Optimal Design of Hysteretic Dampers Connecting 2-MDOF Adjacent Structures for Random Excitations

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Abstract

The dynamic behaviour of two adjacent multi-degrees-of-freedom (MDOF) structures connected with a hysteretic damper is studied under base acceleration. The base acceleration is modeled as stationary white-noise random process. The governing equations of motion of the connected system are derived and solved using stochastic linearization technique. This study is concerned on the optimum design of the connecting dampers based on the minimization of the stored energy in the entire system. This procedure is applied on three models. The first is two adjacent three stories buildings, the second is ten stories buildings and the third is ten-story building adjacent to twenty-story building. The connecting damper is installed in three cases; in all floors, double dampers in different floors, and single damper. The results show that at these optimum properties of the connecting dampers, the response of each structure is reduced and the energy of the entire system is reduced.

Keywords: Connected Structures, Stochastic Linearization, Hysteretic Damper, White-noise

1. INTRODUCTION

Increasing the population and growing social and commercial activities along with the limited land resources available in the modern cities lead to more and more buildings being built close to each other. These buildings, in most cases, are separated without any structural connections or are connected only at the ground level. Hence, wind-resistant or earthquake-resistant capacity of each building mainly depends on itself. If the separation distances between adjacent buildings are not sufficient, mutual pounding may occur during an earthquake, which has been observed in the 1985 Mexico City earthquake, the 1989 Loma Prieta earthquake, and many others [1]. Many researchers studied the suitable distance between two buildings to prevent pounding [2, 3, 4, 5].
To prevent mutual pounding between adjacent buildings during an earthquake, many devices are implemented between the neighboring floors of the adjacent buildings as hinged links suggested by Westermo [6], bell-shaped hollow connectors developed by Kobori et al. [7], viscoelastic damper by Zhu and Iemura [8], Zhang And Xu [9], Kima et al. [10], fluid dampers by Xu et al. [11], Zhang And Xu [12], MR dampers by Bharti et al. [13] , friction dampers by Bhaskararao and Jangid [14, 15], non-linear hysteretic Dampers by Ni et al. [16], and shared tuned mass damper by Abdullah et al. [17].

Many criteria were studied to reach to the optimum parameters of the connected dampers as Luco and De Barros [18] by reducing the first and second modes of the taller building, Aida and Aso [19] which try to maximize the damping in the system, Zhu and Y.L. Xu [20] by minimizing the averaged vibration energy, Yong et al. [21] using multi-objective genetic algorithm and stochastic linearization method, Bhaskararao and Jangid [22] by minimizing the mean square displacement or acceleration responses, and Basili, Angelis [23, 24] by minimizing the energy in the structures. The previous researches focused on the study of two single degree of freedom (SDOF) systems or two multi-degrees-of-freedom (MDOF) systems transformed into two SDOF [24] using the principle of virtual displacements. But the pounding occurs actually between two MDOF systems and the transferred SDOF systems cannot identify the original MDOF systems especially when the connecting dampers have varied properties or are installed in different floors.

This paper aims to perform an optimal design for nonlinear hysteretic devices connecting two adjacent structures described by 2-MDOF system excited by seismic input which is modeled as a Gaussian white-noise stationary stochastic process. As the entire system is nonlinear, a stochastic linearization technique is applied in order to simplify the problem. The optimal design of such devices generally implies that a large number of equations which solved iteratively. A parametric study is performed to obtain the optimum properties of the interconnecting hysteretic devices.

2. PROBLEM DEFINITION

When two MDOF systems having n_1 and n_2 floors respectively connected with n_d hysteretic passive devices as shown in Fig. 1, the equation of motion of the two buildings are

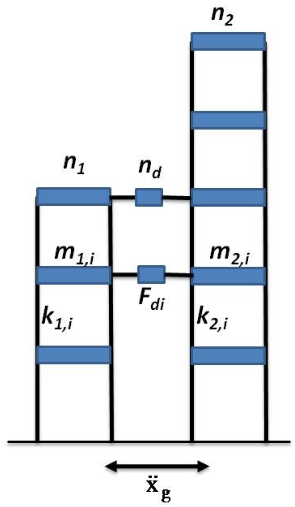

FIGURE 1: Problem model

$$m_{1,i}(\ddot{x}_{1,i}+\ddot{x}_g)+c_{1,i}\,\dot{x}_{1,i}-c_{1,i+1}(\dot{x}_{1,i+1}-\dot{x}_{1,i})+k_{1,i}\,x_{1,i}-k_{1,i+1}(x_{1,i+1}-x_{1,i})-F_{di}=0 \quad (1)$$

and

$$m_{2,i}(\ddot{x}_{2,i}+\ddot{x}_g)+c_{2,i}\,\dot{x}_{2,i}-c_{2,i+1}(\dot{x}_{2,i+1}-\dot{x}_{2,i})+k_{2,i}\,x_{2,i}-k_{2,i+1}(x_{2,i+1}-x_{2,i})+F_{di}=0 \quad (2)$$

where $m_{j,i}$, $c_{j,i}$ and $k_{j,i}$ are the mass, damping and stiffness of the i^{th} floor in the j^{th} building, $\ddot{x}_{j,i}$, $\dot{x}_{j,i}, x_{j,i}$ are the acceleration, velocity and displacement of the i^{th} floor in the j^{th} building, $\ddot{x}_g$ is the ground acceleration, and F_{di} is the force exerted by the damper that fix in the i^{th} floor. The damper force is given by [25]

$$F_{di}=\alpha_i\,k_{3,i}(x_{2,i}-x_{1,i})+(1-\alpha_i)k_{3,i}\,x_{Yi}\,z_i \quad (3)$$

where α_i, k_{3i} and x_{Yi} are the post-pre yield stiffness ratio, pre-yield stiffness and yield displacement of the damper that fix in the i^{th} floor. This force is taken zero if no damper is implemented in this floor. z_i is the auxiliary argument that defined by the Bouc–Wen model through the nonlinear first-order differential equation [26]:

$$\dot{z}_i=-\gamma\left|\Delta\dot{x}_i\right|z_i\left|z_i\right|^{n-1}-\upsilon\,\Delta\dot{x}_i\left|z_i\right|^{n}+A\,\Delta\dot{x}_i \quad (4)$$

where $\Delta\dot{x}_i=\dot{x}_{2,i}-\dot{x}_{1,i}$ is the relative velocity between the two corresponding ith floors when the device is located and the parameters γ, υ, A determine the shape of the hysteresis loops; n controls the smoothness of the transition from the pre-yield to the post-yield region and $\dot{z}_i$ is the derivative of the auxiliary arguments z. This study uses the parameter values as listed in Table 1.

Parameter	Value	Parameter	Value
□	0.5	A	1
□	0.5	n	1

TABLE 1: Bouc–Wen model parameters for hysteresis damper

This non-linear equation can be replaced by other linearized equation as follow [25]

$$x_{Yi}\dot{z}_i + c_{ei}\,\Delta\dot{x}_i + k_{ei}\,z_i = 0 \tag{5}$$

The equivalent linearized parameters (c_{ei}, k_{ei}) can be evaluated in terms of the second moments of $\Delta\dot{x}_i$ and z_i by statistical optimization to minimize the difference between the original and the linearized equations, and they are given by [25];

$$c_{ei} = \sqrt{\frac{2}{\pi}}\left[\upsilon\sqrt{E[z_i^2]} + \gamma\frac{E[\Delta\dot{x}_i z_i]}{\sqrt{E[\Delta\dot{x}_i^2]}}\right] - A \tag{6}$$

$$k_{ei} = \sqrt{\frac{2}{\pi}}\left[\gamma\sqrt{E[\Delta\dot{x}_i^2]} + \upsilon\frac{E[\Delta\dot{x}_i z_i]}{\sqrt{E[z_i^2]}}\right] \tag{7}$$

where the terms $E[z_i^2]$ and $E[\Delta\dot{x}_i^2]$ are, respectively, the variance of variables z_i and $\Delta\dot{x}_i$, and $E[\Delta\dot{x}_i z_i]$ is the covariance of the mentioned variables.
Equations 1, 2 and 5 are expressed in matrix form, combined and rearranged in single equation as follow

$$M\ddot{X} + C\dot{X} + K\,X = M\{1\}\,\ddot{x}_g \tag{8}$$

where X is the variable vector which takes the form

$$X = [X_1^T \quad X_2^T \quad Z^T]^T \tag{9}$$

and M, C and K are the mass, damping and stiffness matrices which take the following forms

$$M = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & 0 \end{bmatrix},\quad C = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ -C^e & C^e & I \end{bmatrix} \text{ and } K = \begin{bmatrix} K_1 + K^\alpha & -K^\alpha & -K^{\alpha 1} \\ -K^\alpha & K_2 + K^\alpha & K^{\alpha 1} \\ 0 & 0 & K^e \end{bmatrix} \tag{10}$$

where the sub-matrices C^e, K^α, $K^{\alpha 1}$ and K^e are diagonal matrices with size n_d and their expressions are given by:

$$C_{i,i}^e = c_{ei} / x_{yi},\ K_{i,i}^\alpha = \alpha k_{3,i},\ K_{i,i}^{\alpha 1} = (1-\alpha)\,k_{3,i}\,x_{yi},\ K_{i,i}^e = k_{ei} / x_{yi} \tag{11}$$

The equations of motion can be generalized and rewritten in space state form as a system of first-order differential equations as follow:

$$\dot{Y} = AY + B\ddot{x}_g \tag{12}$$

where Y is the state vector and takes the form

$$Y = \begin{bmatrix} X_1^T & X_2^T & \dot{X}_1^T & \dot{X}_2^T & Z^T \end{bmatrix}^T \tag{13}$$

and

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \tag{14}$$

$$B = -\begin{bmatrix} \{0\} \\ \{1\} \end{bmatrix} \tag{15}$$

The seismic input is modeled as a Gaussian white-noise stationary stochastic process, which is characterized by the power spectral density So. The white-noise process is chosen in the analysis for its simplicity.
Since the excitation is stationary, this problem can be solved through the Lyapunov equation:

$$AS_{YY} + S_{YY}A^T + 2\pi S_o BB^T = \dot{S}_{YY} \tag{16}$$

where S_{YY} is the covariance matrix of the zero-mean state vector Y and is given by:

$$S_{YY} = E[YY^T] \tag{17}$$

For the stationary response of the system, the covariance matrix is constant over time and the preceding equation reduces to the algebraic Lyapunov equation:

$$AS_{YY} + S_{YY}A^T + 2\pi S_o BB^T = 0 \tag{18}$$

It must be stated that the matrix A involves the equivalent damping and stiffness parameters that are in the same time are functions of the covariance elements of the matrix S. Then, an iterative scheme is required to obtain the solution to Eq. (18). A Matlab program is prepared to make this iterative solution to obtain the stochastic values of the structures.

The aim of this study is to obtain the optimal design of nonlinear hysteretic devices interconnecting two adjacent structures described by 2-MDOF system (Fig. 1). In order to perform the optimal design of nonlinear dissipative control systems, different criteria may be followed. Among the large number of design methodologies, the criterion used in this work refers to energy-based approach. Such criterion is associated with the concept of optimal performance of the dissipative connection.
In order to consider the energy balance in the system, it is started from Eq. (2) where the relative energy balance of the structural system is defined

$$E_k(t) + E_{dc}(t) + E_e(t) = E_F(t) + E_i(t) \tag{19}$$

where E_k, E_{dc}, E_e, E_f and E_i are kinetic energy, energy dissipated by linear viscous damping, elastic energy, energy associated with the devices and input energy. This equation can be rewritten in the stochastic form as a term of the mean values

$$E[E_k] + E[E_{dc}] + E[E_e] = E[E_F] + E[E_i] \tag{20}$$

where

$$E[E_k] = \tfrac{1}{2} E[\dot{x}_i^2],\ E[E_{dc}] = 2[\zeta_i\ \omega_i\ E[\dot{x}_i^2]\ ]\,t\,,\ \ E[E_e] = \tfrac{1}{2}[\ \omega_i^2\ E[x_i^2]\ ] \tag{21}$$

To obtain optimum design for the devices, some references [23, 24] try to maximize the dissipated energy in the devices. In this paper, another way is followed to minimize the total energy stored in the two buildings. An energy ratio index (ERI) is defined as the ratio between the sum of the total energy in the controlled buildings to that of the uncontrolled buildings as follow:

$$ERI = \frac{\sum_{i=1}^{2}\sum_{j=1}^{n_i} E[E_{t\,controlled}]_{i,j}}{\sum_{i=1}^{2}\sum_{j=1}^{n_i} E[E_{t\,uncontrolled}]_{i,j}} \tag{22}$$

where $E[E_t]_{i,j}$ is the expectation of the total energy in the j^{th} floor of i^{th} building and is defined as

$$E[E_t] = E[E_k] + E[E_{dc}] + E[E_e] \tag{23}$$

The main parameter that control the performance of the hysteretic damper is the yielding force F_y which can be normalized as follow

$$\eta_y = \frac{F_y}{\sqrt{2 S_o\, \omega_1}} \tag{24}$$

where □$_1$ is the fundamental natural frequency of the first building, and h_y is the normalized yielding force.
To clarify the effect of hysteretic damper on the reduction of the buildings response, a response index (Y_{ij}) is defined as follow:

$$Y_{ij} = \frac{E[(x_{ij})^2]}{E[(x_{ij}^o)^2]} \tag{25}$$

where $E[(x_{ij})^2]$, $E[(x_{ij}^o)^2]$ are the variances of the displacement of the j^{th} floor of the i^{th} building of the controlled and uncontrolled structures respectively.
The two buildings are well defined by the relative mass ratio □ (m_2/m_1) and relative stiffness ratio □ (k_2/k_1). During the study the mass ratio was varied between 0.1 and 1, while the relative stiffness ratio was varied between 1 and 10. Damping ratio □ for the two buildings is assumed to be 5%.

3. RESULTS AND DISCUSSION

During this research three different models were developed. The first model consisted of two multi-story adjacent buildings each of them have three stories. The second model consisted of two ten-stories buildings connected with dampers. The third model consisted of ten-story building adjacent to twenty-story building. The details of each model and the main findings are presented in the following sections.

3.1 The First Model

Two multi-story adjacent buildings each of them have three stories are analyzed when a hysteretic dampers are provided in each story when the mass and stiffness ratios take the values (□=1, □ = 2). The response index of each floor in the two buildings and the ERI are shown in Fig. (2) for different values of damper normalized yielding force (η_y). The figure shows three stages of the dampers. The first stage when no damper is provided (η_y=0). For this stage the response indices and ERI are unit. The second stage when the dampers dissipated energy efficiently ($0 < \eta_y < 100$) and in this stage the change of □$_y$ changes the response indices and ERI. The third stage when the dampers are worked linearly ($100 < \eta_y$) and in this stage the change of η_y changes slightly the response indices and ERI. The obtained results agree with that obtained previously by Basili and De Angelis [23] when they analyzed two single story buildings connected with dampers.

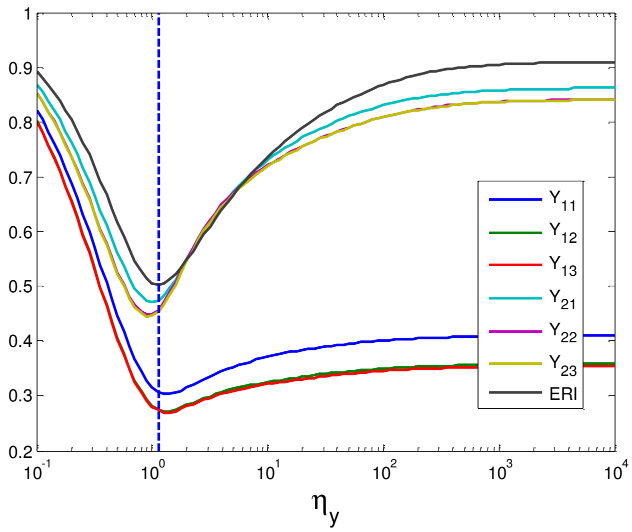

FIGURE 2: ERI and response indices versus □$_y$ for first model (□ = 1, □ = 2, □$_s$ = 5%)

It is found that there is a minimum value of ERI at (η_y = 1.26). It is also found that the response index for each floor in the two buildings under unit, i.e. the connected dampers reduce the response of all floors, and this reduction is maximum at the point of minimum energy. The response indices of the first building (softer) are smaller than that of the second building (stiffer) because the dominator of the response index is the variance of the uncontrolled buildings which is smaller in the second building than that of the first one.

Some references that dealt with two SDOF adjacent buildings [23] stated that this result is not always true especially when there is high difference between the properties of the two buildings as (□=0.1, □ = 5) as shown in Fig. (3a). This figure shows the response indices of the second building which are dramatically increased. But this is not true note because when the figure is expanded, the same figure is found as shown in Fig. (3b) but the response indices of the second building skip the unit and reach to 35 for the linear damper. This is not because the response variance of the second controlled building is increased strong but because the second building is very stiff, then the variance of the uncontrolled (η_y=0) is very small as shown in Fig. (3c).

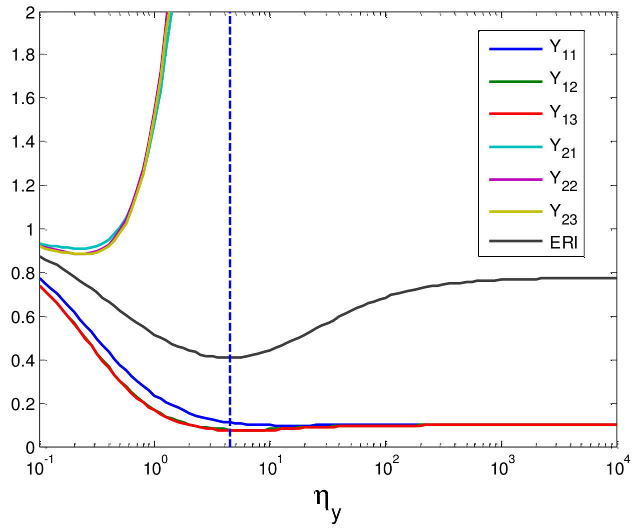

FIGURE 3a : Part of ERI and response indices versus □y for first model (□ = 0.1, □ = 5, □s = 5%)

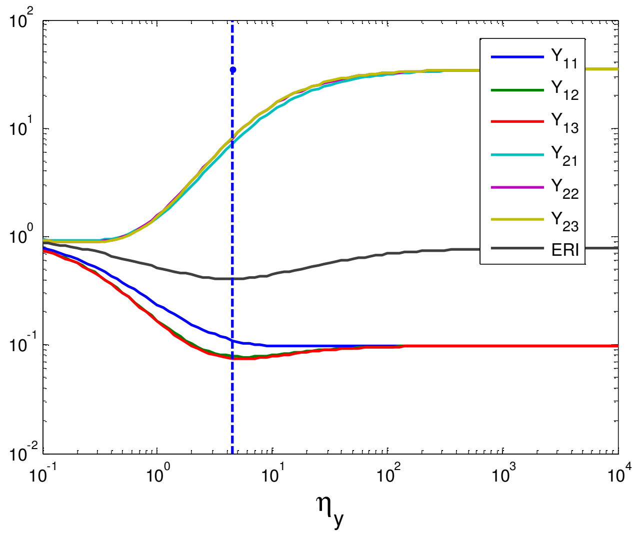

FIGURE 3b : Complete ERI and response indices versus □y for first model (□ = 0.1, □ = 5, □s = 5%)

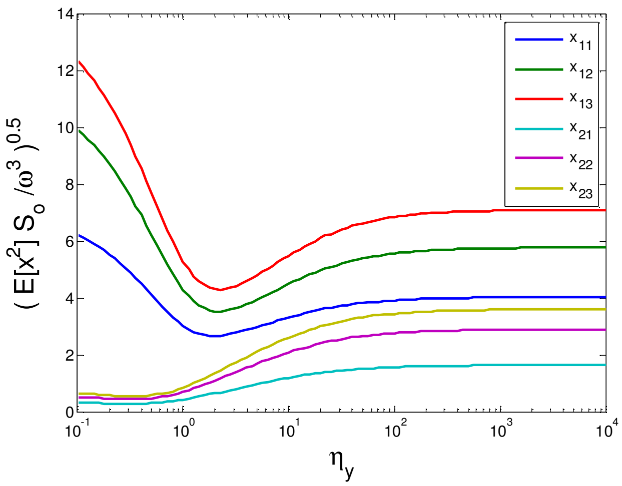

FIGURE 3c : Response indices versus □$_y$ for first model (□ = 0.1, □ = 5, □$_s$ = 5%)

The previous results for a specific properties of the two buildings (□ = 1, □ = 2). A huge study is performed for wide ranges of the buildings properties to conclude the point the minimum ERI and the corresponding optimum η_y as shown in figures (4, 5). It is noted that the minimum ERI is smaller than the unit in all cases, i.e. there is a dissipation of the energy at this optimum point, and the optimum η_y fluctuates round the unit. The expectation of the response indices of the first and second buildings at the optimum point is shown in figures (6,7) respectively. It is found that the response indices expectation is under until except the second building when become very stiffer (□ < 0.17).

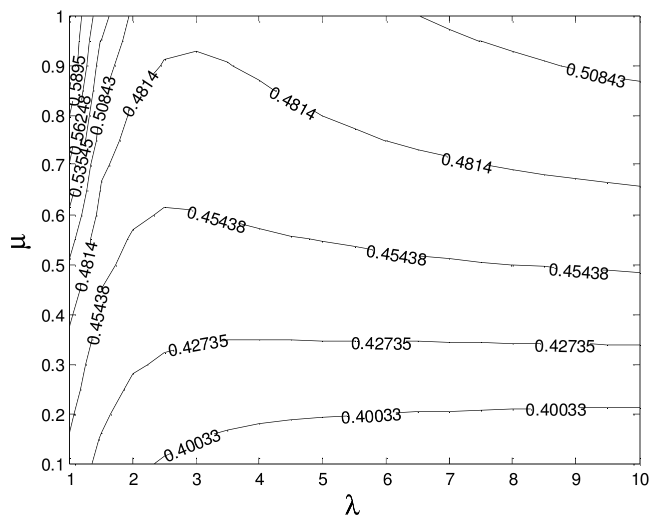

FIGURE 4: Minimum values of ERI for different values of □□

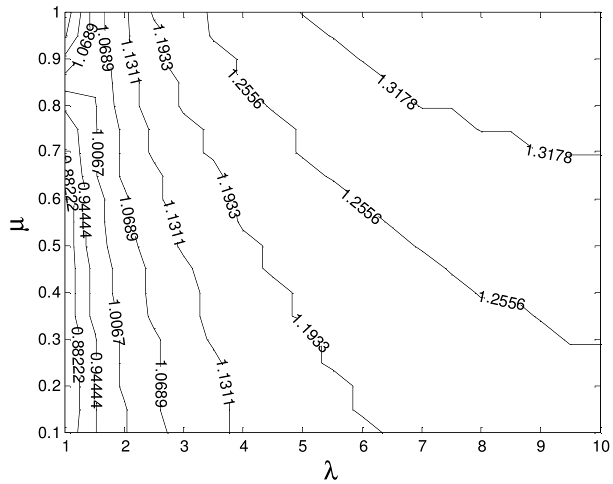

FIGURE 5: Optimum values of $\square_y$ for different values of $\square\square$

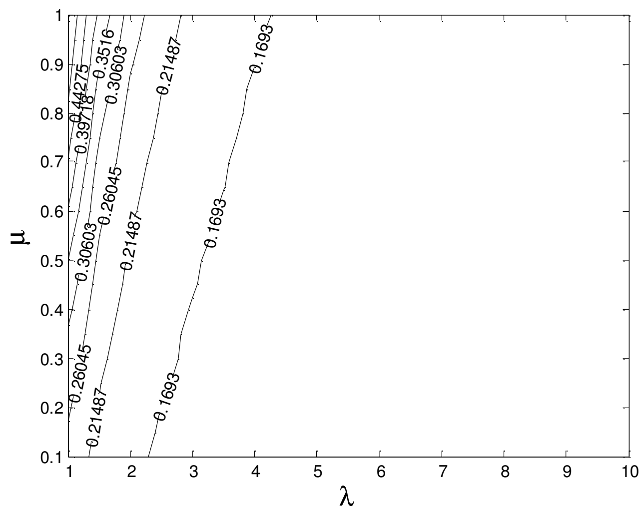

FIGURE 6: Expectation of response index of the first building at Optimum $\square_y$

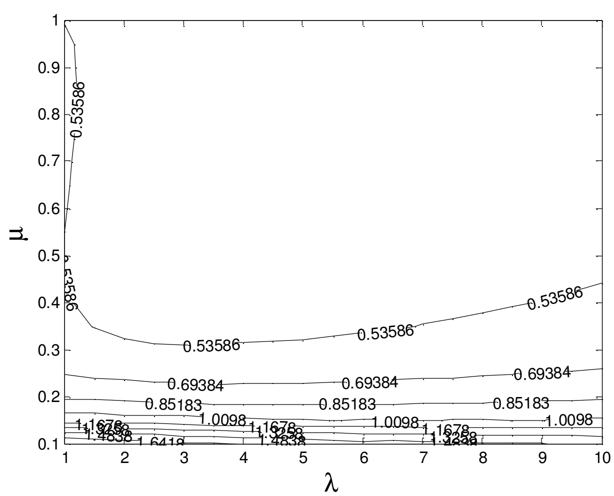

FIGURE 7: Expectation of response index of the second building at Optimum $\square_y$

3.2 Effect of Damper Stiffness Ratio (□$_c$)

The previous work is concerned on the dampers that have a stiffness equals to that of the first buildings (□$_c$=1). When the damper stiffness ratio is changed and ERI is obtained for many cases of buildings properties as shown in Fig. (8). It is shown that after a unit damper stiffness ratio, the variation of the ERI is slight, i.e. the increase of the dampers stiffness does not affect clearly the energy dissipation.

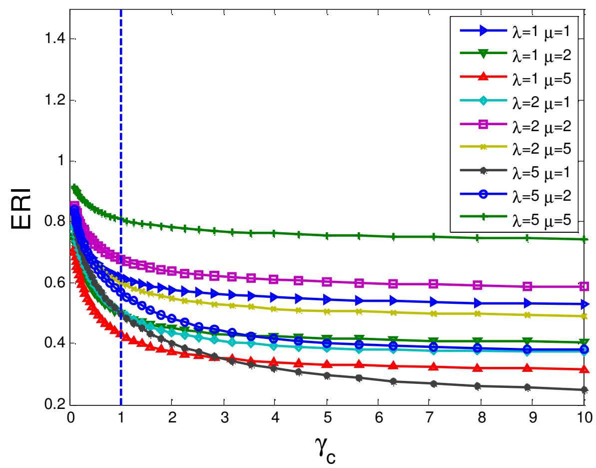

FIGURE 8: ERI of the first model against □$_c$

3.3 Second Model

The second model consisted of two ten-stories buildings connected with dampers is analyzed for specific properties (□ = 2, □ = 1). The normalized damper yield force is changed, and the ERI for the two buildings is calculated and plotted as shown in Fig. (9). The extreme left point close to unit when η_y =0 indicates no damper installed, the extreme right point indicates linear rigid damper, and between them the ERI is varied with the change of η_y and have optimum value at η_y =1. The normalized variances of the two buildings are shown in figures (10 a,b) for the three mentioned cases. The two figures show the normalized variances of the uncontrolled structures, structures connected with elastic rigid damper, and the structures connected with optimum hysteretic damper. It is clear that the optimum hysteretic dampers dissipate the energy and reduce the response of the two buildings. For adjacent buildings with varied relative properties, Fig. (11) shows the minimum values of ERI. It is shown that all values of ERI is less than unit which indicates that the hysteretic dampers reduce the energy in the adjacent buildings in all cases, and its efficiency is pronounced when there is a clear difference between the properties of the two buildings (the lower right corner point).

When the two adjacent buildings are similar (□ = 1, □ = 1, i.e. the upper left corner), the dampers don't work efficiently. The optimum normalized damper yielding force is around the unit as shown in Fig. (12). The expectation of response indices of the two building at Optimum η_y are shown in figures (13, 14) for varied values of system properties. It is shown that the response of the first building (softer) is less than unit specially when there is high difference between the two buildings (lower right point), while the response of the second building (stiffer) is less than unit until the mass ratio have a value 0.3. Above this value of mass ratio, the hysteretic dampers dissipate the energy and the response of the two buildings are reduces, while under this ratio, the mass of the second building is very light and then the response of the original second building that compared with it is very small. The change of the damper stiffness ratio (□$_c$) affects sharply before the unit and then the change will be slight as shown in Fig. (15) which coincide with that concluded for the first model.

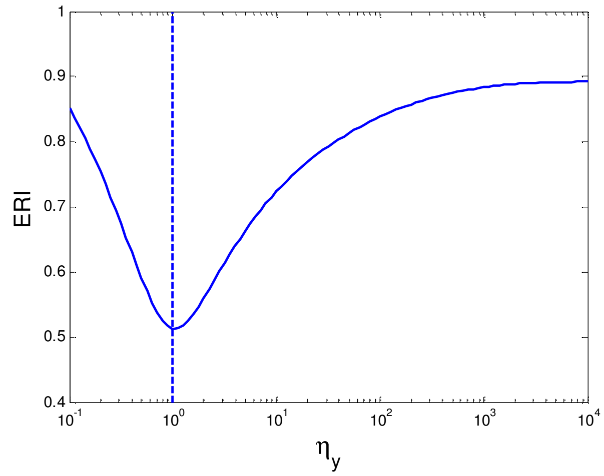

FIGURE 9: ERI of the second model against $\square_y$ ($\square$ = 1, $\square$ = 2, $\square_s$ = 5%)

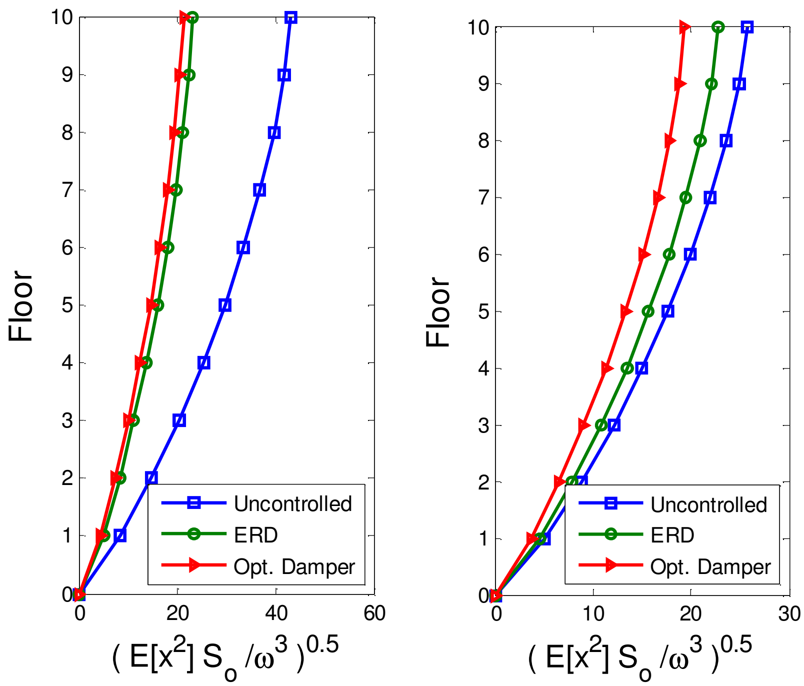

FIGURE 10: Normalized variance of the two buildings in the second mdoel

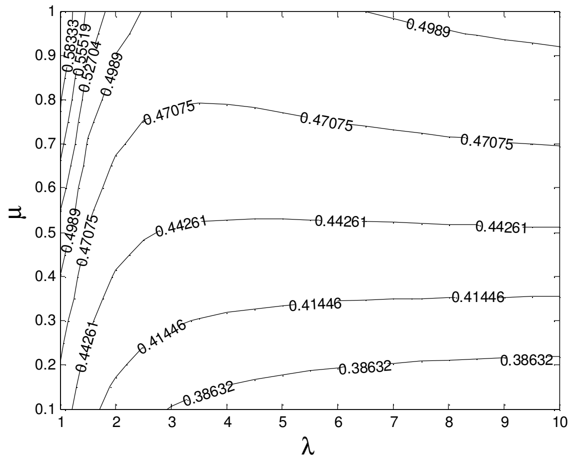

FIGURE 11: Minimum values of ERI of the second model for different values of □□

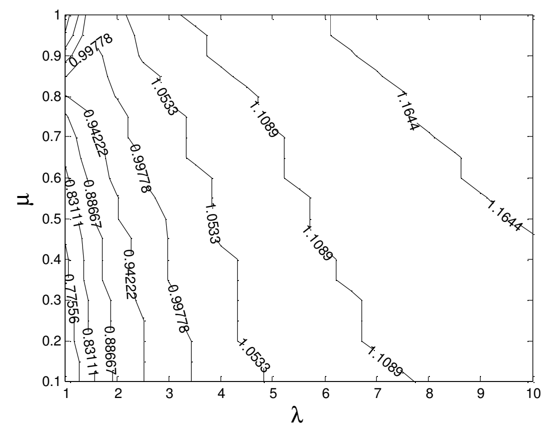

FIGURE 12: Optimum values of □y for different values of □□

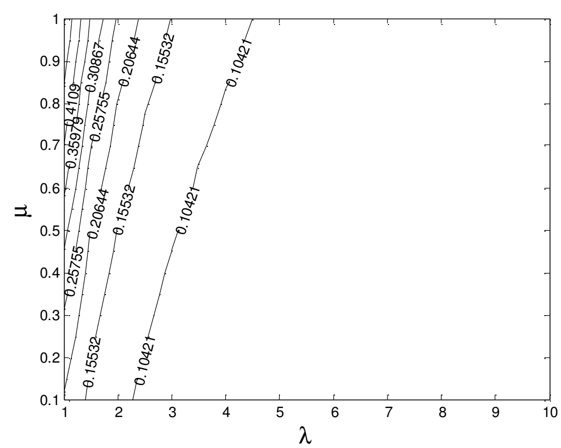

FIGURE 13: Expectation of response index of the first building at Optimum □y

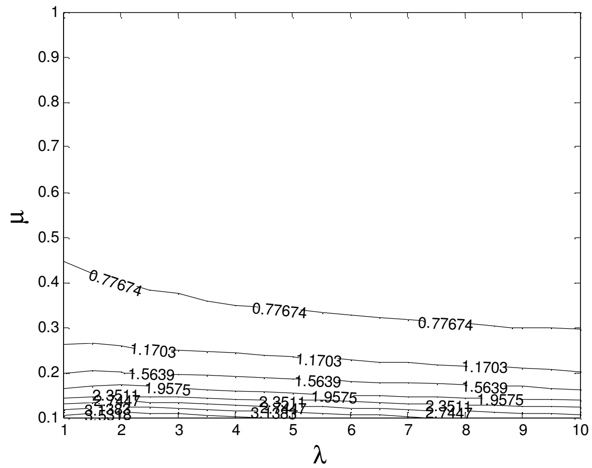

FIGURE 14: Expectation of response index of the second building at Optimum η_y

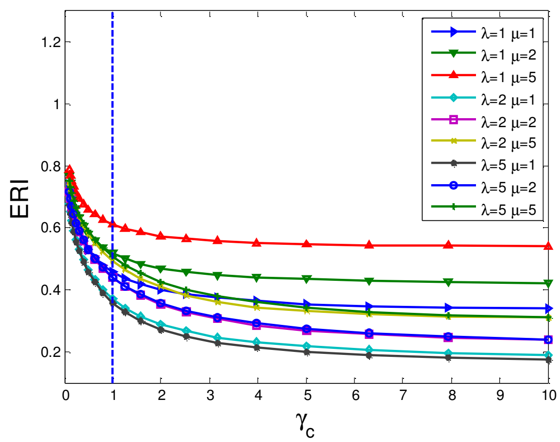

FIGURE 15: ERI of the second model against γ_c

3.4 Different Buildings Heights

The third model consisted of ten-story building adjacent to twenty-story building. This model was analyzed when the two buildings are connected with hysteretic damper at each story with varied normalized yielding force. Fig. (16) shows the ERI and the expectation of the response quantities of the two buildings. It is clear that the dampers dissipate the energy in the system and reduce the response of the two buildings where all curves under unit. This response reduction for all stories of the two buildings as shown in Fig. (17). It is noticed that the response of the first building at the minimum ERI is near this at the elastic rigid damper, which is shown in Fig. (16) where the value at the extreme right point is near that at the dashed line. The response of the second building at the optimum η_y is less than that of the uncontrolled and the elastic rigid dampers.

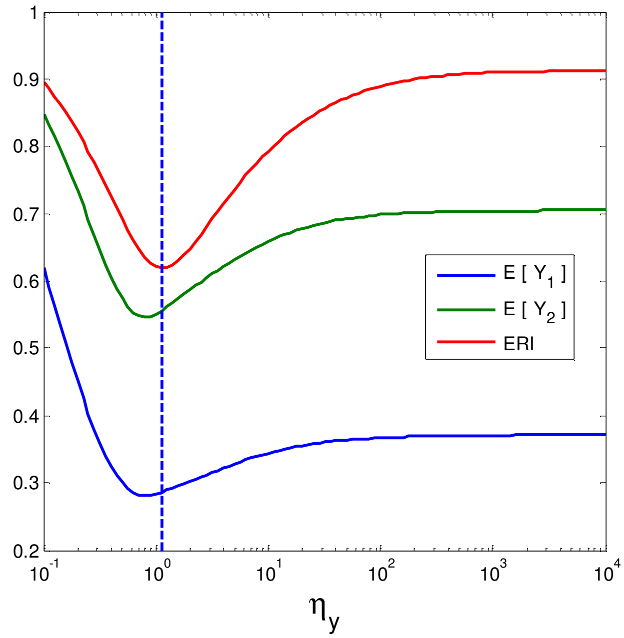

FIGURE 16: ERI and expectation of response quantities of the third model against hy (□ = 1, □ = 2, $\square_s$ = 5%)

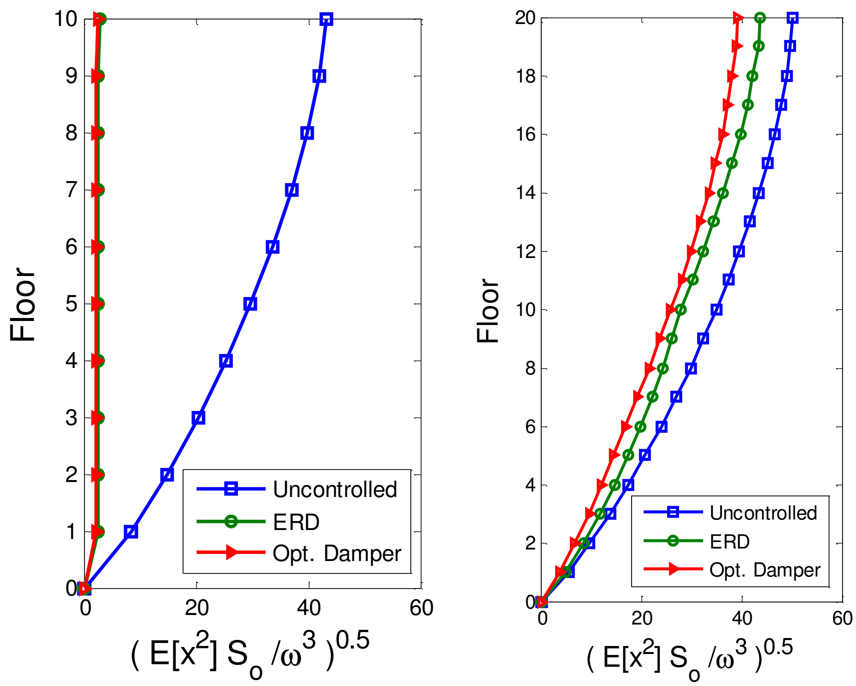

FIGURE 17: Normalized variance of the two buildings in the third mdoel

3.5 Double Dampers Installation

The second model was also analyzed when double dampers are installed in two floors simultaneously to find the optimum damper properties. Table (2) shows the values of the minimum ERI (above diagonal) and the optimum damper normalized yielding force η_y (under diagonal). It is shown that the double dampers dissipate the energy in the two buildings where its values are under unit especially when one of the two dampers is installed at the top floor while the other damper is installed at any floor (the last column in the table). Table (3) shows the expectation of the response indices for the first building (above diagonal) and second one (under diagonal) when double dampers installed. It is clear that the responses of the two buildings are reduced sharply.

	1	2	3	4	5	6	7	8	9	10
1	X	0.79	0.75	0.73	0.72	0.72	0.73	0.72	0.72	0.70
2	2.65	X	0.75	0.73	0.71	0.71	0.71	0.71	0.71	0.69
3	2.75	2.40	X	0.72	0.71	0.71	0.71	0.71	0.71	0.70
4	2.70	2.45	2.25	x	0.72	0.71	0.71	0.71	0.71	0.70
5	2.60	2.40	2.25	2.15	X	0.72	0.71	0.71	0.72	0.71
6	2.50	2.40	2.25	2.20	2.10	x	0.73	0.72	0.72	0.71
7	2.25	2.25	2.25	2.15	2.15	1.95	x	0.74	0.73	0.72
8	2.25	2.20	2.10	2.05	2.05	2.00	1.90	x	0.73	0.73
9	2.25	2.20	2.10	2.05	1.95	1.95	1.95	1.85	x	0.73
10	2.35	2.30	2.20	2.15	2.05	2.00	2.00	1.85	1.70	x

TABLE 2: Minimum ERI and optimum ζ_y for Double dampers installed

	1	2	3	4	5	6	7	8	9	10
1	X	0.55	0.44	0.37	0.34	0.33	0.33	0.32	0.31	0.31
2	0.67	x	0.42	0.36	0.33	0.31	0.31	0.30	0.30	0.30
3	0.56	0.54	X	0.35	0.32	0.31	0.30	0.30	0.29	0.29
4	0.50	0.49	0.48	x	0.32	0.30	0.30	0.30	0.30	0.29
5	0.47	0.47	0.47	0.47	X	0.31	0.30	0.30	0.30	0.30
6	0.47	0.46	0.47	0.47	0.49	x	0.32	0.31	0.30	0.30
7	0.48	0.48	0.48	0.48	0.50	0.51	x	0.32	0.31	0.31
8	0.48	0.48	0.48	0.49	0.50	0.51	0.52	x	0.32	0.31
9	0.47	0.47	0.48	0.49	0.50	0.51	0.52	0.52	x	0.32
10	0.46	0.46	0.47	0.48	0.49	0.51	0.52	0.51	0.50	x

TABLE 3: Expectation of the response indices for the two buildings for Double dampers installed

3.6 Single Damper Installation

A single damper is installed alone in each floor of the two adjacent buildings and they are analyzed to obtain the optimum location and properties of this damper. Table(4) shows the minimum ERI of each case, the corresponding optimum damper normalized yielding force and the expectation of the two buildings in each case. It is shown that the best case when the damper is installed at the top floor.

Floor	1	2	3	4	5	6	7	8	9	10
ERI	0.90	0.83	0.79	0.78	0.77	0.78	0.79	0.78	0.77	0.76
Zetay	3.30	3.60	3.45	3.25	3.15	2.90	2.50	2.65	2.65	2.65
E(E[x1,i])	7.81	5.54	4.38	3.81	3.50	3.46	3.60	3.42	3.33	3.35
E(E[x2,i])	8.57	6.75	5.58	4.97	4.82	4.94	5.03	4.99	4.87	4.79

TABLE 4: Single damper results

4. SUMMARY AND CONCLUSIONS

The dynamic behaviour of two adjacent MDOF structures connected with a hysteretic damper was studied to obtain the optimum properties of the connecting dampers when subjected to white-noise random excitation based on the minimization of the stored energy in the entire system. It is concluded that:

1. The connecting dampers reduce the response of the two adjacent structures sharply.

2. The benefit of the dampers is pronounced when there is clear difference between the two structures.
3. The optimum damper stiffness is equal to that of the first structure.
4. When two dampers only are installed, it is preferred to install one at of them at the top floor and the other in any floor.
5. When single damper only are installed, it is preferred to install it at the top floor.

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