Algorithm for Edge Antimagic Labeling for Specific Classes of Graphs

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Abstract

Graph labeling is a remarkable field having direct and indirect involvement in resolving numerous issues in varied fields. During this paper we tend to planned new algorithms to construct edge antimagic vertex labeling, edge antimagic total labeling, (a, d)-edge antimagic vertex labeling, (a, d)-edge antimagic total labeling and super edge antimagic labeling of varied classes of graphs like paths, cycles, wheels, fans, friendship graphs. With these solutions several open issues during this space can be solved.

Keywords: Graph Labeling, Edge Antimagic Vertex Labelling, Edge Antimagic Total Labelling, Super Edge Antimagic Labelling, Paths, Cycles, Fan Graphs, Wheels, Friendship Graphs.

1. INTRODUCTION

A graph G = (V, E) is finite, straightforward and un-directed. G denotes graph then G includes a vertex and edge sets. Vertex set denoted by V = V (G) and edge set E = E (G). We followed the standard notations m = |E| and n = |V|. A typical graph theoretical notation is followed refer [13]. Labelled graphs are getting associate more and more helpful family of Mathematical Models for a broad vary of applications [4, 17]. It's terribly crucial impact in network communications explained in [5]. Several latest applications presented its usage to image authentication and frequency allocation.

Graph Labeling is a method of mapping that maps several set of elements of graph to a collection of numbers (usually +ve or non -ve integers). The foremost complete graph labeling latest survey is in [3] [14]. Sedlacek [9] introduce labelings that simplify the thought of a magic labeling. The magic labelings is outlined as a bijection of graph component to set of successive integers ranging from one, satisfying some reasonably "constant sum" property. If this Bijection involves vertices or edges or both as graph elements to a collection of integers yielding a constant sum known as magic constant, it'll be known as Vertex or Edge or Total Magic Labeling. Hartsfield along with Ringel in [6] introduced the idea of an Antimagic graphs. In step with them associate Antimagic labeling is a process of edge labeling of the graph with integers 1,2,…., m in order that weight at every vertex is totally different from the weight at every vertex.

In [7] Bodendiek with Walter outlined the thought of an (a, d)-Antimagic labeling as edge labeling during which the weights of vertex from an AP(arithmetic progression) ranging from a and have common distinction d. Martin Baca, Francois Bertault and MacDougall [8] initiated the notions of the Vertex Antimagic Total Labeling [VATL] and (a, d)-Vertex Antimagic Total Labeling [(a, d)-VATL], and conjuncture that every regular graphs are (a, d)- VATL. In the year 2004, K.A.Sugeng et al. [10] presented the concept of SVMTL (super vertex magic total labeling) & SEMTL (super edge magic total labeling). The existence of In [11] Antimagic vertex labeling of categories of hyper graphs like Cycles, Wheels and therefore the existence and non-existence of the Antimagic vertex labeling of Wheels are mentioned in theorems. An idea to get antimagic labeling for trees given in [12]. In reference [2] they projected the procedure (algorithms) to build (a, d)-Antimagic labeling, Antimagic labeling, (a, d)-vertex Antimagic total labeling of complete graphs and vertex Antimagic total labeling that could be a generalization of many different kinds of labelings.

In [15], we deal with the magic labeling of vertices and edges of a graph. Again magic labeling is expressed in-terms of Vertex Magic Total Labeling (VMTL), Edge Total Magic Labeling (EMTL) and Total Magic Labeling (TML). We have studied existing approaches for magic labeling and we found some improvements can be done over existing VMTL algorithms and we design algorithm to find EMTLs. We proposed new and enhanced algorithms for VMTL, EMTL and TML. We applied these algorithms on different kinds of graphs like cycles, wheels, fans and friendship graphs. In[16] we proposed new algorithms to construct vertex antimagic edge labeling, (a, d)-vertex antimagic labeling, vertex antimagic total labeling and (a, d)-vertex antimagic total labeling and super vertex-antimagic total labeling of various classes of graphs like paths, cycles, wheels, Fan Graph and Friend Graphs.

In continuation to this, we have projected algorithms for EATL (Edge Antimagic Total labeling) on different categories of graphs like cycles, paths, wheels, and fan and Friendship graphs. These algorithms are similar all categories with minor changes because of structural variations. With these we have to study the behaviour of the graphs with specific graph size. For given Graph with size we will determine the attainable labelings, possible values of a and d to create (a, d) Edge opposed magic Total Labelings and also the chance of forming SEMTL (super edge magic total labeling).

2. PRELIMINARIES

Standard definitions of paths and wheels and cycles, fan and Friendship graphs are as follows. A Path (P_n) could be a cycle without an edge from initial vertex to final vertex. Cycle could be a graph wherever there's an edge amid the neighbouring vertices solely and therefore the vertex is adjacent to final one (Fig1a). Wheel could be a Cycle with central hub, wherever all vertices of cycle are neighbouring to that (Fig1b) Fans & Friendship graphs are variations of wheels. If a path is linked to central hub it's a Fan (Fig1c). A Friendship graph has n triangles with 1 common vertex known as hub and n is size of Friendship graph (Fig 1d)

FIGURE 1a: Path (P_4)

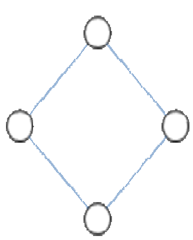

FIGURE 1b: Cycle(C4)

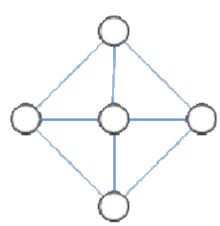

FIGURE 1c: Wheel(W4)

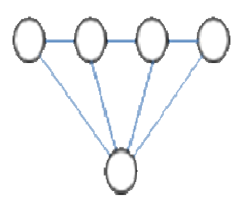

FIGURE 1d: Fan Graph(F4)

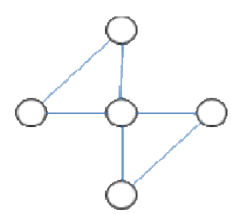

FIGURE 1e: FriendshipGraph(T2)

Labeling is the method of distribution integers to graph elements is often called as a mapping function from integers to Elements of Graph. Magic Labeling is outlined as a bijection from {1, 2, 3, 4, 5… n-1,n} to n - graph elements such the total of every element could be a magic constant K . If the element is vertex or edge or both it's referred to as Vertex or Edge or Total magic Total Labeling.

Let a G be a Graph with vertices v and edges e if there's a 1 to 1 function from set of integers {1,2,3…..,n-2,n-1,n} to edges of Graph & vertices are appointed label as total of edges incident to that. If the edge weights are completely different then it'll be EAVL (Edge Antimagic Vertex

Labeling) .For any 2 integers a, d if the sides are assign with labels is { a , a + d ,……… , a+(e-1)d }, where a is greater than 0 i.e. a>0 and d greater than equal to 0 d>=0 is termed (a, d) EATL. If the vertices are appointed with then successive numbers then, it'll be SEATL (Super Edge Antimagic Labeling).

Likewise for a G graph with vertices V and edges E if there's a 1 to 1 operate from set of integers {1,2,3,4,…………, v + e} to vertex set V & edge set E of graph and edge weight is total of labels appointed to edge & labels appointed to its (V) vertices . If edge weights completely different then it'll be Edge Antimagic Total Labeling (EATL). For any 2 integers a, d if the vertices appointed with labels then , wherever a is greater than 0 i.e. a>0 and d greater than equal to 0 d>=0 is termed as (a, d) EATL. If the vertices are indicated with successive numbers then, it'll be Super Edge Antimagic Total Labeling (SEATL). Following section provides procedure to spot above all for various topologies of Graphs. Accumulative variety of all such potentialities is calculated. And open issues are solved & observed properties of graphs

3. PROPOSED WORK

During this section we have a tendency to discuss algorithmic rules to spot varied features of edge Antimagic labeling. We have a tendency to offer generalized algorithmic rule in common to any or all forms of Graphs mentioned during this paper. As a result of topological variations every graph structure needs some modifications to algorithmic rule.1st we have a tendency to discuss algorithmic rule then needed changes for every kind of graph are given during this section. the subsequent functions are utilized in designing algorithmic rule.

npx : Generates various sizes x from set of n to set of numbers that are obtainable and unused labels.

is_Duplicate (array, size) : The function returns Boolean value if array have duplicate values other wise false

is_RegDiff (array, size) : The functions returns change of rate if the array have adjacent values along with a common difference given by AP else it returns negative one (-).

is_SEATL (array) : The function returns Boolean value if is_RegDiff (array, size) function returns true then it also returns true otherwise false

Input : Graph G with vertices V & Edges E

Output : Possible variety of total Edge Anti Magic Vertex Labeling, Edge Anti Magic Vertex Labeling, (a, d) Edge Anti Magic Vertex Labeling with a, d values, Total Edge Anti Magic Total Labeling, Edge Anti Magic Total Labeling, (a, d) Edge Anti Magic Total Labeling with a, d values and checks existence of super Edge Antimagic Labeling

Algorithmic procedure for EAVL

```
Read Graph with size n & set labels range {1,2,3,4….,r}.
for (i=1; i<=r ;i++)
        If there exist rPn & it's not an isomorphic then set them as labels of hub and spokes.
        for (j=1;j<=r;j++)
        if there exist rPn, then continue process
        Otherwise display "all possible assignments are checked".
        Set them as labels of edges.
        For all edges calculate weight.
        Wei_ght[e] = addition of labels of its vertices.
        If( is_Duplicate(wei_ght, n)) EAVL_cnt++;
        Set d= is_RegDiff(wei_ght, n)
        If(d>=1) adEAVL_cnt++;
        If(d==1) SEAVL_cnt++;
Stop.
```

Algorithm for EATL

```
Read Graph with size n and set labels range {1,2,3,4,5….r}.
For(i=1;i<=r;i++)
        If there exist  rPn &  it's not an isomorphic then set them as labels of hub & spokes.
        for (j=1;j<=r;j++)
        if there exist ar rP3 then continue process.
        Otherwise display "all possible assignments are checked".
        Link1: Set them as labels of last edge, 1st vertex and 1st edge.
         Previous vertex=1 current vertex=2
         for (i=1;i<=r;i++)
                if there exist a rp2  then continue process
                else go to Link1.
                Link2: Set them as labels of current vertex and current edge.
                Previous vertex=current vertex    Current vertex= curr_vertex+1
                If current vertex=n then go to Link2.
                else
                reset labels assigned to previous vertex as available.
                Previous vertex=prev_vertex-1   Current vertex= curr_vertex-1
         For all edges calculate weight.
                Wei_ght[e] = addition of labels assigned to it and labels assigned to its vertices.
                If (  is_Duplicate(wei_ght, n))  EATL_cnt++;
                Set d= is_RegDiff(wei_ght, n)
                If (d>=1)  adEATL_cnt++;
                If (d==1)  SEATL_cnt++;
 Stop.
```

Modifications to be done For Path:

For EAVL Range R is {1,2,.....n}.
For EATL Range R is {1,2,.....2n-1}.
We can avoid Step 2 for Paths and set label of first edge as zero.

Modifications to be done For Cycle:

For EAVL Range R is {1,2,.....n}.
For EAT Range R is {1,2,.....2n}.
We can avoid Step 2 for Cycle.

Modifications to be done For Wheel:

For EAVL Range R is {1,2,.....n+1}.
For EATL Range R is {1,2,.....3n+1}.

Modifications to be done For Fan Graph:

For EAVL Range R is {1,2,.....n}.
For EATL Range R is {1,2,.....3n}.
Set label of first edge as zero.

Modifications to be done For Friendship Graph:

For EAVL Range R is {1,2,.....2*n+1}.
For EATL Range R is {1,2,.....5n+1}.
Set labels of even edges as zero.

4. RESULTS

Here we tend to designed algorithms to supply additive variety of edge anti magic vertex or total labelings. we tend to conjointly known doable variety of (a, d) EAVL or Edge anti magic total labelings for various values of a and d. Also we tend to calculated so many of super Antimagic labelings if exists. Many authors analyzed behaviour of a specific graph structure for a few a and d values. However here these algorithms turn out all such sequence of arrangement of vertices and edges labels. So, we will simply and visually perceive behaviour of any structure. Observations made are as follows.

Paths:
All paths of size n>=3 is Anti magic. For Associate in Nursing instance if the path size is eight there are 6024 (EAVL) Edge anti magic vertex labelings are possible and it's 64 (a, d) Edge anti magic vertex labelings and fifty six among them are super edge magic. It has (3, 2) and (6, 1) Edge anti magic vertex labelings for several sequences. Paths with any length consists several Edge anti magic total labelings. For instance the path with length five has 233520 Edge anti magic total labelings. it's 4680 (a, d) Edge anti magic total labelings among 1840 are super. This sort of study is often done on any path. for each path there's no (a, d)-EAT labeling with d > 6.

Cycles:
In cycles, all cycles with size >=3 is antimagic. These algorithmic rules are estimated for many values of n. For n=5 it resulted thirty Edge anti magic vertex labelings. It also has 10 (a, d) Edge anti magic vertex labelings for a few values of a and d and each one of those are super. For a cycle of size five we tend to observed 1858600 Edge anti magic total labelings. It also has 10460 (a ,d) Edge anti magic total labelings among 4980 are super. feasible values of (a, d) are (6,5), (7,5), (8,4)etc.
In the similar way we will analyze any cycle with any given size.

Wheels:
Wheel with size≥ three, has no edge magic vertex labeling. But it has edge magic total labelings. For given wheel size we are able to show the attainable Edge anti magic total labelings. If the wheel size is three, then there exist 1128768 Edge anti magic total labelings. It's 4512 (a, d) Edge anti magic total labelings for values among 3840 are super Edge anti magic total labelings. we've got (15,1), (16,1), (17,1), (18,1), (10,3) and (11,3) etc attainable (a, d) Edge anti magic total labelings

Fan Graphs:
Fan Graphs are Antimagic with size three to six. Although the number of such sequences are very less all those are (a, d) Edge anti magic vertex labelings. Fan graphs with size three, four and five are giving 8 (a, d) Edge Anti Magic Vertex Labelings however all are super Edge anti magic vertex labelings & all are of (3,1) Edge anti magic vertex labelings. For Fan Graphs we tend to indicate some shocking results. If the fan size is three, then there exist 152152 Edge anti magic total labelings. It's 1344(a, d) Edge anti magic total labeling for various values among 672 are super Edge anti magic total labelings. we've got (14,2),(9,3) ,(16,1),(8,3) (11,3), (15,1), (12,2) and (11,3) etc attainable (a, d) Edge anti magic total labelings.

Friendship Graphs:
Friendship graph also has edge anti magic labeling. Friendship graphs also are observed as every (a, d) edge anti magic vertex labeling is super. Friendship graph with size 2 is having 24 edge anti magic vertex labelings and none of them is (a, d) edge anti magic vertex labeling. However Friendship graphs with size 1,3,4,5 are producing edge anti magic vertex labelings,(a, d) edge anti magic vertex labelings. For these graphs all are super edge magic. Friendship graph with size three producing 192 edge anti magic vertex labelings and 48 (a, d) edge anti magic vertex labelings and every one area unit super. Friendship graph with size four manufacturing 3072 edge anti magic vertex labelings and 2304 (a, d) edge anti magic vertex labelings and all of them are super edge anti magic vertex labelings. Each Friendship graph could be a super edge anti magic total labeling. For Friendship graphs with size 3 has huge number of edge anti magic total labeling (a, d) edge anti magic total labeling for d=2. These are some observations created by us.

5. CONCLUSIONS
In this paper, we tend to provide algorithms to enumerate all Edge Antimagic labelings on wheel graphs, Fan Graphs, cycle Graphs and Friendship graphs. The thought of the algorithms can be applied to alternative categories of graphs or adopted to develop algorithms for other form of labeling. within the in the meantime, we still engaged on algorithm for other form of labeling like edge anti magic and total, harmonious, swish etc. we tend to present the number of non

isomorphic completely different anti magic labelings on every graph for a some small size graphs. The number of non-isomorphic labeling on larger size of the remaining graphs is still an open problem.

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