} S^T M S \quad \textbf{(6)}$$

Based on the above discussion, the control law for a multi DOF robot manipulator can be written as:

$$U = U_{eq} + U_{dis} \quad \textbf{(7)}$$

Where, the model-based component U_{eq} compensate for the nominal dynamics of the systems. So U_{eq} can be calculated as follows:

$$U_{eq} = \left[M^{-1}(B + C + G) + \dot{S}\right]M \quad \textbf{(8)}$$

A simple solution to get the sliding condition when the dynamic parameters have uncertainty is the switching control law:

$$U_{dis} = K(\vec{x},t).\,sgn(s) \qquad sgn(s) = \begin{cases} 1 & s > 0 \\ -1 & s < 0 \\ 0 & s = 0 \end{cases} \tag{9}$$

Where the $K(\vec{x},t)$ is the positive constant. Since the control input U has to be a discontinuous term, the control switching could not to be perfect and this will have chattering. Chattering can caused the high frequency oscillation of the controllers output and fast breakdown of mechanical elements in actuators. Chattering is one of the most important challenging in sliding mode controllers which, many papers have been presented to solve this problems [39]. To reduce chattering many researchers introduced the boundary layer methods, which in this method the basic idea is to replace the discontinuous method by saturation (linear) method with small neighbourhood of the switching surface. Several papers have been presented about reduce the chattering [27]; [18]; [40]. Therefore the saturation function $Sat(S/_{\emptyset})$ added to the control law:

$$U = K(\vec{x},t).\,Sat\left(S/_{\emptyset}\right) \qquad sat\left(S/_{\emptyset}\right) = \begin{cases} 1 & (S/_{\emptyset} > 1) \\ -1 & (S/_{\emptyset} < 1) \\ S/_{\emptyset} & (-1 < S/_{\emptyset} < 1) \end{cases} \tag{10}$$

where $\emptyset$ is the width of the boundary layer, therefore the control output can be write as

$$U = U_{eq} + K.\,sat\left(S/_{\emptyset}\right) = \begin{cases} U_{eq} + K.\,sgn(S) & ,|S| \ge \emptyset \\ U_{eq} + K.\,S/_{\emptyset} & ,|S| < \emptyset \end{cases} \tag{11}$$

Suppose that the dynamic formulation of robot manipulate is written by the following equation [61-68]:

$$\tau = M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) \tag{12}$$

the lyapunov formulation can be written as follows,

$$V = \frac{1}{2}S^T.M.S \tag{13}$$

the derivation of V can be determined as,

$$\dot{V} = \frac{1}{2}S^T.\dot{M}.S + S^T\ M\dot{S} \tag{14}$$

the dynamic equation of robot manipulator can be written based on the sliding surface as

$$M\dot{S} = -VS + M\dot{S} + VS + G - \tau \tag{15}$$

it is assumed that

$$S^T(\dot{M} - 2V)S = 0 \tag{16}$$

by substituting (15) in (14)

$$\dot{V} = \frac{1}{2}S^T\dot{M}S - S^TVS + S^T(M\dot{S} + VS + G - \tau) = S^T(M\dot{S} + VS + G - \tau) \tag{17}$$

suppose the control input is written as follows

$$\hat{\tau} = \widehat{\tau_{eq}} + \widehat{\tau_{dis}} = [\widehat{M^{-1}}(\hat{V} + \hat{G}) + \dot{S}]\hat{M} + K.\,sgn(S) + K_vS \tag{18}$$

by replacing the equation (18) in (17)

$$\dot{V} = S^T(M\dot{S} + VS + G - \hat{M}\dot{S} - \hat{V}S - \hat{G} - K_vS - Ksgn(S)) = S^T\left(\tilde{M}\dot{S} + \tilde{V}S + \tilde{G} - K_vS - Ksg\right. \tag{19}$$

it is obvious that

$$\left|\hat{M}\dot{S} + \tilde{V}S + \tilde{G} - K_v S\right| \leq \left|\hat{M}\dot{S}\right| + \left|\tilde{V}S\right| + \left|\tilde{G}\right| + |K_v S| \tag{20}$$

the Lemma equation in robot manipulator system can be written as follows

$$K_u = \left[\left|\hat{M}\dot{S}\right| + |VS| + |G| + |K_v S| + \eta\right]_i, i = 1, 2, 3, 4, \ldots \tag{21}$$

the equation (33) can be written as

$$K_u \geq \left|\left[\hat{M}\dot{S} + VS + G - K_v S\right]_i\right| + \eta_i \tag{22}$$

therefore, it can be shown that [63-68]

$$\dot{V} \leq -\sum_{i=1}^{n} \eta_i |S_i| \tag{23}$$

Consequently the equation (40) guaranties the stability of the Lyapunov equation

Combinations of fuzzy logic systems with sliding mode method have been proposed by several researchers. As mention previously, SMFC is fuzzy controller based on sliding mode method for easy implementation, stability, and robustness. The SMFC initially proposed by Palm to design nonlinear approximation boundary layer instead of linear approximation [27]. The main drawback of SMFC is the value of sliding surface λ which must be pre-defined. The most important advantage of SMFC compare to pure SMC is design a nonlinearity boundary layer. The system performance is sensitive to the sliding surface sloop λ for both sliding mode controller and sliding mode fuzzy controller application. For instance, if large value of λ are chosen the response is very fast but the system is very unstable and conversely, if small value of λ is considered the response of the system is very slow but the system is usually stable. Therefore, calculation the optimum value of λ for a system is one of the most important challenges. Even though most of time the control performance of FLC and SMFC is similar, the SMFC has two most important advantages;

The number of rule base is smaller and
better robustness and stability.
Several papers have been proposed on this method and several researchers' works in this area [41-46]. To compensate the nonlinearity for dynamic equivalent control several researchers used model base fuzzy controller instead of classical equivalent controller that was employed to obtain the desired control behaviour and a fuzzy switching control was applied to reinforce system performance. In the proposed fuzzy sliding mode control fuzzy rule base was designed to estimate the dynamic equivalent part. A block diagram for proposed fuzzy sliding mode controller is shown in Figure 1. In this method fuzzy rule for sliding surface (S) to design fuzzy error base-like equivalent control was obtained the rules whereused instead of nonlinear dynamic equation of equivalent control to reduce the chattering and also to eliminate the nonlinear formulation of dynamic equivalent control term.

$$\begin{gathered} 1 > \text{if } S \text{ is } N.B \text{ then } \tau \text{ is } N.B \\ 2 > \text{if } S \text{ is } Z \text{ then } \tau \text{ is } Z \end{gathered} \tag{24}$$

In FSMC the tracking error is defined as:

$$e = q_d - q_a \tag{25}$$

where $q_d = [q_{1d}, q_{2d}, q_{3d}]^T$ is desired output and $q_a = [q_{1a}, q_{2a}, q_{3a}]^T$ is an actual output. The sliding surface is defined as follows:

$$S = \dot{e} + \lambda e \tag{26}$$

where $\lambda = diag[\lambda_1, \lambda_2, \lambda_3]$ is chosen as the bandwidth of the robot manipulator controller. The time derivative of S can be calculated by the following equation

$\dot{S} = \ddot{q}_d + \lambda_1 \dot{e}$ **(27)**

Based on classical SMC the FSMC can be calculated as

$\hat{U} = U_{fuzzy} + U_r$ **(28)**

Where, the model-based component $\hat{U}_{eq}$ compensate for the nominal dynamics of systems. So $\hat{\tau}_{eq}$ can be calculated as

$U_{fuzzy} = [M^{-1}(B + C + G) + \dot{S}]M$ **(29)**

and U_r is

$U_{sat} = K.sat(S)$ **(30)**

To eliminate the chattering fuzzy inference system is used instead of saturation function to design nonlinear sliding function which as a summary the design of fuzzy logic controller for FSMC has five steps:

1. **Determine inputs and outputs:** This controller has one input (S) and one output (α). The input is sliding function (S) and the output is coefficient which estimate the saturation function (α).
2. **Find membership function and linguistic variable:** The linguistic variables for sliding surface (S) are; Negative Big (NB), Negative Medium (NM), Negative Small (NS), Zero (Z), Positive Small (PS), Positive Medium (PM), Positive Big (PB), and the linguistic variables to find the saturation coefficient (α) are; Large Left (LL), Medium Left (ML), Small Left (SL), Zero (Z), Small Right (SR), Medium Right (MR), Large Right (LR).
3. **Choice of shape of membership function:** In this work triangular membership function was selected.
4. **Design fuzzy rule table:** design the rule base of fuzzy logic controller can play important role to design best performance FSMC, suppose that two fuzzy rules in this controller are

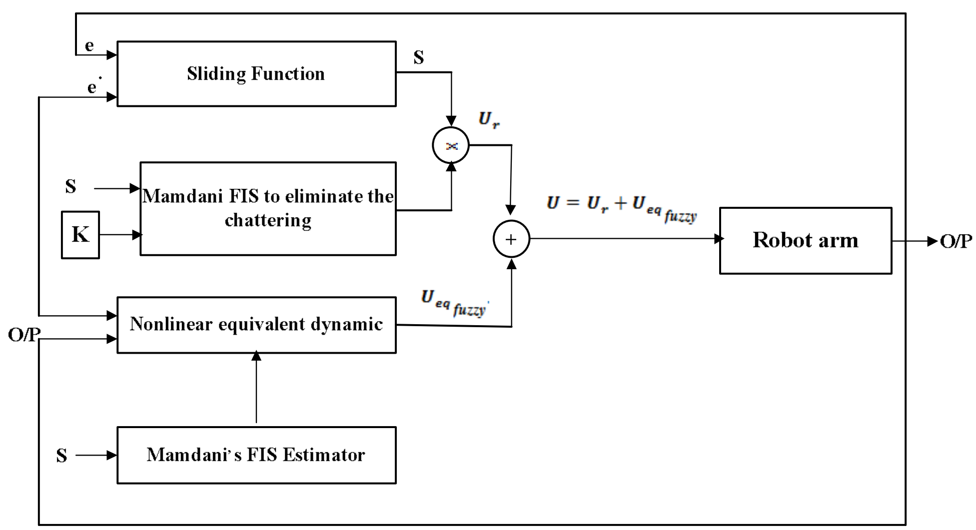

FIGURE 1: Block diagram of proposed artificial chattering free FSMC with minimum rule base

F.R[1]: ***IF S is Z, THEN α is Z.*** **(31)**
F.R[2]: ***IF S is (PB) THEN α is (LR).***

The complete rule base for this controller is shown in Table 1.

TABLE 1: Rule table for proposed FSMC

S	NB	NM	NS	Z	PS	PM	PB
τ	LL	ML	SL	Z	SR	MR	LR

The control strategy that deduced by Table1 are

- If sliding surface (S) is N.B, the control applied is N.B for moving S to S=0.
- If sliding surface (S) is Z, the control applied is Z for moving S to S=0.

5. **Defuzzification**: The final step to design fuzzy logic controller is deffuzification , there are many deffuzzification methods in the literature, in this controller the COG method will be used, where this is given by

$$COG(x_k,y_k)=\frac{\sum_i U_i \sum_{j=1}^{r} \mu_u(x_k,y_k,U_i)}{\sum_i \sum_{j=1}^{r} \mu_u(x_k,y_k,U_i)} \quad (32)$$

The fuzzy system can be defined as below

$$f(x)=\tau_{fuzzy}=\sum_{l=1}^{M}\theta^T\zeta(x)=\psi(e,\dot{e}) \quad (33)$$

where $\theta=(\theta^1,\theta^2,\theta^3,\dots\dots,\theta^M)^T,\zeta(x)=(\zeta^1(x),\zeta^2(x),\zeta^3(x),\dots\dots,\zeta^M(x))^T$

$$\zeta^1(x)=\frac{\sum_i \mu_{(xi)}x_i}{\sum_i \mu_{(xi)}} \quad (34)$$

where $\theta=(\theta^1,\theta^2,\theta^3,\dots\dots,\theta^M)$ is adjustable parameter in (B.1) and $\mu_{(xi)}$ is membership function. error base fuzzy controller can be defined as

$$\tau_{fuzzy}=\psi(e,\dot{e}) \quad (35)$$

According to the formulation (43)

$$if\ S=0\ then\ -\dot{e}=\lambda e \quad (36)$$

the fuzzy division can be reached the best state when $S.\dot{S}<0$ and the error is minimum by the following formulation

$$\theta^*=\arg\min\left[Sup_{x\in U}\left|\sum_{l=1}^{M}\theta^T\zeta(x)-\tau_{equ}\right|\right] \quad (37)$$

Where $\theta^*$is the minimum error, $sup_{x\in U}\left|\sum_{l=1}^{M}\theta^T\zeta(x)-\tau_{equ}\right|$ is the minimum approximation error. Figure 2 is shown the fuzzy instead of saturation function.

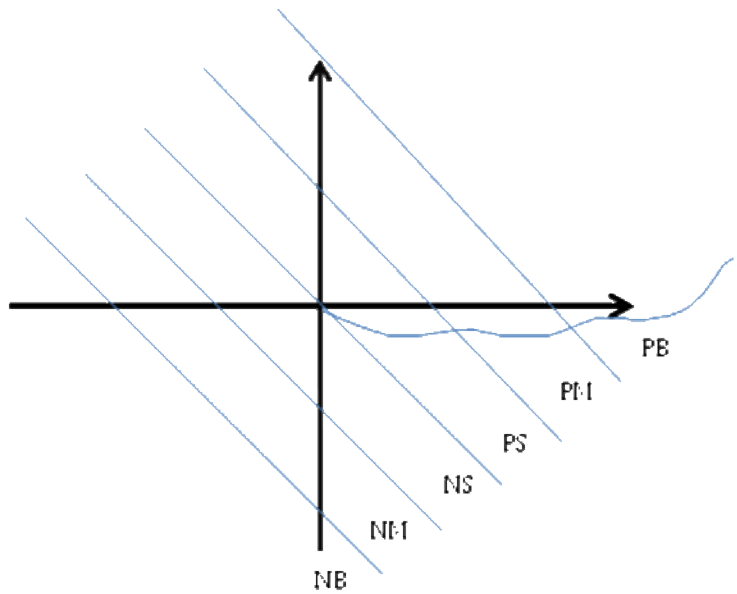

FIGURE 2: Nonlinear fuzzy inference system instead of saturation function

The system performance in FSMC is sensitive to sliding surface slope, λ. Thus, determination of an optimum λ value for a system is an important problem. If the system parameters are unknown or uncertain, the problem becomes more highlighted. This problem may be solved by adjusting the surface slope and boundary layer thickness of the sliding mode controller continuously in real-time. Several researchers' works on adaptive sliding mode control and their applications in robotic manipulator has been investigated in [30-35]; [47-58]. In this way, the performance of the overall system can be improved with respect to the classical sliding mode controller.

This section focuses on, self tuning gain updating factor for two most important factor in FSMC, namely, sliding surface slop ($\boldsymbol{\lambda}$) and boundary layer thickness ($\emptyset$). Self tuning-FSMC has strong resistance and can solve the uncertainty problems. Several researchers work on AFSMC in robot manipulator [24-28]; [59-60]. The block diagram for this method is shown in Figure 3. In this controller the actual sliding surface gain ($\boldsymbol{\lambda}$) is obtained by multiplying the sliding surface with gain updating factor$(\boldsymbol{\alpha})$. The gain updating factor$(\boldsymbol{\alpha})$ is calculated on-line by fuzzy dynamic model independent which has sliding surface (S) as its inputs. The gain updating factor is independent of any dynamic model of robotic manipulator parameters. Assuming that$\boldsymbol{\alpha} = \mathbf{1}$, following steps used to tune the controller: adjust the value of $\boldsymbol{\lambda}$, $\boldsymbol{\emptyset}$ and $\boldsymbol{\alpha}$ to have an acceptable performance for any one trajectory by using trial and error. Some researcher design MIMO adaptive fuzzy sliding mode controller [30-31]; [35] and also someone design SISO adaptive fuzzy sliding mode controller [32]; [34].

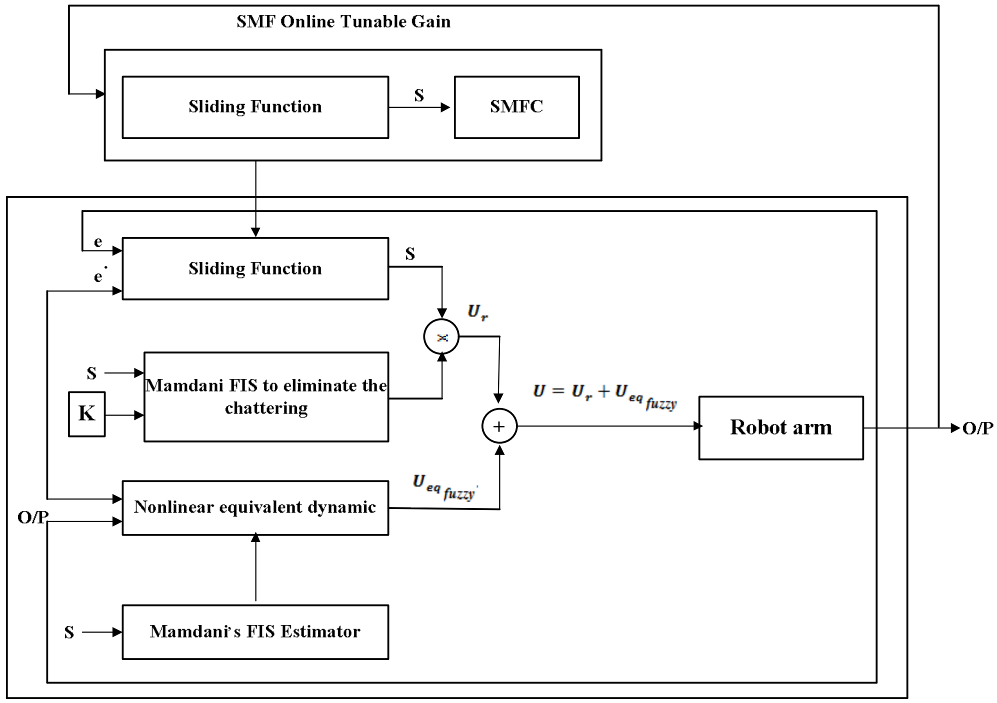

FIGURE 3: Block diagram of proposed artificial chattering free self tuning fuzzy sliding mode controller with minimum rule base in fuzzy equivalent part and fuzzy supervisory.

The adaptive controller is used to find the minimum errors of $\theta - \theta^*$.

suppose K_j is defined as follows

$$K_j = \frac{\sum_{l=1}^{M} \theta_j^l [\mu_A(S_j)]}{\sum_{l=1}^{M} [\mu_A(S_j)]} = \theta_j^T \zeta_j(S_j) \tag{38}$$

Where $\zeta_j(S_j) = [\zeta_j^1(S_j), \zeta_j^2(S_j), \zeta_j^3(S_j), \ldots\ldots, \zeta_j^M(S_j)]^T$

$$\zeta_j^1(S_j) = \frac{\mu_{(A)}{}_j^l(S_j)}{\sum_i \mu_{(A)}{}_j^l(S_j)} \tag{39}$$

the adaption low is defined as

$$\dot{\theta}_j = \gamma_{sj} S_j \zeta_j(S_j) \tag{40}$$

where the γ_{sj} is the positive constant.

According to the formulation (11) and (12) in addition from (10) and (40)

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) = \sum_{l=1}^{M} \theta^T \zeta(x) - \lambda S - K \tag{41}$$

The dynamic equation of robot manipulator can be written based on the sliding surface as;

$$M\dot{S} = -VS + M\dot{S} + VS + G - \tau \tag{42}$$

It is supposed that

$$S^T(\dot{M} - 2V)S = 0 \tag{43}$$

it can be shown that

$$M\dot{S} + (V + \lambda)S = \Delta f - K \quad \textbf{(44)}$$

where $\Delta f = [M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q)] - \sum_{l=1}^{M} \theta^T \zeta(x)$
as a result $\dot{V}$ is became

$$\dot{V} = \frac{1}{2}S^T \dot{M} S - S^T V S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j$$
$$= S^T(-\lambda S + \Delta f - K) + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j$$
$$= \sum_{j=1}^{m} [S_j (\Delta f_j - K_j)] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j$$
$$= \sum_{j=1}^{m} [S_j (\Delta f_j - \theta_j^T \zeta_j(S_j))] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j$$
$$= \sum_{j=1}^{m} [S_j (\Delta f_j - (\theta_j^*)^T \zeta_j(S_j) + \phi_j^T \zeta_j(S_j))] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j$$
$$= \sum_{j=1}^{m} [S_j (\Delta f_j - \left((\theta_j^*)^T \zeta_j(S_j)\right)] - S^T \lambda S)] + \sum (\frac{1}{\gamma_{sj}} \phi_j^T [\gamma_{sj} \zeta_j(S_j) S_j + \dot{\phi}_j])$$

where $\dot{\theta}_j = \gamma_{sj} S_j \zeta_j(S_j)$ is adaption law, $\dot{\phi}_j = -\dot{\theta}_j = -\gamma_{sj} S_j \zeta_j(S_j)$,
consequently $\dot{V}$ can be considered by

$$\dot{V} = \sum_{j=1}^{m} [S_j \Delta f_j - \left((\theta_j^*)^T \zeta_j(S_j)\right)] - S^T \lambda S \quad \textbf{(45)}$$

the minimum error can be defined by

$$e_{mj} = \Delta f_j - \left((\theta_j^*)^T \zeta_j(S_j)\right) \quad \textbf{(46)}$$

$\dot{V}$ is intended as follows

$$\dot{V} = \sum_{j=1}^{m} [S_j e_{mj}] - S^T \lambda S$$
$$\leq \sum_{j=1}^{m} |S_j||e_{mj}| - S^T \lambda S$$
$$= \sum_{j=1}^{m} |S_j||e_{mj}| - \lambda_j S_j^{\,2}$$
$$= \sum_{j=1}^{m} |S_j|(|e_{mj}| - \lambda_j S_j) \quad \textbf{(47)}$$

For continuous function $g(x)$, and suppose $\varepsilon > 0$ it is defined the fuzzy logic system in form of (36) such that

$$Sup_{x \in U} |f(x) - g(x)| < \epsilon \quad \textbf{(48)}$$

the minimum approximation error (e_{mj}) is very small.

$$if \;\; \lambda_j = \alpha \quad that \quad \alpha|S_j| > e_{mj} \; (S_j \neq 0) \quad then \quad \dot{V} < 0 \; for \; (S_j \neq 0) \quad \textbf{(49)}$$

3. APPLICATION: PUMA ROBOT MANIPULATOR

Dynamic modelling of robot manipulators is used to describe the behaviour of robot manipulator, design of model based controller, and simulation results. The dynamic modelling describe the relationship between joint motion, velocity, and accelerations to force/torque or current/voltage and also it can be used to describe the particular dynamic effects (e.g., inertia, coriolios, centrifugal, and the other parameters) to behaviour of system. It is well known that the equation of an *n-DOF* robot manipulator governed by the following equation [36]; [2]:

$$M(q)\ddot{q} + N(q,\dot{q}) = \tau \quad \textbf{(50)}$$

Where τ is actuation torque, $M(q)$ is a symmetric and positive define inertia matrix, $N(q,\dot{q})$ is the vector of nonlinearity term. This robot manipulator dynamic equation can also be written in a following form:

$$\tau = M(q)\ddot{q} + B(q)[\dot{q}\,\dot{q}] + C(q)[\dot{q}]^2 + G(q) \quad \textbf{(51)}$$

Where $B(q)$ is the matrix of coriolios torques, $C(q)$ is the matrix of centrifugal torques, and $G(q)$ is the vector of gravity force. The dynamic terms in equation (50) are only manipulator position. This is a decoupled system with simple second order linear differential dynamics. In other words, the component $\ddot{q}$ influences, with a double integrator relationship, only the joint variableq_i, independently of the motion of the other joints. Therefore, the angular acceleration is found as to be [2]:

$$\ddot{q} = M^{-1}(q).\{\tau - N(q,\dot{q})\} \quad \textbf{(52)}$$

This technique is very attractive from a control point of view. The three degrees of freedom PUMA robot has the same configuration space equation general form as its 6-DOF convenient. In this type, the last three joints are blocked, so, only three links of PUMA robot are used in this paper, $q_4 = q_5 = q_6 = 0$. The dynamic equation of PUMA robot manipulator is given by [61-68]

$$M(\theta)\begin{bmatrix}\ddot{\theta}1\\ \ddot{\theta}_2\\ \ddot{\theta}_3\end{bmatrix} + B(\theta)\begin{bmatrix}\dot{\theta}_1\dot{\theta}_2\\ \dot{\theta}_1\dot{\theta}_3\\ \dot{\theta}_2\dot{\theta}_3\end{bmatrix} + C(\theta)\begin{bmatrix}\dot{\theta}_1^2\\ \dot{\theta}_2^2\\ \dot{\theta}_3^2\end{bmatrix} + G(\theta) = \begin{bmatrix}\tau_1\\ \tau_2\\ \tau_3\end{bmatrix} \quad \textbf{(53)}$$

Where

$$M(q) = \begin{bmatrix} M_{11} & M_{12} & M_{13} & 0 & 0 & 0\\ M_{21} & M_{22} & M_{23} & 0 & 0 & 0\\ M_{31} & M_{32} & M_{33} & 0 & M_{35} & 0\\ 0 & 0 & 0 & M_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & M_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & M_{66}\end{bmatrix} \quad \textbf{(54)}$$

$$B(q) = \begin{bmatrix} b_{112} & b_{113} & 0 & b_{115} & 0 & b_{123} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & b_{214} & 0 & 0 & b_{223} & 0 & b_{225} & 0 & 0 & b_{235} & 0 & 0 & 0 & 0\\ 0 & 0 & b_{314} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ b_{412} & b_{412} & 0 & b_{415} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & b_{514} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \quad \textbf{(55)}$$

$$C(q) = \begin{bmatrix} 0 & C_{12} & C_{13} & 0 & 0 & 0\\ C_{21} & 0 & C_{23} & 0 & 0 & 0\\ C_{31} & C_{32} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ C_{51} & C_{52} & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \quad \textbf{(56)}$$

$$G(q) = \begin{bmatrix} 0\\ g_2\\ g_3\\ 0\\ g_5\\ 0\end{bmatrix} \quad \textbf{(57)}$$

Suppose $\ddot{q}$ is written as follows

$$\ddot{q} = M^{-1}(q).\{\tau - [B(q)\dot{q}\dot{q} + C(q)\dot{q}^2 + g(q)]\} \quad \textbf{(58)}$$

and K is introduced as

$$I = \{\tau - [B(q)\dot{q}\dot{q} + C(q)\dot{q}^2 + g(q)]\} \quad (59)$$

$\ddot{q}$ can be written as

$$\ddot{q} = M^{-1}(q).I \quad (60)$$

Therefore I for PUMA robot manipulator can be calculated by the following equation

$$I_1 = \tau_1 - [\, b_{112}\dot{q}_1\dot{q}_2 + b_{113}\dot{q}_1\dot{q}_3 + 0 + b_{123}\dot{q}_2\dot{q}_3] - [\, C_{12}\dot{q}_2{}^2 + C_{13}\dot{q}_3{}^2] - g_1 \quad (60)$$

$$I_2 = \tau_2 - [\, b_{223}\dot{q}_2\dot{q}_3] - [\, C_{21}\dot{q}_1{}^2 + C_{23}\dot{q}_3{}^2] - g_2 \quad (61)$$

$$I_3 = \tau_3 - [C_{31}\dot{q}_1{}^2 + C_{32}\dot{q}_2{}^2] - g_3 \quad (62)$$

$$I_4 = \tau_4 - [\, b_{412}\dot{q}_1\dot{q}_2 + b_{413}\dot{q}_1\dot{q}_3] - g_4 \quad (63)$$

$$I_5 = \tau_5 - [\, C_{51}\dot{q}_1{}^2 + C_{52}\dot{q}_2{}^2] - g_5 \quad (64)$$

$$I_6 = \tau_6 \quad (65)$$

4. RESULTS

Classical sliding mode control (SMC), fuzzy sliding mode control (FSMC) and artificial chattering free adaptive FSMC are implemented in Matlab/Simulink environment. Changing updating factor performance, tracking performance, error, and robustness are compared.

• Changing Sliding Surface Slope Performance

For various value of sliding surface slope (λ) in SMC, AFSMC and ASMFC, the error and trajectory performances are shown in Figures 4 to 7.

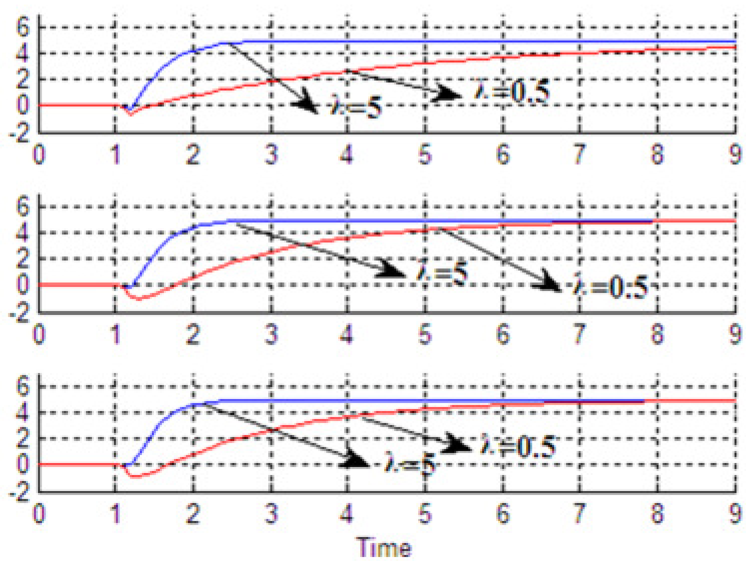

FIGURE 4: SMC trajectory performance, first; second and third link

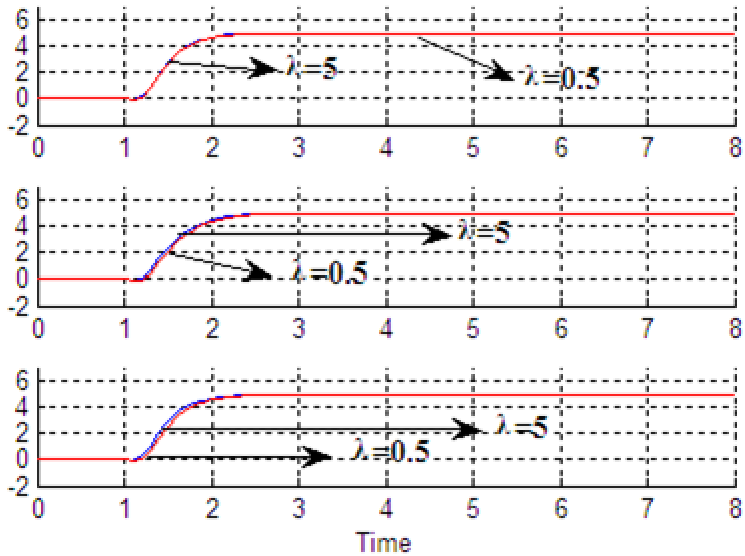

FIGURE 5: Artificial chattering free adaptive FSMC trajectory performance, first; second and third link

Figures 4 and 5 are shown trajectory performance with different sliding function for, Artificial chattering free adaptive FSMC and SMC. It is shown that the sensitivity in Artificial chattering free adaptive FSMC to sliding function is lower than SMC. Figures 6 and 7 are shown the error performance with different sliding surface slope in classical SMC and Artificial chattering free adaptive FSMC. For various sliding surface slope (λ), Artificial chattering free adaptive FSMC, has better error performance compare to classical SMC.

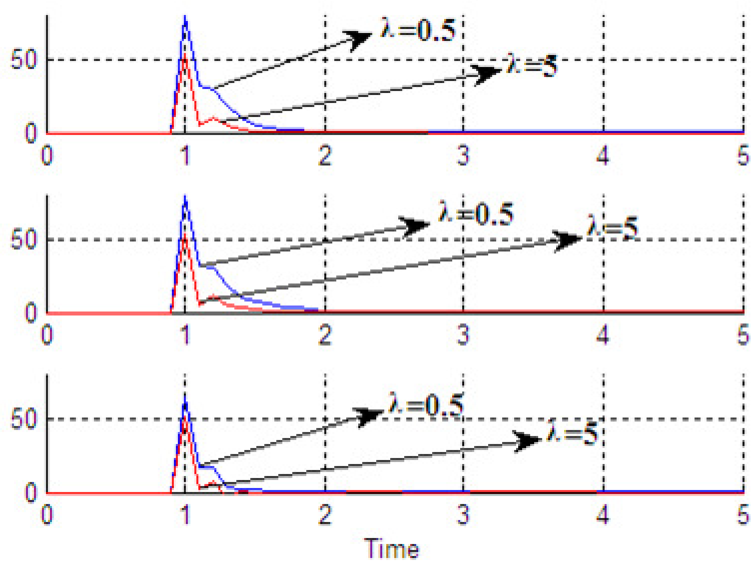

FIGURE 6: Error performance: SMC (first; second and third link)

The new sliding surface slope coefficient is updated by multiplying the error new coefficient (K_e) with predetermined slope value **(λ).**

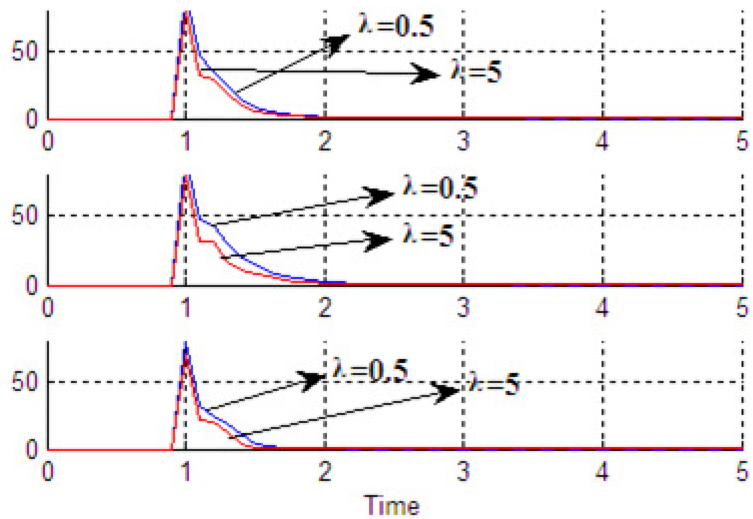

FIGURE 7: Error performance: Artificial chattering free adaptive FSMC (first; second and third link)

- **Tracking Performances**

From the simulation for first, second, and third trajectory without any disturbance, it can be seen that Artificial chattering free adaptive FSMC and classical SMC have same performance. This is primarily due to the constant parameters in simulation. Figure 8 is shown tracking performance in certain system for SMC, FSMC and proposed method.

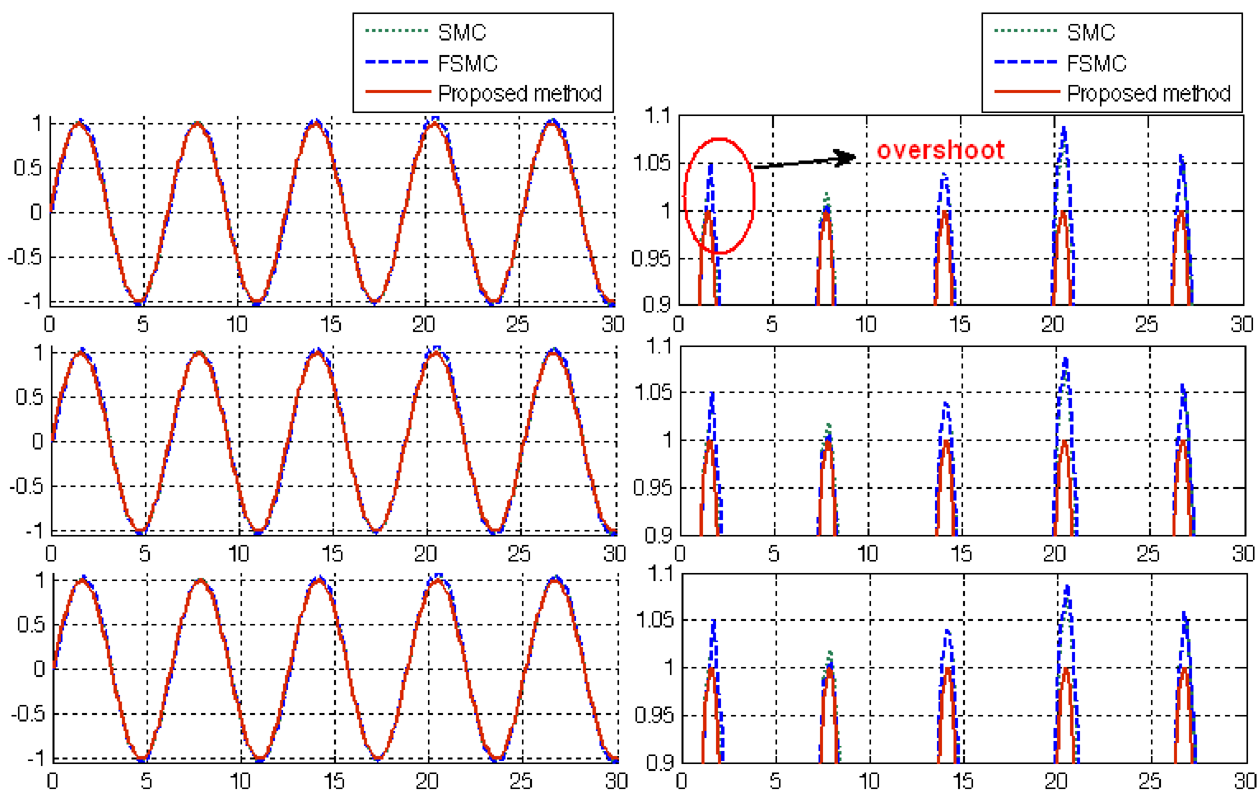

FIGURE 8: Trajectory performance: Artificial chattering free adaptive FSMC, SMC and FSMC (first; second and third link)

- **Disturbance Rejection**

A band limited white noise with predefined of 40% the power of input signal is applied to the Sinuse response. Figure 9 is shown disturbance rejection for Artificial chattering free adaptive FSMC, SMC and FSMC.

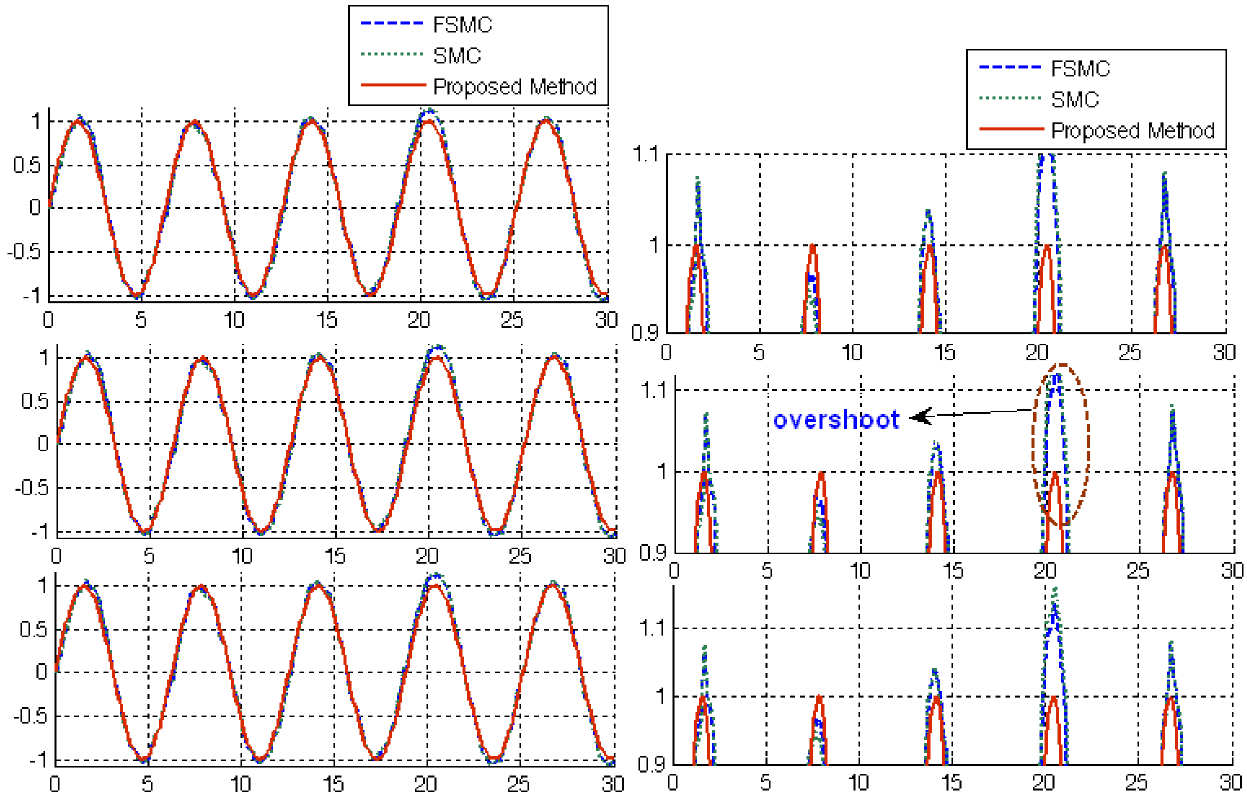

FIGURE 9: Disturbance rejection: Artificial chattering free adaptive FSMC, SMC and FSMC (first; second and third link)

- **Errors in the Model**

Although SMC and FSMC have the same error rate (refer to Table:2), they have oscillation tracking which causes chattering phenomenon at the presence of disturbances. As it is obvious in Table: 2 proposed methods is a FSMC which tuning on-line and FSMC is a SMC which estimate the equivalent part therefore FSMC have acceptable performance with regard to SMC in presence of certain and uncertainty but the best performance is in Artificial chattering free adaptive FSMC.

TABLE 2 : RMS Error Rate of Presented controllers

RMS Error Rate	**SMC**	**FSMC**	**Proposed method**
Without Noise	**1e-3**	**1.2e-3**	**1e-7**
With Noise	**0.012**	**0.013**	**1.12e-6**

- **Chattering Phenomenon**

  As mentioned in previous, chattering is one of the most important challenges in sliding mode controller which one of the major objectives in this research is reduce or remove the chattering in system's output. To reduce the chattering researcher is used fuzzy inference method instead of swithing function. Figure 10 has shown the power of fuzzy boundary layer (fuzzy saturation) method to reduce the chattering in proposed method.

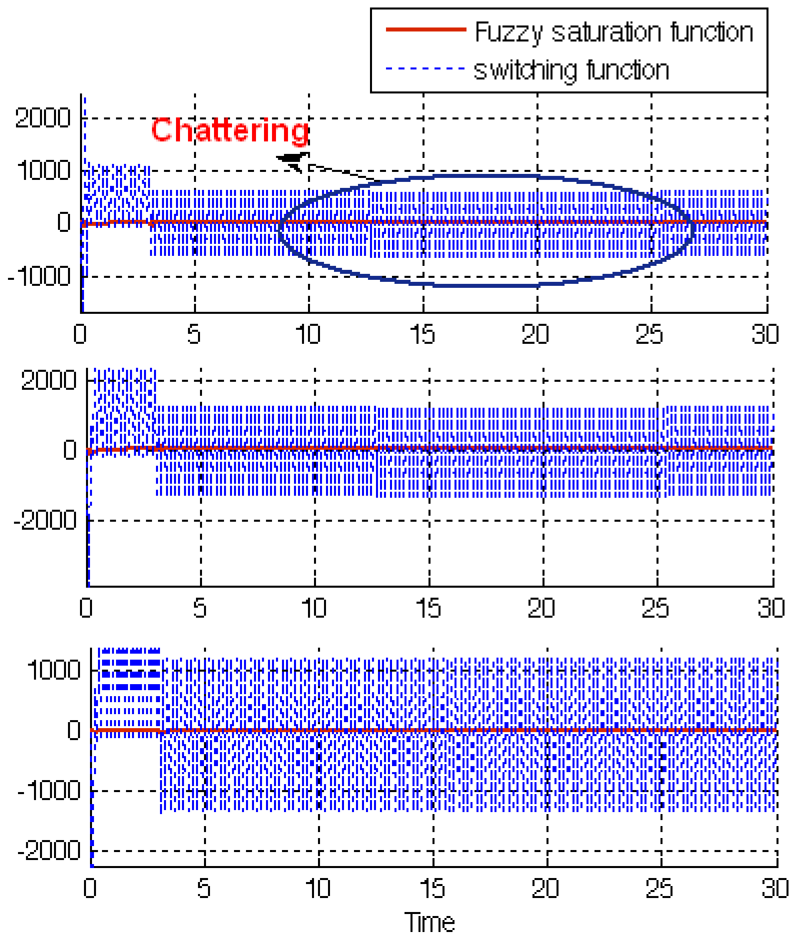

FIGURE 10: chattering phenomenon: Artificial chattering free adaptive FSMC with switching function and fuzzy saturation function (first; second and third link)

5. CONCLUSION

Refer to the research, a 7 rules Mamdani's artificial sliding mode fuzzy chattering free fuzzy sliding mode control and this suitability for use in the control of robot manipulator has proposed in order to design high performance nonlinear controller in the presence of uncertainties and external disturbances. Sliding mode control methodology is selected as a frame work to construct the control law and address the stability and robustness of the close-loop system. The proposed approach effectively combines the design techniques from sliding mode control, fuzzy logic and adaptive control to improve the performance (e.g., trajectory, disturbance rejection, error and chattering) and enhance the robustness property of the controller. Each method by adding to the previous controller has covered negative points. The system performance in sliding mode controller and fuzzy sliding mode controller are sensitive to the sliding function. Therefore, compute the optimum value of sliding function for a system is the important which this problem has solved by adjusting surface slope of the sliding function continuously in real-time. The chattering phenomenon is eliminate by fuzzy method when estimate the saturation function with 7 rule base. In this way, the overall system performance has improved with respect to the classical sliding mode controller. This controller solved chattering phenomenon as well as mathematical nonlinear equivalent part by applied fuzzy supervisory method in fuzzy sliding mode controller and artificial chattering free adaptive FSMC.

REFERENCES

[1] T. R. Kurfess, *Robotics and automation handbook*: CRC, 2005.

[2] J. J. E. Slotine and W. Li, *Applied nonlinear control* vol. 461: Prentice hall Englewood Cliffs, NJ, 1991.

[3] K. Ogata, *Modern control engineering*: Prentice Hall, 2009.

[4] L. Cheng*, et al.*, "Multi-agent based adaptive consensus control for multiple manipulators with kinematic uncertainties," 2008, pp. 189-194.

[5] J. J. D'Azzo*, et al.*, *Linear control system analysis and design with MATLAB*: CRC, 2003.

[6] B. Siciliano and O. Khatib, *Springer handbook of robotics*: Springer-Verlag New York Inc, 2008.

[7] I. Boiko*, et al.*, "Analysis of chattering in systems with second-order sliding modes," *IEEE Transactions on Automatic Control,* vol. 52, pp. 2085-2102, 2007.

[8] J. Wang*, et al.*, "Indirect adaptive fuzzy sliding mode control: Part I: fuzzy switching," *Fuzzy Sets and Systems,* vol. 122, pp. 21-30, 2001.

[9] C. Wu, "Robot accuracy analysis based on kinematics," *IEEE Journal of Robotics and Automation,* vol. 2, pp. 171-179, 1986.

[10] H. Zhang and R. P. Paul, "A parallel solution to robot inverse kinematics," 2002, pp. 1140-1145.

[11] J. Kieffer, "A path following algorithm for manipulator inverse kinematics," 2002, pp. 475-480.

[12] Z. Ahmad and A. Guez, "On the solution to the inverse kinematic problem(of robot)," 1990, pp. 1692-1697.

[13] F. T. Cheng*, et al.*, "Study and resolution of singularities for a 6-DOF PUMA manipulator," *Systems, Man, and Cybernetics, Part B: Cybernetics, IEEE Transactions on,* vol. 27, pp. 332-343, 2002.

[14] M. W. Spong and M. Vidyasagar, *Robot dynamics and control*: Wiley-India, 2009.

[15] A. Vivas and V. Mosquera, "Predictive functional control of a PUMA robot," 2005.

[16] D. Nguyen-Tuong*, et al.*, "Computed torque control with nonparametric regression models," 2008, pp. 212-217.

[17] V. Utkin, "Variable structure systems with sliding modes," *Automatic Control, IEEE Transactions on,* vol. 22, pp. 212-222, 2002.

[18] R. A. DeCarlo*, et al.*, "Variable structure control of nonlinear multivariable systems: a tutorial," *Proceedings of the IEEE,* vol. 76, pp. 212-232, 2002.

[19] K. D. Young*, et al.*, "A control engineer's guide to sliding mode control," 2002, pp. 1-14.

[20] O. Kaynak, "Guest editorial special section on computationally intelligent methodologies and sliding-mode control," *IEEE Transactions on Industrial Electronics,* vol. 48, pp. 2-3, 2001.

[21] J. J. Slotine and S. Sastry, "Tracking control of non-linear systems using sliding surfaces, with application to robot manipulators†," *International Journal of Control,* vol. 38, pp. 465-492, 1983.

[22] J. J. E. Slotine, "Sliding controller design for non-linear systems," *International Journal of Control,* vol. 40, pp. 421-434, 1984.

[23] R. Palm, "Sliding mode fuzzy control," 2002, pp. 519-526.

[24] C. C. Weng and W. S. Yu, "Adaptive fuzzy sliding mode control for linear time-varying uncertain systems," 2008, pp. 1483-1490.

[25] M. Ertugrul and O. Kaynak, "Neuro sliding mode control of robotic manipulators," *Mechatronics,* vol. 10, pp. 239-263, 2000.

[26] P. Kachroo and M. Tomizuka, "Chattering reduction and error convergence in the sliding-mode control of a class of nonlinear systems," *Automatic Control, IEEE Transactions on,* vol. 41, pp. 1063-1068, 2002.

[27] H. Elmali and N. Olgac, "Implementation of sliding mode control with perturbation estimation (SMCPE)," *Control Systems Technology, IEEE Transactions on,* vol. 4, pp. 79-85, 2002.

[28] J. Moura and N. Olgac, "A comparative study on simulations vs. experiments of SMCPE," 2002, pp. 996-1000.

[29] Y. Li and Q. Xu, "Adaptive Sliding Mode Control With Perturbation Estimation and PID Sliding Surface for Motion Tracking of a Piezo-Driven Micromanipulator," *Control Systems Technology, IEEE Transactions on,* vol. 18, pp. 798-810, 2010.

[30] B. Wu*, et al.*, "An integral variable structure controller with fuzzy tuning design for electro-hydraulic driving Stewart platform," 2006, pp. 5-945.

[31] L. A. Zadeh, "Toward a theory of fuzzy information granulation and its centrality in human reasoning and fuzzy logic," *Fuzzy Sets and Systems,* vol. 90, pp. 111-127, 1997.

[32] L. Reznik, *Fuzzy controllers*: Butterworth-Heinemann, 1997.

[33] J. Zhou and P. Coiffet, "Fuzzy control of robots," 2002, pp. 1357-1364.

[34] S. Banerjee and P. Y. Woo, "Fuzzy logic control of robot manipulator," 2002, pp. 87-88.

[35] K. Kumbla*, et al.*, "Soft computing for autonomous robotic systems," *Computers and Electrical Engineering,* vol. 26, pp. 5-32, 2000.

[36] C. C. Lee, "Fuzzy logic in control systems: fuzzy logic controller. I," *IEEE Transactions on systems, man and cybernetics,* vol. 20, pp. 404-418, 1990.

[37] R. J. Wai*, et al.*, "Implementation of artificial intelligent control in single-link flexible robot arm," 2003, pp. 1270-1275.

[38] R. J. Wai and M. C. Lee, "Intelligent optimal control of single-link flexible robot arm," *Industrial Electronics, IEEE Transactions on,* vol. 51, pp. 201-220, 2004.

[39] M. B. Menhaj and M. Rouhani, "A novel neuro-based model reference adaptive control for a two link robot arm," 2002, pp. 47-52.

[40] S. Mohan and S. Bhanot, "Comparative study of some adaptive fuzzy algorithms for manipulator control," *International Journal of Computational Intelligence,* vol. 3, pp. 303–311, 2006.

[41] F. Barrero*, et al.*, "Speed control of induction motors using a novel fuzzy sliding-mode structure," *Fuzzy Systems, IEEE Transactions on,* vol. 10, pp. 375-383, 2002.

[42] Y. C. Hsu and H. A. Malki, "Fuzzy variable structure control for MIMO systems," 2002, pp. 280-285.

[43] Y. C. Hsueh*, et al.*, "Self-tuning sliding mode controller design for a class of nonlinear control systems," 2009, pp. 2337-2342.

[44] R. Shahnazi*, et al.*, "Position control of induction and DC servomotors: a novel adaptive fuzzy PI sliding mode control," *Energy Conversion, IEEE Transactions on,* vol. 23, pp. 138-147, 2008.

[45] C. C. Chiang and C. H. Wu, "Observer-Based Adaptive Fuzzy Sliding Mode Control of Uncertain Multiple-Input Multiple-Output Nonlinear Systems," 2007, pp. 1-6.

[46] H. Temeltas, "A fuzzy adaptation technique for sliding mode controllers," 2002, pp. 110-115.

[47] C. L. Hwang and S. F. Chao, "A fuzzy-model-based variable structure control for robot arms: theory and experiments," 2005, pp. 5252-5258.

[48] C. G. Lhee*, et al.*, "Sliding mode-like fuzzy logic control with self-tuning the dead zone parameters," *Fuzzy Systems, IEEE Transactions on,* vol. 9, pp. 343-348, 2002.

[49] "Sliding-Like Fuzzy Logic Control with Self-tuning the Dead Zone Parameters," 1999.

[50] X. Zhang*, et al.*, "Adaptive sliding mode-like fuzzy logic control for high order nonlinear systems," pp. 788-792.

[51] M. R. Emami*, et al.*, "Development of a systematic methodology of fuzzy logic modeling," *IEEE Transactions on Fuzzy Systems,* vol. 6, 1998.

[52] *1994,* vol. 4, pp. 212-218, 1994.

[53] Z. Kovacic and S. Bogdan, *Fuzzy controller design: theory and applications*: CRC/Taylor & Francis, 2006.

[54] F. Y. Hsu and L. C. Fu, "Nonlinear control of robot manipulators using adaptive fuzzy sliding mode control," 2002, pp. 156-161.

[55] R. G. Berstecher*, et al.*, "An adaptive fuzzy sliding-mode controller," *Industrial Electronics, IEEE Transactions on,* vol. 48, pp. 18-31, 2002.

[56] V. Kim, "Independent joint adaptive fuzzy control of robot manipulator," 2002, pp. 645-652.

[57] Y. Wang and T. Chai, "Robust adaptive fuzzy observer design in robot arms," 2005, pp. 857-862.

[58] B. K. Yoo and W. C. Ham, "Adaptive control of robot manipulator using fuzzy compensator," *Fuzzy Systems, IEEE Transactions on,* vol. 8, pp. 186-199, 2002.

[59] H. Medhaffar*, et al.*, "A decoupled fuzzy indirect adaptive sliding mode controller with application to robot manipulator," *International Journal of Modelling, Identification and Control,* vol. 1, pp. 23-29, 2006.

[60] Y. Guo and P. Y. Woo, "An adaptive fuzzy sliding mode controller for robotic manipulators," *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on,* vol. 33, pp. 149-159, 2003.

[61] Piltan, F., et al., "Design sliding mode controller for robot manipulator with artificial tunable gain," Canaidian Journal of pure and applied science, 5 (2): 1573-1579, 2011.

[62] Farzin Piltan, A. R. Salehi and Nasri B Sulaiman.," Design artificial robust control of second order system based on adaptive fuzzy gain scheduling," world applied science journal (WASJ), 13 (5): 1085-1092, 2011

[63] F. Piltan*, et al.*, "Artificial Control of Nonlinear Second Order Systems Based on AFGSMC," *Australian Journal of Basic and Applied Sciences, 5(6)*, pp. 509-522, 2011.

[64] Piltan, F., et al., "Design Artificial Nonlinear Robust Controller Based on CTLC and FSMC with Tunable Gain," International Journal of Robotic and Automation, 2 (3): 205-220, 2011.

[65] Piltan, F., et al., "Design Mathematical Tunable Gain PID-Like Sliding Mode Fuzzy Controller with Minimum Rule Base," International Journal of Robotic and Automation, 2 (3): 146-156, 2011.

[66] Piltan, F., et al., "Design of FPGA based sliding mode controller for robot manipulator," International Journal of Robotic and Automation, 2 (3): 183-204, 2011.

[67] Piltan, F., et al., "A Model Free Robust Sliding Surface Slope Adjustment in Sliding Mode Control for Robot Manipulator," World Applied Science Journal, 12 (12): 2330-2336, 2011.

[68] Piltan, F., et al., "Design Adaptive Fuzzy Robust Controllers for Robot Manipulator," World Applied Science Journal, 12 (12): 2317-2329, 2011.