### Evolutionary Design of Backstepping Artificial Sliding Mode Based Position Algorithm: Applied to Robot Manipulator

**Farzin Piltan** SSP.ROBOTIC@yahoo.com Department of Electrical and Electronic Engineering, Faculty of Engineering, Universiti Putra Malaysia 43400 Serdang, Selangor, Malaysia N. Sulaiman nasri@eng.upm.edu.my Department of Electrical and Electronic Engineering, Faculty of Engineering, Universiti Putra Malaysia 43400 Serdang, Selangor, Malaysia Samaneh Roosta SSP.ROBOTIC@yahoo.com Industrial Electrical and Electronic Engineering SanatkadeheSabze Pasargad. CO (S.S.P. Co), NO:16, PO.Code 71347-66773, Fourth floor Dena Apr, Seven Tir Ave , Shiraz , Iran Atefeh Gavahian SSP.ROBOTIC@yahoo.com Industrial Electrical and Electronic Engineering SanatkadeheSabze Pasargad. CO (S.S.P. Co), NO:16, PO.Code 71347-66773, Fourth floor Dena Apr, Seven Tir Ave , Shiraz , Iran Samira Soltani SSP.ROBOTIC@yahoo.com Industrial Electrical and Electronic Engineering SanatkadeheSabze Pasargad. CO (S.S.P. Co), NO:16, PO.Code 71347-66773, Fourth floor Dena Apr, Seven Tir Ave . Shiraz . Iran

#### Abstract

This paper expands a fuzzy sliding mode based position controller whose sliding function is on-line tuned by backstepping methodology. The main goal is to guarantee acceptable position trajectories tracking between the robot manipulator end-effector and the input desired position. The fuzzy controller in proposed fuzzy sliding mode controller is based on Mamdani's fuzzy inference system (FIS) and it has one input and one output. The input represents the function between sliding function, error and the rate of error. The second input is the angle formed by the straight line defined with the orientation of the robot, and the straight line that connects the robot with the reference cart. The outputs represent angular position, velocity and acceleration commands, respectively. The backstepping methodology is on-line tune the sliding function based on self tuning methodology. The performance of the backstepping on-line tune fuzzy sliding mode controller (TBsFSMC) is validated through comparison with previously developed robot manipulator position controller based on adaptive fuzzy sliding mode control theory (AFSMC). Simulation results signify good performance of position tracking in presence of uncertainty and external disturbance.

**Keywords**: Fuzzy Sliding Mode Controller, Backstepping Controller, Robot Manipulator, Backstepping on-Line Tune Fuzzy Sliding Mode Controller

#### 1. INTRODUCTION

In the recent years robot manipulators not only have been used in manufacturing but also used in vast area such as medical area and working in International Space Station. Control methodologies and the

mechanical design of robot manipulators have started in the last two decades and the most of researchers work in these methodologies [1]. PUMA 560 robot manipulator is an articulated 6 DOF serial robot manipulator. This robot is widely used in industrial and academic area and also dynamic parameters have been identified and documented in the literature [2-3]. There are several methods for controlling a robot manipulator (e.g., PUMA robot manipulator), which all of them follow two common goals, namely, hardware/software implementation and acceptable performance. However, the mechanical design of robot manipulator is very important to select the best controller but in general two types schemes can be presented, namely, a joint space control schemes and an operation space control schemes[1]. Both of these controllers are closed loop which they have been used to provide robustness and rejection of disturbance effect. One of the simplest ways to analysis control of multiple DOF's robot manipulators are analyzed each joint separately such as SISO systems and design an independent joint controller for each joint. In this controller, inputs only depends on the velocity and displacement of the corresponding joint and the other parameters between joints such as coupling presented by disturbance input. Joint space controller has many advantages such as one type controllers design for all joints with the same formulation, low cost hardware, and simple structure [1, 4].

Sliding mode controller (SMC) is one of the influential nonlinear controllers in certain and uncertain systems which are used to present a methodical solution for two main important controllers' challenges, which named: stability and robustness. Conversely, this controller is used in different applications; sliding mode controller has subsequent drawbacks, the first one is chattering phenomenon, which it is caused to some problems such as saturation and heat for mechanical parts of robot manipulators or drivers and the second one is nonlinear equivalent dynamic formulation in uncertain systems[1, 5-12]. In order to solve the chattering in the systems output, boundary layer method should be applied so beginning able to recommended model in the main motivation which in this method the basic idea is replace the discontinuous method by saturation (linear) method with small neighborhood of the switching surface. Slotine and Sastry have introduced boundary layer method instead of discontinuous method to reduce the chattering[13]. Slotine has presented sliding mode with boundary layer to improve the industry application [14]. R. Palm has presented a fuzzy method to nonlinear approximation instead of linear approximation inside the boundary layer to improve the chattering and control the result performance[15]. Moreover, C. C. Weng and W. S. Yu improved the previous method by using a new method in fuzzy nonlinear approximation inside the boundary layer and adaptive method[16]. As mentioned [16]sliding mode fuzzy controller (SMFC) is fuzzy controller based on sliding mode technique to simple implement, most exceptional stability and robustness. Conversely above method has the following advantages; reducing the number of fuzzy rule base and increasing robustness and stability, the main disadvantage of SMFC is need to define the sliding surface slope coefficient very carefully. To eliminate the above problems control researchers have applied artificial intelligence method (e.g., fuzzy logic) in nonlinear robust controller (e.g., sliding mode controller) besides this technique is very useful in order to implement easily. Estimated uncertainty method is used in term of uncertainty estimator to compensation of the system uncertainties. It has been used to solve the chattering phenomenon and also nonlinear equivalent dynamic. If estimator has an acceptable performance to compensate the uncertainties, the chattering is reduced. Research on estimated uncertainty to reduce the chattering is significantly growing as their applications such as industrial automation and robot manipulator. For instance, the applications of artificial intelligence, neural networks and fuzzy logic on estimated uncertainty method have been reported in [17-20]. Wu et al. [22] have proposed a simple fuzzy estimator controller beside the discontinuous and equivalent control terms to reduce the chattering. Elmali et al. [19]and Li and Xu [21]have addressed sliding mode control with perturbation estimation method (SMCPE) to reduce the classical sliding mode chattering. This method was tested for the tracking control of the first two links of a SCARA type HITACHI robot. In this technique, digital controller is used to increase the system's response quality. Conversely this method has the following advantages; increasing the controller's response speed and reducing dependence on dynamic system model by on-line control, the main disadvantage are chattering phenomenon and need to improve the performance.

In recent years, artificial intelligence theory has been used in sliding mode control systems. Neural network, fuzzy logic and neuro-fuzzy are synergically combined with nonlinear classical controller and used in nonlinear, time variant and uncertainty plant (e.g., robot manipulator). Fuzzy logic controller (FLC) is one of the most important applications of fuzzy logic theory. This controller can be used to control nonlinear, uncertain and noisy systems. This method is free of some model-based techniques as in classical controllers. As mentioned that fuzzy logic application is not only limited to the modelling of nonlinear systems [23-28]but also this method can help engineers to design easier controller. The main reasons to use fuzzy logic technology are able to give approximate recommended solution for unclear and complicated systems to easy understanding and flexible. Fuzzy logic provides a method which is able to model a controller for nonlinear plant with a set of IF-THEN rules, or it can identify the control actions and describe them by using fuzzy rules. The applications of artificial intelligence such as neural networks and fuzzy logic in modelling and control are significantly growing especially in recent years. For instance, the applications of artificial intelligence, neural networks and fuzzy logic, on robot arm control have reported in [29-31]. Wai et al. [29-30]have proposed a fuzzy neural network (FNN) optimal control system to learn a nonlinear function in the optimal control law. This controller is divided into three main groups: arterial intelligence controller (fuzzy neural network) which it is used to compensate the system's nonlinearity and improves by adaptive method, robust controller to reduce the error and optimal controller which is the main part of this controller. Mohan and Bhanot [32] have addressed comparative study between some adaptive fuzzy, and a new hybrid fuzzy control algorithm for manipulator control. They found that self-organizing fuzzy logic controller and proposed hybrid integrator fuzzy give the best performance as well as simple structure. Research on combinations of fuzzy logic systems with sliding mode method is significantly growing as nonlinear control applications. For instance, the applications of fuzzy logic on sliding mode controller have reported in [11, 33-37].

Research on applied fuzzy logic methodology in sliding mode controller (FSMC) to reduce or eliminate the high frequency oscillation (chattering), to compensate the unknown system dynamics and also to adjust the linear sliding surface slope in pure sliding mode controller considerably improves the robot manipulator control process [34-35]. H.Temeltas [38] has proposed fuzzy adaption techniques for SMC to achieve robust tracking of nonlinear systems and solves the chattering problem. Conversely system's performance is better than sliding mode controller; it is depended on nonlinear dynamic equgation. C. L. Hwang et al. [39]have proposed a Tagaki-Sugeno (TS) fuzzy model based sliding mode control based on N fuzzy based linear state-space to estimate the uncertainties. A multi-input multi-output FSMC reduces the chattering phenomenon and reconstructs the approximate the unknown system has been presented for a robot manipulator [34]. Investigation on applied sliding mode methodology in fuzzy logic controller (SMFC) to reduce the fuzzy rules and refine the stability of close loop system in fuzzy logic controller has grown specially in recent years as the robot manipulator control [10]; [40-42]. Lhee et al. [40]have presented a fuzzy logic controller based on sliding mode controller to more formalize and boundary layer thickness. Emami et al. [43] have proposed a fuzzy logic approximate inside the boundary layer. H.K.Lee et al. [44] have presented self tuning SMFC to reduce the fuzzy rules, increase the stability and to adjust control parameters control automatically. However the application of FSMC and SMFC are growing but the main SMFC drawback compared to FSMC is calculation the value of sliding surface & pri-defined very carefully. Moreover, the advantages of SMFC compared to FLC reduce the number of fuzzy rule base and increase the robustness and stability. At last FSMC compare to the SMFC is more suitable for implementation action.

In various dynamic parameters systems that need to be training on-line tuneable gain control methodology is used. On-line tuneable control methodology can be classified into two main groups, namely, traditional adaptive method and fuzzy adaptive method. Fuzzy adaptive method is used in systems which want to training parameters by expert knowledge. Traditional adaptive method is used in systems which some dynamic parameters are known. In this research in order to solve disturbance rejection and uncertainty dynamic parameter, on-line tuneable method is applied to artificial sliding mode controller. F Y Hsu et al. [45] have presented adaptive fuzzy sliding mode control which can update fuzzy rules to compensate nonlinear parameters and guarantee the stability robot manipulator controller. Y.C. Hsueh et al. [35] have presented self tuning sliding mode controller which can resolve the chattering problem without to using saturation function. For nonlinear dynamic systems (e.g., robot manipulators) with various parameters, online control technique can train the dynamic parameter to have satisfactory performance. Calculate sliding surface slope is common challenge in classical sliding mode controller and fuzzy sliding mode controller. Research on adaptive (on-line tuneable) fuzzy control is significantly growing, for instance, different adaptive fuzzy controllers have been reported in [32, 46-48]. The adaptive sliding mode controller is used to estimate the unknown dynamic parameters and external disturbances. For instance, the applications of adaptive fuzzy sliding mode controller to control the robot manipulators have been reported in [11, 16, 37]. Yoo and Ham [49]have proposed a MIMO fuzzy system to help the compensation and estimation the torque coupling. In n - DOF robot manipulator with k membership function for each input variable, the number of fuzzy rules for each joint is equal to  $3k^{2n}$  that causes to high computation load and also this controller has chattering. This method can only tune the consequence part of the fuzzy rules. Medhafer et al. [50] have proposed an indirect adaptive fuzzy sliding mode controller to control robot manipulator. This MIMO algorithm, applies to estimate the nonlinear dynamic parameters. If each input variable have  $K_2$ membership functions, the number of fuzzy rules for each joint is  $(m + 1)K_2^m + K_2$ . Compared with the previous algorithm the number of fuzzy rules have reduced by introducing the sliding surface as inputs of fuzzy systems. Y. Guo and P. Y. Woo [51]have proposed a SISO fuzzy system compensate and reduce the chattering. First suppose each input variable with  $K_2$  membership function the number of fuzzy rules for each joint is  $K_2$  which decreases the fuzzy rules and the chattering is also removed. C. M. Lin and C. F. Hsu [52] can tune both systems by fuzzy rules. In this method the number of fuzzy rules equal to  $K_{2}$ with low computational load but it has chattering. Shahnazi et al., have proposed a SISO PI direct adaptive fuzzy sliding mode controller based on Lin and Hsu algorithm to reduce or eliminate chattering with  $K_2$  fuzzy rules numbers. The bounds of PI controller and the parameters are online adjusted by low adaption computation [36]. Table 1 is illustrated a comparison between sliding mode controller [1, 5-11, 13], fuzzy logic controller (FLC)[23-32], applied sliding mode in fuzzy logic controller (SMFC)[10, 40-42], applied fuzzy logic method in sliding mode controller (FSMC)[45-46, 51] and adaptive fuzzy sliding mode controller [5-11].

#### This paper is organized as follows:

In section 2, design proposed backstepping on-line tunable gain in fuzzy sliding mode controller is presented. Detail of dynamic equation of robot arm is presented in section 3. In section 4, the simulation result is presented and finally in section 5, the conclusion is presented.

# 2. DESIGN PROPOSED BACKSTEPPING ON-LINE TUNE FUZZY SLIDING MODE CONTROLLER

Sliding mode controller (SMC) is a influential nonlinear, stable and robust controller which it was first proposed in the early 1950 by Emelyanov and several co-workers and has been extensively developed since then with the invention of high speed control devices [1, 5-11]. A time-varying sliding surface s(x, t) is given by the following equation:

$$s(x,t) = \left(\frac{d}{dt} + \lambda\right)^{n-1} \tilde{x} = 0 \tag{1}$$

where  $\lambda$  is the constant and it is positive. The derivation of S, namely,  $\frac{1}{2}$  can be calculated as the following formulation [5-11]:

$$\mathbf{\dot{S}} = (\ddot{\mathbf{x}} - \ddot{\mathbf{x}}_d) + \lambda(\dot{\mathbf{x}} - \dot{\mathbf{x}}_d) \tag{2}$$

The control law for a multi degrees of freedom robot manipulator is written as:  $U = U_{eq} + U_r$ 

Where, the model-based component  $U_{eq}$  is the nominal dynamics of systems and it can be calculate as follows:

$$U_{eq} = [M^{-1}(B + C + G) + S]M$$
(4)

Where M(q) is an inertia matrix which it is symmetric and positive,  $V(q, \dot{q}) = B + C$  is the vector of nonlinearity term and G(q) is the vector of gravity force and  $U_r$  with minimum chattering based on [5-11] is computed as;

$$U_r = K \cdot (\mathrm{mu} + \mathrm{b}) \left( \frac{S}{\mathrm{o}} \right)$$
<sup>(5)</sup>

Where  $\phi_{u} = mu + b = saturation_{fuction}$  is a dead zone (saturation) function and, u and b are unlimited coefficient, by replace the formulation (5) in (3) the control output can be written as;

$$\boldsymbol{U} = \boldsymbol{U}_{eq} + \boldsymbol{K}.\left(\mathbf{mu} + \mathbf{b}\right) \begin{pmatrix} \boldsymbol{S}/\boldsymbol{\varphi} \end{pmatrix} = \begin{cases} \boldsymbol{U}_{eq} + \boldsymbol{K}.\operatorname{sgn}(\boldsymbol{S}) & |\boldsymbol{S}| \ge \boldsymbol{\varphi} \\ \boldsymbol{U}_{eq} + \boldsymbol{K}.\boldsymbol{S}/\boldsymbol{\varphi} & , |\boldsymbol{S}| < \boldsymbol{\varphi} \end{cases}$$
(6)

Where the function of sgn(S) defined as;

$$sgn(s) = \begin{cases} 1 & s > 0 \\ -1 & s < 0 \\ 0 & s = 0 \end{cases}$$
(7)

The fuzzy system can be defined as below

$$f(x) = U_{fuzzy} = \sum_{l=1}^{M} \theta^{T} \zeta(x) = \psi(S)$$
(8)
where  $\theta = (\theta^{1}, \theta^{2}, \theta^{3}, \dots, \theta^{M})^{T}, \zeta(x) = (\zeta^{1}(x), \zeta^{2}(x), \zeta^{3}(x), \dots, \zeta^{M}(x))^{T}$ 

$$\sum_{k=1}^{N} \theta^{k} = (\xi^{1}, \xi^{k}) = (\xi^{1}(x), \xi^{k}) = (\xi^{1}($$

$$\zeta^{1}(x) = \frac{\sum_{i} \mu_{(xi)x_{i}}}{\sum_{i} \mu_{(xi)}}$$
(9)

where  $\theta = (\theta^1, \theta^2, \theta^3, \dots, \theta^N)$  is adjustable parameter in (8) and  $\mu_{(xi)}$  is membership function. error base fuzzy controller can be defined as  $U_{fuzzy} = \psi(S)$  (10)

The fuzzy division can be reached the best state when 5.5 < 0 and the error is minimum by the following formulation

$$\theta^* = \arg\min\left[Sup_{x\in U}\right| \sum_{l=1}^{M} \theta^T \zeta(x) - U_{equ}|]$$
(11)

Where  $\theta^*$  is the minimum error,  $\sup_{x \in U} |\sum_{l=1}^M \theta^T \zeta(x) - \tau_{equ}|$  is the minimum approximation error.

(3)

Type of method	Advantages	Disadvantages	What to do?
1.SMC 2.FLC	<ul> <li>Good control performance for nonlinear systems</li> <li>In MIMO systems</li> <li>In discrete time circuit</li> <li>Used in unclear and</li> </ul>	<ul> <li>Equivalent dynamic formulation</li> <li>Chattering</li> <li>It has limitation under condition of : uncertain system and external disturbance</li> <li>Quality of design</li> </ul>	Applied artificial intelligent method in SMC (e.g., FSMC or SMFC) Applied adaptive
0.0ME0	uncertain systems <ul> <li>Flexible</li> <li>Easy to understand</li> <li>Shortened in design</li> </ul>	<ul> <li>Should be to defined fuzzy coefficient very carefully</li> <li>Cannot guarantee the stability</li> <li>reliability</li> </ul>	method in FLC, tuning parameters and applied to classical linear or nonlinear controller
3.SMFC	<ul> <li>Reduce the rule base</li> <li>Reduce the chattering</li> <li>Increase the stability and robustness</li> </ul>	<ul> <li>Equivalent part</li> <li>Defined sliding surface slope coefficient very carefully</li> <li>Difficult to implement</li> <li>Limitation in noisy and uncertain system</li> </ul>	Applied adaptive method, self learning and self organizing method in SMFC
4.FSMC	<ul> <li>More robust</li> <li>Reduce the chattering</li> <li>Estimate the equivalent</li> <li>Easy to implement</li> </ul>	<ul> <li>Model base estimate the equivalent part</li> <li>Limitation in noisy and uncertain system</li> </ul>	Design fuzzy error base like equivalent controller and applied adaptive method
5.Adaptive FSMC	<ul> <li>More robust</li> <li>eliminate the chattering</li> <li>Estimate the equivalent</li> </ul>	<ul> <li>Model base estimate the equivalent part</li> </ul>	

TABLE 1: Comparison of	f six important	algorithms
------------------------	-----------------	------------

suppose  $K_j$  is defined as follows

$$K_j = \frac{\sum_{l=1}^M \theta_j^l[\mu_A(S_j)]}{\sum_{l=1}^M [\mu_A(S_j)]} = \theta_j^T \zeta_j(S_j)$$

(12)

Where 
$$\zeta_{j}(S_{j}) = [\zeta_{j}^{1}(S_{j}), \zeta_{j}^{2}(S_{j}), \zeta_{j}^{2}(S_{j}), \ldots, \zeta_{j}^{M}(S_{j})]^{T}$$
  
 $\zeta_{j}^{1}(S_{j}) = \frac{\mu_{(A)}_{j}^{l}(S_{j})}{\sum_{i} \mu_{(A)}_{i}^{l}(S_{j})}$ 
(13)

where the  $\gamma_{si}$  is the positive constant.

According to the nonlinear dynamic equivalent formulation of robot manipulator the nonlinear equivalent part is estimated by (8)

$$\left[M^{-1}(B+C+G)+S\right]M = \sum_{i=1}^{M} \theta^{T} \zeta(x) - \lambda S - K$$
<sup>(14)</sup>

Based on (3) the formulation of proposed fuzzy sliding mode controller can be written as;

$$U = U_{eq_{fuzzy}} + U_r \tag{15}$$

## Where $U_{eq_{fuzzy}} = [M^{-1}(B + C + G) + S]M + \sum_{l=1}^{M} \theta^{T} \zeta(x) + K$

Figure 1 is shown the proposed fuzzy sliding mode controller.





As mentioned above pure sliding mode controller has nonlinear dynamic equivalent limitations in presence of uncertainty and external disturbances in order to solve these challenges this work applied Mamdani's fuzzy inference engine estimator in sliding mode controller. However proposed FSMC has satisfactory performance but calculate the sliding surface slope by try and error or experience knowledge is very difficult, particularly when system has structure or unstructured uncertainties; backstepping self tuning sliding function fuzzy sliding mode controller is recommended. The backstepping method is based on mathematical formulation which this method is introduced new variables into it in form depending on the dynamic equation of robot manipulator. This method is used as feedback linearization in order to solve nonlinearities in the system. To use of nonlinear fuzzy filter this method in this research makes it possible to create dynamic nonlinear equivalent backstepping estimator into the online tunable fuzzy sliding control process to eliminate or reduce the challenge of uncertainty in this part. The backstepping controller is calculated by;

\*\*

$$U_{B,S} = U_{eq_{B,S}} + M.I \tag{15}$$

Where  $U_{BS}$  is backstepping output function,  $U_{eq_{BS}}$  is backstepping nonlinear equivalent function which can be written as (16) and I is backstepping control law which calculated by (17)

(16)

(19)

$$U_{eq_{B,S}} = [(B + C + G)]$$
(16)  
$$I = [\ddot{\theta} + K_1(K_1 - 1) \cdot e + (K_1 + K_2) \cdot \dot{e}]$$
(17)

Based on (10) and (16) the fuzzy backstepping filter is considered as

$$(B+C+G) = \sum_{i=1}^{M} \theta^{T} \zeta(x) - \lambda S - K$$
<sup>(18)</sup>

Based on (15) the formulation of fuzzy backstepping filter can be written as;  $U = U_{eqB.S_{fuzzy}} + MI$ 

Where 
$$U_{eq_{B,3furry}} = [(B + C + G)] + \sum_{i=1}^{M} \theta^{T} \zeta(x) + K$$
  
The adaption low is defined as  
 $\theta_{i} = \gamma_{ei} S_{i} \zeta_{i} (S_{i})$  (20)

where the  $\gamma_{si}$  is the positive constant and  $\zeta_j(S_j) = [\zeta_j^1(S_j), \zeta_j^2(S_j), \zeta_j^3(S_j), \dots, \zeta_j^M(S_j)]^T$ 

$$\zeta_j^1(S_j) = \frac{\mu_{(A)_j}^l(S_j)}{\sum_i \mu_{(A)_j}^l(S_j)}$$
(21)

The dynamic equation of robot manipulator can be written based on the sliding surface as;  $MS = -VS + MS + VS + G - \tau$ (22)

It is supposed that  

$$S^{T}(M-2V)S = 0$$
(23)

The derivation of Lyapunov function (  ${f P}$  ) is written as

$$\begin{split} \Psi &= \frac{1}{2} S^T \dot{M} S - S^T V S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\ &= S^T (-\lambda S + \Delta f - K) + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\ &= \sum_{j=1}^m [S_j (\Delta f_j - K_j)] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\ &= \sum_{j=1}^m [S_j (\Delta f_j - \theta_j^T \zeta_j(S_j))] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\ &= \sum_{j=1}^m [S_j (\Delta f_j - (\theta_j^*)^T \zeta_j(S_j) + \phi_j^T \zeta_j(S_j))] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \dot{\phi}_j \\ &= \sum_{j=1}^m [S_j (\Delta f_j - (\theta_j^*)^T \zeta_j(S_j) + \phi_j^T \zeta_j(S_j))] - S^T \lambda S + \sum \frac{1}{\gamma_{sj}} \phi_j^T \phi_j \\ &= \sum_{j=1}^m [S_j (\Delta f_j - ((\theta_j^*)^T \zeta_j(S_j))] - S^T \lambda S)] + \sum (\frac{1}{\gamma_{sj}} \phi_j^T [\gamma_{sj} \zeta_j(S_j) S_j + \phi_j]) \end{split}$$

Where  $\theta_j = \gamma_{aj} S_j \zeta_j (S_j)$  is adaption law and  $\phi_j = -\theta_j = -\gamma_{aj} S_j \zeta_j (S_j)$ , consequently V can be considered by (24)  $\hat{V} = \sum_{i=1}^{m} [S_{j} \Delta f_{j} - \left( (\theta_{j}^{*})^{T} \zeta_{j} (S_{j}) \right)] - S^{T} \lambda S$ 

The minimum error can be defined by

Farzin Piltan, N. Sulaiman, S. Roosta, A. Gavahian & S. Soltani

$$\boldsymbol{\varepsilon}_{mj} = \Delta f_j - \left( (\boldsymbol{\theta}_j^*)^T \boldsymbol{\zeta}_j(\boldsymbol{S}_j) \right)$$
(25)

V is intended as follows

$$\begin{aligned}
\mathcal{V} &= \sum_{j=1}^{m} [S_j e_{mj}] - S^T \lambda S \\
&\leq \sum_{j=1}^{m} |S_j| |e_{mj}| - S^T \lambda S \\
&= \sum_{j=1}^{m} |S_j| |e_{mj}| - \lambda_j S_j^2 \\
&= \sum_{j=1}^{m} |S_j| (|e_{mj}| - \lambda_j S_j)
\end{aligned}$$
(26)

For continuous function  $U_{eqB.S_{fuzzy}}$  and suppose  $\varepsilon > 0$  it is defined the fuzzy backstepping controller in form of (19) such that

$$Sup_{x \in U} \left| U_{eqB, S_{fuzzy}} + MI \right| < \epsilon$$

(27)

As a result TBsFSMC is very stable which it is one of the most important challenges to design a controller with suitable response. Figure 2 is shown the block diagram of proposed TBsFSMC.





#### **3.** APPLICATION: DYNAMIC OF ROBOT MANIPULATOR

It is well known that the equation of an *n-DOF* robot manipulator governed by the following equation [1-3]:  $M(q)\ddot{q} + N(q,\dot{q}) = \tau$ (28)

Where  $\tau$  is actuation torque, M(q) is a symmetric and positive define inertia matrix,  $N(q, \dot{q})$  is the vector of nonlinearity term. This robot manipulator dynamic equation can also be written in a following form:

#### $\boldsymbol{v} = \boldsymbol{M}(\boldsymbol{q})\boldsymbol{\ddot{q}} + \boldsymbol{B}(\boldsymbol{q})[\boldsymbol{\dot{q}} \boldsymbol{\dot{q}}] + \boldsymbol{C}(\boldsymbol{q})[\boldsymbol{\dot{q}}]^2 + \boldsymbol{G}(\boldsymbol{q})$

Where the matrix of coriolios torque is B(q), C(q) is the matrix of centrifugal torques, and G(q) is the vector of gravity force. The dynamic terms in equation (2) are only manipulator position. This is a decoupled system with simple second order linear differential dynamics. In other words, the component  $\mathbf{\ddot{q}}$  influences, with a double integrator relationship, only the joint variable  $q_i$ , independently of the motion of the other joints. Therefore, the angular acceleration is found as to be [2-3, 5-11]:

#### $\ddot{q} = M^{-1}(q).\{\tau - N(q, \dot{q})\}$

This technique is very attractive from a control point of view.

Position control of PUMA-560 robot manipulator is analyzed in this paper; as a result the last three joints are blocked. The dynamic equation of PUMA-560 robot manipulator is given as

$$M(\theta) \begin{bmatrix} \theta' 1 \\ \theta_2 \\ \theta_3 \end{bmatrix} + B(\theta) \begin{bmatrix} \theta_1 \theta_2 \\ \theta_1 \theta_3 \\ \theta_2 \theta_3 \end{bmatrix} + C(\theta) \begin{bmatrix} \theta_1^2 \\ \theta_2^2 \\ \theta_3^2 \end{bmatrix} + G(\theta) = \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}$$
(31)

Where

Suppose *q* is written as follows

(29)

(30)

$$\ddot{q} = M^{-1}(q) \left\{ \tau - [B(q)\dot{q}\dot{q} + C(q)\dot{q}^2 + g(q)] \right\}$$
(36)

and K is introduced as  $K = \{\tau - [B(q)\dot{q}\dot{q} + C(q)\dot{q}^2 + g(q)]\}$ 

$$\vec{q}$$
 can be written as  
 $\vec{q} = M^{-1}(q)$ . K (38)

#### 4. RESULT: VALIDITY CHECKING BETWEEN TBSFSMC, SMC AND FSMC

To validation of this work it is used 6-DOF's PUMA robot manipulator and implements proposed TBsFSMC, SMC and FSMC in this robot manipulator.

**Tracking performances** Figure 3 is shown tracking performance in TBsFSMC, SMC and FSMC without disturbance for proposed trajectory.



FIGURE 3: TBsFSMC, SMC and FSMC: without disturbance

By comparing this response, Figure 3, conversely the TBsFSMS and FSMC's overshoot are lower than SMC's, SMC's response is faster than TBsFSMC. The Settling time in TBsFSMC is fairly lower than SMC and FSMC.

(37)

**Disturbance rejection:** Figure 4 is indicated the power disturbance removal in TBsFSMC, SMC and FSMC. Besides a band limited white noise with predefined of 40% the power of input signal is applied to above controllers; it found slight oscillations in SMC and FSMC trajectory responses.



FIGURE 4: TBsFSMC, SMC and FSMC: with disturbance.

Among above graph, relating to step trajectory following with external disturbance, SMC and FSMC have slightly fluctuations. By comparing overshoot and rise time; SMC's overshoot (4.4%) is higher than FSMC and TBsFSMC, SMC's rise time (0.6 sec) is considerably lower than FSMC and TBsFSMC. As mentioned in previous section, chattering is one of the most important challenges in sliding mode controller which one of the major objectives in this research is reduce or remove the chattering in system's output. Figure 4 also has shown the power of boundary layer (saturation) method to reduce the chattering in above controllers. Overall in this research with regard to the step response, TBsFSMC has the steady chattering compared to the SMC and FSMC.

#### **Errors in The Model**

Although SMC and FSMC have the same error rate (refer to Table.2), they have high oscillation tracking which causes instability and chattering phenomenon at the presence of disturbances. As it is obvious in Table.2 proposed TBsFSMC has error reduction in noisy environment compared to the other controllers and displays smoother trend in above profiles.

RMS Error Rate	SMC	FSMC	TBsFSMC
Without Noise	1e-3	1.2e-3	1e-5
With Noise	0.012	0.013	1e-5

TABLE2: RMS Error	Rate of Presented controllers
-------------------	-------------------------------

#### 5. CONCLUSION

Refer to the research, a position backstepping on-line tuning fuzzy sliding mode control (TBsFSMC) design and application to 6 DOF's robot manipulator has proposed in order to design high performance nonlinear controller in the presence of uncertainties. Regarding to the positive points in backstepping algorithm, sliding mode methodology, estimate the equivalent nonlinear part by applied fuzzy logic methodology and on-line tunable method, the output has improved. Each method by adding to the previous algorithms has covered negative points. In this work in order to solve uncertainty challenge in pure SMC, fuzzy logic estimator method is applied to sliding mode controller. In this paper Mamdani's fuzzy inference system has considered with one input (sliding function) fuzzy logic controller instead of mathematical nonlinear dynamic equivalent part. The system performance in fuzzy sliding mode controller is sensitive to the sliding function especially in presence of external disturbance. This problem is solved by adjusting sliding function of the fuzzy sliding mode controller continuously in real-time by on-line fuzzy like backstepping algorithm. In this way, the overall system performance has improved with respect to the fuzzy sliding mode controller and sliding mode controller. As mentioned in result, this controller solved chattering phenomenon as well as mathematical nonlinear equivalent part in presence of uncertainty and external disturbance by applied backstepping like fuzzy supervisory method in fuzzy sliding mode controller and on-line tuning the sliding function.

#### REFERENCES

- [1] T. R. Kurfess, *Robotics and automation handbook*: CRC, 2005.
- [2] B. Armstrong, *et al.*, "The explicit dynamic model and inertial parameters of the PUMA 560 arm," 2002, pp. 510-518.
- [3] P. I. Corke and B. Armstrong-Helouvry, "A search for consensus among model parameters reported for the PUMA 560 robot," 2002, pp. 1608-1613.
- [4] K. Ogata, *Modern control engineering*: Prentice Hall, 2009.
- [5] F. Piltan, et al., "Artificial Control of Nonlinear Second Order Systems Based on AFGSMC," Australian Journal of Basic and Applied Sciences, 5(6), pp. 509-522, 2011.
- [6] Piltan, F., et al., 2011. Design sliding mode controller for robot manipulator with artificial tunable gain. Canaidian Journal of pure and applied science, 5 (2): 1573-1579.
- [7] Piltan, F., et al., 2011. Design Artificial Nonlinear Robust Controller Based on CTLC and FSMC with Tunable Gain, International Journal of Robotic and Automation, 2 (3): 205-220.
- [8] Piltan, F., et al., 2011. Design Mathematical Tunable Gain PID-Like Sliding Mode Fuzzy Controller with Minimum Rule Base, International Journal of Robotic and Automation, 2 (3): 146-156.

- [9] Piltan, F., et al., 2011. Design of FPGA based sliding mode controller for robot manipulator, International Journal of Robotic and Automation, 2 (3): 183-204.
- [10] Piltan, F., et al., 2011. A Model Free Robust Sliding Surface Slope Adjustment in Sliding Mode Control for Robot Manipulator, World Applied Science Journal, 12 (12): 2330-2336.
- [11] Piltan, F., et al., 2011. Design Adaptive Fuzzy Robust Controllers for Robot Manipulator, World Applied Science Journal, 12 (12): 2317-2329.
- [12] J. J. E. Slotine and W. Li, *Applied nonlinear control* vol. 461: Prentice hall Englewood Cliffs, NJ, 1991.
- [13] J. J. Slotine and S. Sastry, "Tracking control of non-linear systems using sliding surfaces, with application to robot manipulators<sup>†</sup>," *International Journal of Control,* vol. 38, pp. 465-492, 1983.
- [14] J. J. E. Slotine, "Sliding controller design for non-linear systems," *International Journal of Control,* vol. 40, pp. 421-434, 1984.
- [15] R. Palm, "Sliding mode fuzzy control," 2002, pp. 519-526.
- [16] C. C. Weng and W. S. Yu, "Adaptive fuzzy sliding mode control for linear time-varying uncertain systems," 2008, pp. 1483-1490.
- [17] M. Ertugrul and O. Kaynak, "Neuro sliding mode control of robotic manipulators," *Mechatronics,* vol. 10, pp. 239-263, 2000.
- [18] P. Kachroo and M. Tomizuka, "Chattering reduction and error convergence in the sliding-mode control of a class of nonlinear systems," *Automatic Control, IEEE Transactions on,* vol. 41, pp. 1063-1068, 2002.
- [19] H. Elmali and N. Olgac, "Implementation of sliding mode control with perturbation estimation (SMCPE)," *Control Systems Technology, IEEE Transactions on,* vol. 4, pp. 79-85, 2002.
- [20] J. Moura and N. Olgac, "A comparative study on simulations vs. experiments of SMCPE," 2002, pp. 996-1000.
- [21] Y. Li and Q. Xu, "Adaptive Sliding Mode Control With Perturbation Estimation and PID Sliding Surface for Motion Tracking of a Piezo-Driven Micromanipulator," *Control Systems Technology, IEEE Transactions on,* vol. 18, pp. 798-810, 2010.
- [22] B. Wu, *et al.*, "An integral variable structure controller with fuzzy tuning design for electro-hydraulic driving Stewart platform," 2006, pp. 5-945.
- [23] L. A. Zadeh, "Toward a theory of fuzzy information granulation and its centrality in human reasoning and fuzzy logic," *Fuzzy Sets and Systems*, vol. 90, pp. 111-127, 1997.
- [24] L. Reznik, *Fuzzy controllers*: Butterworth-Heinemann, 1997.
- [25] J. Zhou and P. Coiffet, "Fuzzy control of robots," 2002, pp. 1357-1364.
- [26] S. Banerjee and P. Y. Woo, "Fuzzy logic control of robot manipulator," 2002, pp. 87-88.
- [27] K. Kumbla, et al., "Soft computing for autonomous robotic systems," *Computers and Electrical Engineering*, vol. 26, pp. 5-32, 2000.

- [28] C. C. Lee, "Fuzzy logic in control systems: fuzzy logic controller. I," *IEEE Transactions on systems, man and cybernetics,* vol. 20, pp. 404-418, 1990.
- [29] R. J. Wai, et al., "Implementation of artificial intelligent control in single-link flexible robot arm," 2003, pp. 1270-1275.
- [30] R. J. Wai and M. C. Lee, "Intelligent optimal control of single-link flexible robot arm," *Industrial Electronics, IEEE Transactions on,* vol. 51, pp. 201-220, 2004.
- [31] M. B. Menhaj and M. Rouhani, "A novel neuro-based model reference adaptive control for a two link robot arm," 2002, pp. 47-52.
- [32] S. Mohan and S. Bhanot, "Comparative study of some adaptive fuzzy algorithms for manipulator control," *International Journal of Computational Intelligence*, vol. 3, pp. 303–311, 2006.
- [33] F. Barrero, et al., "Speed control of induction motors using a novel fuzzy sliding-mode structure," *Fuzzy Systems, IEEE Transactions on,* vol. 10, pp. 375-383, 2002.
- [34] Y. C. Hsu and H. A. Malki, "Fuzzy variable structure control for MIMO systems," 2002, pp. 280-285.
- [35] Y. C. Hsueh, *et al.*, "Self-tuning sliding mode controller design for a class of nonlinear control systems," 2009, pp. 2337-2342.
- [36] R. Shahnazi, *et al.*, "Position control of induction and DC servomotors: a novel adaptive fuzzy PI sliding mode control," *Energy Conversion, IEEE Transactions on,* vol. 23, pp. 138-147, 2008.
- [37] C. C. Chiang and C. H. Wu, "Observer-Based Adaptive Fuzzy Sliding Mode Control of Uncertain Multiple-Input Multiple-Output Nonlinear Systems," 2007, pp. 1-6.
- [38] H. Temeltas, "A fuzzy adaptation technique for sliding mode controllers," 2002, pp. 110-115.
- [39] C. L. Hwang and S. F. Chao, "A fuzzy-model-based variable structure control for robot arms: theory and experiments," 2005, pp. 5252-5258.
- [40] C. G. Lhee, et al., "Sliding mode-like fuzzy logic control with self-tuning the dead zone parameters," *Fuzzy Systems, IEEE Transactions on*, vol. 9, pp. 343-348, 2002.
- [41] Lhee. C. G., J. S. Park, H. S. Ahn, and D. H. Kim, "Sliding-Like Fuzzy Logic Control with Selftuning the Dead Zone Parameters," *IEEE International fuzzy systems conference proceeding*, 1999,pp.544-549.
- [42] X. Zhang, *et al.*, "Adaptive sliding mode-like fuzzy logic control for high order nonlinear systems," pp. 788-792.
- [43] M. R. Emami, et al., "Development of a systematic methodology of fuzzy logic modeling," *IEEE Transactions on Fuzzy Systems*, vol. 6, 1998.
- [44] H.K.Lee, K.Fms, "A Study on the Design of Self-Tuning Sliding Mode Fuzzy Controller. Domestic conference," *IEEE Conference*, *1994*, vol. 4, pp. 212-218.
- [45] F. Y. Hsu and L. C. Fu, "Nonlinear control of robot manipulators using adaptive fuzzy sliding mode control," 2002, pp. 156-161.

- [46] R. G. Berstecher, et al., "An adaptive fuzzy sliding-mode controller," *Industrial Electronics, IEEE Transactions on,* vol. 48, pp. 18-31, 2002.
- [47] V. Kim, "Independent joint adaptive fuzzy control of robot manipulator," 2002, pp. 645-652.
- [48] Y. Wang and T. Chai, "Robust adaptive fuzzy observer design in robot arms," 2005, pp. 857-862.
- [49] B. K. Yoo and W. C. Ham, "Adaptive control of robot manipulator using fuzzy compensator," *Fuzzy Systems, IEEE Transactions on,* vol. 8, pp. 186-199, 2002.
- [50] H. Medhaffar, *et al.*, "A decoupled fuzzy indirect adaptive sliding mode controller with application to robot manipulator," *International Journal of Modelling, Identification and Control*, vol. 1, pp. 23-29, 2006.
- [51] Y. Guo and P. Y. Woo, "An adaptive fuzzy sliding mode controller for robotic manipulators," *Systems, Man and Cybernetics, Part A: Systems and Humans, IEEE Transactions on,* vol. 33, pp. 149-159, 2003.
- [52] C. M. Lin and C. F. Hsu, "Adaptive fuzzy sliding-mode control for induction servomotor systems," *Energy Conversion, IEEE Transactions on*, vol. 19, pp. 362-368, 2004.