Interpolation Technique using Non Linear Partial Differential Equation with Edge Directed Bicubic

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Abstract

With the large use of images for the communication, image zooming plays an important role. Image zooming is the process of enlarging the image with some factor of magnification, where the factor can be integer or non-integer. Applying zooming algorithm to an image generally results in aliasing; edge blurring and other artifacts. The main focus of the work presented in this paper is on the reduction of these artifacts. This paper focuses on reduction of these artifacts and presents an image zooming algorithm using non-linear fourth order PDE method combined with edge directed bi-cubic algorithm. The proposed method uses high resolution image obtained from edge directed bi-cubic interpolation algorithm to construct the zoomed image. This technique preserves edges and minimizes blurring and staircase effects in the zoomed image. In order to evaluate image quality obtained after zooming, the objective assessment is performed.

Keywords: Image Zooming, Edge Directed Bicubic, Artifacts, PDE.

1. INTRODUCTION

Digital images are the convenient mode of transferring information. If particular part of image is not clear in its original size, then user may have to zoom the image that means zooming plays an important role in today's world. There are varieties of image zooming techniques. Most of the zooming techniques use image interpolation. Interpolation is to find a set of unknown pixel values from a set of known pixel values in an image. Common techniques are nearest neighbor interpolation [1], bilinear, bi-cubic [2], Edge-forming methods [3], [4], Edge directed bi-cubic interpolation [5], Error-amended sharp edge interpolation [6], linear interpolation [7], locally adaptive zooming [8], Adaptive Zooming (AZ) algorithm [9], and PDE algorithm [10].

Now a day, it has been observed that there are problems while producing an enlarged picture from a given digital image such as blurring and staircase effects. There are linear algorithms such as pixel replication, bilinear interpolation, Bi-cubic interpolation etc. Pixel replication method [1] uses the nearest pixel for image interpolation. Bilinear interpolation [2] uses nearest 2*2 pixels for the interpolation. Bi-cubic technique takes 4*4 neighboring pixels to calculate the interpolation point [3].

In general, the basic problem of linear algorithms is blurring along the edges or artifacts around them [10].These linear technologies do not properly restore zooming images at the edges and areas with high contrast values. The Nonlinear interpolation methods have been suggested to reduce the artifacts of linear methods. The major step in the nonlinear methods is to either fit the edges with some templates or predict edge information for the high resolution image from the low resolution one. The commonly used nonlinear methods for image zooming are locally adaptive zooming, Adaptive zooming, Curvature interpolation technique etc.

In nonlinear techniques, PDE methods have been widely used for de-noising of image. The images obtained from these techniques preserve the edges sharpness and give smooth images. Fourth order PDE method widely used for image de-noising and it reduces the artifacts occurred during zooming.

This research work proposes an image zooming method that combines Fourth order PDE presented in [10] with edge directed bi-cubic interpolation algorithm [5] and studies the performance of this combined technique. It is observed that this new method of image zooming reduces the staircase effect and improves the results compared to existing zooming techniques such as Bilinear interpolation, LAZ interpolation etc.

The paper is arranged as follows. Section II reviews few preliminary concepts. Section III explains proposed work of this paper. Section IV gives simulation results. Section V concludes the paper.

2. PRELIMINARIES

This Section quickly reviews Edge-Directed Bicubic Interpolation algorithm and Non-linear Partial Differential Equation algorithm of image zooming.

2.1 Edge-Directed Bicubic Interpolation (EDBA)

EDBA presented in [5] uses information about varying edge structures of images by a non-linear interpolation method for image zooming. Thus the low resolution (LR) image is magnified by copying original pixels into an enlarged grid and the missing pixels are filled. For every unknown pixel, the 45° and 135°diagonal directional gradients H_1, H_2 are computed as follows

$$H1 = \sum_{p=3,\pm 1} \sum_{q=3,\pm 1} |y(i+p, j-n)| - |y(i+p-2, j-n+2)| (45^o diagonal) \quad (1)$$

$$H2 = \sum_{p=3,\pm 1} \sum_{q=3,\pm 1} |y(i+p, j+n)| - |y(i+p-2, j+n-2)| (135^o diagonal) \quad (2)$$

and the edge direction at the pixel is estimated. Interpolation is performed according to equation (3). For every unknown pixel, q_1 (45° diagonal or horizontal directional interpolation value) and q_2 (135° diagonal or vertical directional interpolation value) are calculated. The edge direction at the pixel is estimated. Interpolation is performed according to equation (3), Then the missing pixel q is estimated as follows [5]. Thus the Initial low resolution (LR) image of the size ($n \times n$) is expanded to high resolution (HR) image of size $(2n-1)\times(2n-1)$.

$$if \quad \frac{(1+H_1)}{(1+H_2)} > S$$

$$q = q_2;$$

$$else\ if \ \frac{(1+H_1)}{(1+H_2)} > S \quad (3)$$

$$q = q_1;$$

else

Compute weights and q.

end

2.2 Non-linear Partial Differential Equation algorithm (PDE)

By using edge directed interpolation algorithm, the LR image of the size (i×j) is converted to the HR image of the size (i.k)*(j.k). High resolution image obtained from edge directed bicubic interpolation algorithm is noisy image. Hence PDE algorithm [10] has been used in this paper for noise removal where HR image is used as input image for PDE algorithm.

PDE algorithm in [10] introduced a method based on partial differential equation. Fourth-order PDE model is a noise removal method to reconstruct the original image $I(x, y)$ from the noisy image $Y(x, y)$.

The non-linear fourth-order PDE formula is as follows [10]:

$$I^{k+1} = I^{k} - \Delta t \left[E_{xx}\left(\frac{E_{xx}\, I^{k}}{|E^{2} I^{k}|}\right) + E_{yx}^{-}\left(\frac{E_{xy}^{+} I^{k}}{|E^{2} I^{k}|}\right) + E_{xy}^{+}\left(\frac{E_{yx}^{-} I^{k}}{|E^{2} I^{k}|}\right) + E_{yy}\left(\frac{E_{yy}\, I^{k}}{|E^{2} I^{k}|}\right) \right] - \Delta t \lambda (I^{k} - Y) \quad (4)$$

where, k= number of iterations = 0, 1, 2…, n

Δx, Δy are the mesh sizes for the x, y variables, ε is the step mesh and Δt is the time step. The HR noisy image obtained by using formula in equation (3) is considered as an initial image (Y). $I^k$is image obtained after zooming.[10]

3. PROPOSED WORK

The proposed image zooming method in this paper is a combination of non linear fourth order PDE method with Edge Directed Bi-cubic algorithm. The work done in papers [10] and [11] inspired the use of non-linear fourth-order PDE model in the proposed method. At first step we down sampled the gray scale input image by the factor of two or four in both row and column dimensions to get the Low resolution (LR) image, from which we obtained the high resolution (HR) zoomed image using different methods such as classical bilinear interpolation method, locally adaptive zooming algorithm and edge directed bi-cubic interpolation algorithm.

The PSNR of zoomed HR image is then measured by comparing it with the original HR image.

First stage: Enlargement of image

The first stage enlarges the input image of (n*n) pixels into a size of (2n-1)*(2n-1). More precisely the size of the image enlarged depends on the zooming factor (2 or 4).

Second stage: Filling the empty pixels

In the second stage the Edge Directed Bi-cubic Interpolation algorithm is used for filling the empty pixels. The image obtained from the edge directed bi-cubic algorithm is given as input to the PDE which is noise removal method given by Equation (2) where Y is the input image for PDE and the output image I is obtained by doing number of iterations until we get the stable output. The flowchart shows this flow of the proposed work.

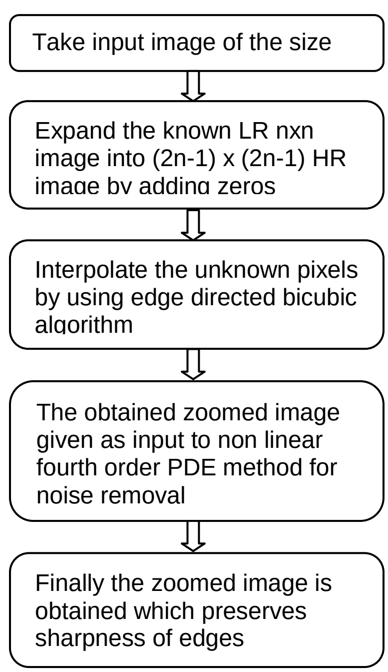


FIGURE1: Flow of the proposed algorithm.

The choice of an appropriate value for the parameters (ε, Δx, Δy) and time step (Δt) used in PDE algorithm is very important for the proposed method because if the value (Δt) in an image is larger than the desired value, the image will not result in stable states through multiple repetitions. Further, if the values of the parameters (ε, Δx, Δy) in an image are not properly given, the image will be blocky. For this purpose, several trial and error experiments have been performed to obtain the proper values on different images.

4. SIMULATION AND RESULTS

For simulation software Matlab (version 8.3.0) has been used to implement image zooming. The best values were designated for different parameters as shown in Table 1 and Image zooming was performed with different methods including proposed method.

Simulation parameters	**Values**
Δt	$10^{-9.5}$
Δx	0.9
Δy	0.9
λ	0
ε	$10^{-9.5}$
Zooming factors	2,4

TABLE 1: Simulation Parameters.

Figures 2, 3, 4 shows the results of simulations of image zooming for three image such as ‘Elaine’, ‘Zelda’ and ‘Trui1’ using various methods. The zooming factors used in simulations were 2 and 4.

(a)Original image

(b)Proposed method

(c) EDBA

(d) BILI

(e) LAZ

FIGURE 2: (a) Original Image (b) Proposed Method, (c) EDBA, (d) BILI, (e) LAZ.

In Figure 2, (a) shows original image of trui1,(b) shows zoomed image obtained by proposed method with zooming factor 2,(c) shows zoomed image obtained by EDBA,(d) shows zoomed image obtained by BILI and (e) shows zoomed image obtained by LAZ.

In similar way, Figures 2 and 3 show the simulated results for test images ‘Elaine’ and ‘Zelda’ using different methods as mentioned earlier.

(a) Original image

(b) Proposed method

(c) EDBA

(d) BILI

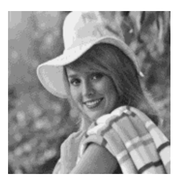
(e) LAZ

FIGURE 3: (a) Original Image (b) Proposed Method, (c) EDBA, (d) BILI , (e) LAZ.

(a)Original image

(b)Proposed method

(c)EDBA



(d)BILI

(e)LAZ

FIGURE 4: (a) Original Image (b) Proposed Method, (c) EDBA, (d) BILI , (e) LAZ.

In this paper, the objective image quality has been assessed to evaluate the performance of the proposed algorithm. In this the Edge Directed Bi-cubic Interpolation algorithm was implemented along with PDE.

The proposed method was compared with the other three interpolation methods. These methods are classical bilinear interpolation method, locally adaptive zooming algorithm and edge directed bi-cubic interpolation algorithm.

The objective evaluation performed shows the better performance of proposed method over bilinear interpolation, locally adaptive zooming algorithm, edge directed bicubic interpolation algorithm in maintaining the edges and quality of zoomed image. In figure 2 the images of 'trui1' have been zoomed for the different magnification factors such as 2 and 4 and results are shown, which indicates that the proposed method reduces the blur effect compared to the other earlier methods. The SSIM and PSNR were calculated using equations (5) and (6).

Structural similarity index metric: The structural similarity index metric (SSIM) proposed in [10] is based on human visual system. It consists of three different metrics. Let *x, y* be the original and the test images, respectively. SSIM is defined as:

$$\begin{aligned} SSIM &= \left(I(x,y) * c(x,y) * s(x,y)\right) \\ I(x,y) &= \frac{2\mu_x\mu_y}{\mu_x^2 + \mu_y^2} \\ C(x,y) &= \frac{2\sigma_x\sigma_y}{\sigma_x^2 + \sigma_y^2} \\ S(x,y) &= \sigma_{xy}/\sigma_x\sigma_y \end{aligned} \qquad (5)$$

I measures how much the *x* and *y* are close in luminance. *C* measures the similarities between the contrasts of the images. *S* is the correlation coefficient between *x* and *y,* which measures the degree of linear correlation between them. where,.

$$\mu_x = \frac{1}{N}\sum_{i=1}^{N} X_i \text{ , } \mu_y = \frac{1}{N}\sum_{i=1}^{N} Y_i \text{ , } \sigma_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \mu_x)^2, \sigma_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \mu_Y)^2,$$

$$\sigma_{xy}^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \mu_x)^2 \, (Y_i - \mu_y)^2.$$

Peak-signal-to-noise ratio-The Peak-signal-to-noise (PSNR) is a well-known metric and is associated with MSE, it is defined as follows:

$$PSNR = \frac{10log_{10} \, 255^2}{MSE} db \qquad (6)$$

Where MSE is mean square error which is defined as:

$$MSE = (\frac{1}{K * I}) \sum_{K} \sum_{I} (M_{KI} - N_{KI})$$

Where *M* and *N* denotes the original image and zoomed image. $K*I$ is the size of both original image and zoomed image.

Table 2 and Table 3 show the values of PSNR and SSIM respectively for seven test images according to magnification factors two and four. The highest PSNR value obtained while using different algorithms is shown in each row.

Factor	2			
PSNR	**BILI**	**LAZ**	**EDBA**	**Proposed method**
Elaine	31.8136	31.2461	31.8052	31.7996
Barbara	30.0997	29.9026	30.0825	30.0837
Rice	29.0972	27.7955	29.0874	29.0833
Lena	30.9486	30.7443	30.9341	30.9365
Trui1	36.4129	35.1123	36.4486	36.4551
Zelda	36.6104	36.0264	36.6062	36.6079
Cameraman	25.5364	24.8564	25.5146	25.5105

TABLE 2: PSNR of Images by Magnification Factor 2.

Factor	4			
PSNR	**BILI**	**LAZ**	**EDBA**	**Proposed method**
Elaine	29.3016	28.4946	29.3016	29.2996
Barbara	28.9864	28.9157	28.9864	28.9880
Rice	24.4693	23.4550	24.4693	24.4704
Lena	29.7818	29.7145	29.7818	29.7856
Trui1	28.7683	27.9542	28.7683	28.7704
Zelda	31.8432	31.3562	31.8432	31.8440
Cameraman	21.5290	21.0812	21.5290	21.5309

TABLE 3: PSNR of Images by Magnification Factor 4.

Factor	2			
SSIM	**BILI**	**LAZ**	**EDBA**	**Proposed method**
Elaine	0.7803	0.7688	0.7801	0.7798
Barbara	0.6143	0.6005	0.6135	0.6138
Rice	0.8071	0.7777	0.8065	0.8063
Lena	0.6530	0.6394	0.6524	0.6528
Trui1	0.9635	0.9548	0.9634	0.9634
Zelda	0.9247	0.9177	0.9245	0.9245
Cameraman	0.8563	0.8411	0.8557	0.8559

TABLE 4: SSIM of Images by Magnification Factor 2.

Factor	4			
SSIM	**BILI**	**LAZ**	**EDBA**	**Proposed method**
Elaine	0.7119	0.6919	0.7119	0.7119
Barbara	0.5099	0.5018	0.5099	0.5110
Rice	0.6658	0.6178	0.6658	0.6658
Lena	0.5580	0.5503	0.5580	0.5584
Trui1	0.8666	0.8451	0.8666	0.8667
Zelda	0.8590	0.8475	0.8590	0.8590
Cameraman	0.7113	0.6940	0.7113	0.7116

TABLE 5: SSIM of Images by Magnification Factor 4.

Through the simulation it has been observed that the proposed method which is combination of edge directed bicubic algorithm and partial differential equation algorithm achieved highest value of PSNR and SSIM for different images. Reduction of artifacts, staircase effect, blurring near the edges is achieved by simulation. After comparing the proposed method with curvature interpolation method[12], error amended sharp edge interpolation[6], new edge directed interpolation[13] and modified edge directed interpolation[14], it has been observed that our results are better than these methods in objective manner and by visual comparison.

5. CONCLUSION

Fourth order PDE method widely used for image de-noising and it reduces the artifacts occurred during zooming. Edge directed bicubic interpolation algorithm is well adapted to the varying edge structure of images. In this paper, the image zooming method has been presented based on fourth order partial differential equation in combination with edge directed bi-cubic interpolation algorithm. It has been found from the results that the proposed method is able to preserve edges and reduce the blur effect compared to the other existing methods explained in the paper.

6. FUTURE SCOPE

The reviewed adaptive scheme is tested with only gray scale images. In future, It may be extended for color images interpolation. Also the use of other interpolation method can make the resulting image better in terms of visual appearance and in terms of computational time.

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