

# Computationally Efficient Methods for Sonar Image Denoising using Fractional Mask

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## Abstract

Sonar images produced due to the coherent nature of scattering phenomenon inherit a multiplicative component called speckle and contain almost homogeneous as well as textured regions with relatively rare edges. Speckle removal is a pre-processing step required in applications like the detection and classification of objects in the sonar image. In this paper computationally efficient Fractional Integral Mask algorithms to remove the speckle noise from sonar images is proposed. Riemann- Liouville definition of fractional calculus is used to create Fractional integral masks in eight directions. The use of a mask incorporated with the significant coefficients from the eight directional masks and a single convolution operation required in such case helps in obtaining the computational efficiency. The sonar image heterogeneous patch classification is based on a new proposed naive homogeneity index which depends on the texture strength of the patches and despeckling filters can be adjusted to these patches. The application of the mask convolution only to the selected patches again reduce the computational complexity. The non-homomorphic approach used in the proposed method avoids the undesired bias occurring in the traditional homomorphic approach. Experiments show that the mask size required directly depends on the fractional order. Mask size can be reduced for lower fractional orders thus ensuring the computation complexity reduction for lower orders. Experimental results substantiate the effectiveness of the despeckling method. The different non reference image performance evaluation criterion are used to evaluate the proposed method.

**Keywords:** Speckle, Fractional Order, Heterogeneous, Patches, Homogeneity.

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## 1. INTRODUCTION

Sonar images are highly affected by speckle noise which reduces spatial resolution. Denoising is required in sonar images to distinguish a number of different regions by analyzing the image. Purely visual interpretation is subjective, qualitative and time consuming. Computer assisted interpretations is used for the identification and interpretation of all distinctive objects in the sonar images [1].

Acoustics is the best means to investigate the water column and sea bed efficiently and accurately. Sonar mapping systems can be roughly divided into three categories: single-beam echo-sounders, multi-beam echo-sounders and side scan sonars [2]. But the tool of choice for high resolution seabed mapping remains the side scan sonar.

Using an important number of transducers, operating at sound or ultrasound frequencies, the sonar systems generate high resolution images. The user can observe, analyzing such an image or some regions. This observation process is perturbed by the speckle noise [3]. This noise is a

result of the coherent addition of the backscatter waves generated by the elementary targets contained in every resolution cell. The speckle reduction filters are very important in the preprocessing phase to increase the detection or classification performances. Such a filter must realize a great speckle reduction in the regions where the reflectivity is constant and the preservation of the details of the scene in the other regions. Denoising of sonar images is for applications like finding the geological aspect of oceanic crusts, manifestations of biological activity, effective monitoring and sustainable management of shallow-water environments, detecting and mapping pipelines and their environments, monitoring of fish and habitats, detecting shipwrecks, mapping oil spills and mine monitoring.

Contrary to the standard filters, the adaptive filters take local image information into consideration while carrying out the filtration process. Adaptive filters can reduce speckle noise in homogeneous areas while preserving texture and high frequency information in heterogeneous areas. Among the best known adaptive classical speckle filters are the Lee [4], the Kuan [5], the Frost [6] filters. These filters use the second-order sample statistics within a minimum mean squared error estimation approach.

In addition to the classical de-speckling filters, more complex filters recently developed include the iterative Speckle Reducing Anisotropic Diffusion (SRAD) filter [7], Gamma MAP filter and multi-scale wavelet decomposition based filters [8] using an adaptive soft-threshold of wavelet coefficients and SRAD filtration of wavelet coefficients.

### 1.1 Side Scan Sonar Imaging

The interpretation of sonar images has traditionally been performed visually by trained interpreters. The interpretation of side-scan imagery is a skilled procedure [9], [10]. This presents the distinct advantage of using the skill of the interpreter to limits which are often unattainable by computers. Computer-assisted interpretation encompasses the fields of image processing and assists the skilled interpreter. It aims at enhancing the visibility of objects, and relations between objects and in some cases, it can also bring information that was invisible to the human eye for physiological reasons like texture-oriented analysis. Most importantly, computer-assisted information brings an objective and quantitative assessment to help the interpreter. The despeckling of the sonar image is the first image processing technique applied on the sonar images after its formation before further interpretations done on such images.

### 1.2 Sonar Image Heterogeneity

Sonar images [11] are characterized by three statistical classes called homogeneous, textured, and target. The best estimator for the first class with a spatially constant reflectivity is the plain averaging of intensity pixel values in a neighborhood. Pixels belonging to the intrinsically noise-free third class should be detected and left unprocessed. The speckle noise affects the intermediate class. Classically the three classes are found by thresholding the local coefficient of variation of the noisy image, defined as the ratio of local standard deviation to local mean. The two thresholds, namely  $C_{\min}$  and  $C_{\max}$  are empirically set equal to noise variance, the standard deviation of speckle, and  $\sqrt[3]{\sigma_n}$ . A patch based noise level estimation method and a patch based homogeneity index calculation is used in the proposed method.

### 1.3 Speckle Noise

Speckle is difficult to distinguish from the real signals at the limit of resolution of the sonar and it proves hard to remove without affecting significantly the image. The multiplicative model for the speckle noise is

$$I(x,y) = R(x,y)u(x,y)$$

where  $I(x,y)$  is the observed image intensity,  $R(x,y)$  denotes the corresponding terrain reflectivity and  $u(x,y)$  is a multiplicative speckle noise statistically independent of  $R(x,y)$ , with mean  $\bar{u}$  and variance  $\sigma^2$ . This model formulates the speckle as a multiplicative modulation of the

scene reflectivity. Hence, the speckle effects are more pronounced in a high intensity area than in a low intensity area.

Prior to attempting to segmentation and feature extraction in the image, the speckle noise must be removed. Denoising is required in sonar images to distinguish a number of different regions by analyzing the image. Two common approaches for de-speckling of images exist. The first approach is to average several images acquired from the same scene, which is called multi-look processing or super-resolution. In that way the speckle noise will be reduced due to its random nature, while the observed scene will not be degraded. The second technique is based on filtering the speckle noise based on a single image using a two-dimensional filter such as standard Low-Pass and Median filters or adaptive ones.

The behavior of speckle noise is similar to Brownian motion[12]. The amplitude of the speckle noise has a Rayleigh distribution and can be described by:

$$A = A_R + jA_I$$

Where  $A_R$  is an in-phase component and  $A_I$  is the quadrature component of the speckle amplitude, both of which are statistically independent random variables with Gaussian distribution, zero mean, and identical variance. The intensity of the speckle noise can be given by:

$$u = A_R^2 + A_I^2$$

and has a negative exponential distribution defined by

$$p(u) = \frac{1}{\bar{u}} e^{-\frac{u}{\bar{u}}}, u > 0$$

Where  $\bar{u}$  is the mean of the speckle intensity  $u$ .

Due to the multiplicative behavior of the speckle noise, an image acquired by a coherent imager can be described by a multiplicative model with exponential distribution.

## 2. FRACTIONAL INTEGRAL MASKS

The classical denoising algorithms uses the integer order integration directly or indirectly and this results in loss of image details. Fractional calculus has proven to be better over integer calculus to analyze and model natural signals [13]. Riemann-Liouville (RL), Grunwald- Letnikov(GL) and Caputo are widely used definitions of fractional calculus. Fractional integral algorithm based on RL definition and GL definition are used for image denoising [14]. The reference [15] uses Riesz fractional differential operator for image enhancement. The method implemented uses the Reimann-Liouville definition to create fractional integral masks in eight directions.

### 2.1 Combined Fractional Integral Mask for Reducing The Computations

The noise which exist as high frequency signals in image is eliminated by using fractional integral mask as low pass filter. The coefficients of mask are calculated using Reimann- Liouville definition for Fractional integral calculus. Fractional order  $\nu > 0$  corresponds to fractional differentiation whereas fractional orders  $\nu < 0$  corresponds to fractional integration. The Reimann-Liouville definition for fractional calculus is as follows:

$$\frac{d^\nu}{dx^\nu} s(x) = \frac{1}{\Gamma(-\nu)} \int_0^x \frac{s(\xi)}{(x-\xi)^{\nu+1}} d\xi = \frac{1}{\Gamma(-\nu)} \int_0^x \frac{s(x-\xi)}{\xi^{\nu+1}} d\xi, \nu < 0$$

The image is a two dimensional signal in which each pixel is represented using its x and y coordinates. The two dimensional equivalent of the above equation used after changing the continuous integral into discrete sum of products and partial differentiation is essential. The equations in [16], [17] are used to estimate the coefficients of masks in eight directions according to the value of order  $\alpha$ . For different fractional orders different eight directional masks are obtained and the significant coefficients in the masks has an inverse relation to the fractional order.

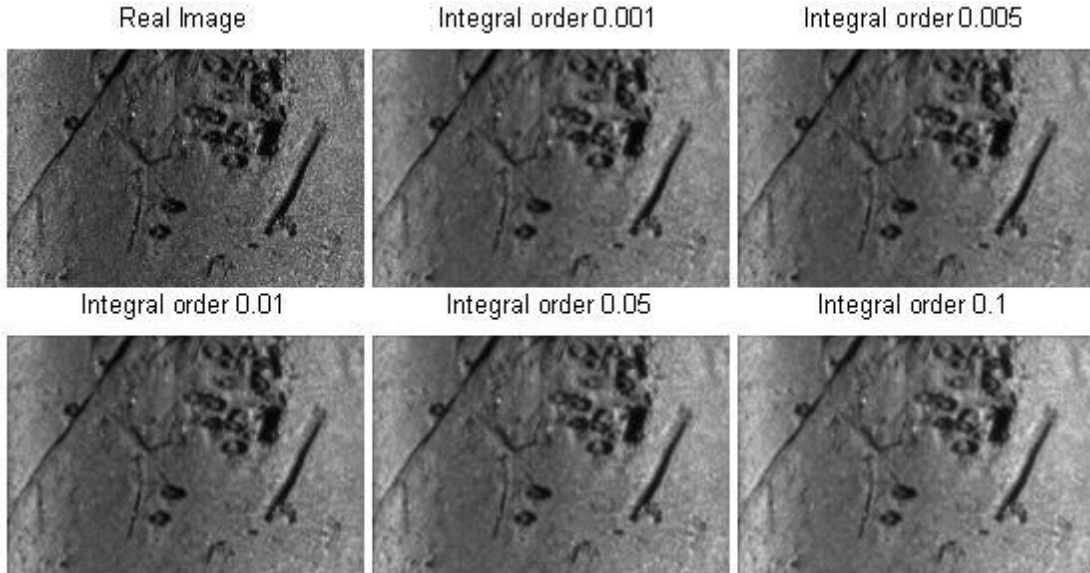
The paper [18] uses a 7x7 masks in eight directions and two types of combining techniques in the process of convolution between image and Fractional Integral masks. In the normal method, the image is convoluted separately with eight directional fractional integral masks. These masks can be implemented for different orders. Mask coefficients are estimated for the required order using equations. Convolution of image with masks of eight directions assures better de-noising as it considers the relationship of the pixel of interest with all its neighbouring pixels of every direction. The individual results of convolution are then combined by taking the average of pixel values corresponding to the respective (x,y) coordinate positions. Thus, the resultant image will carry the effect of adjacent pixels of eight directions and so the method ensures better de-noising performance than the traditional noise removal algorithms.

The technique can be very well utilized in applications which involve directional noise. De-noising of the image can be done in the direction of interest and works well for the range of fractional orders 0.0001 to 0.4.

The algorithm proposed here has utilized the second combining method in the above said paper and is applied on different side scan sonar images affected with speckle noise. The mask used for despeckling is formed by using the eight fractional masks which are arranged and combined according to their directional characteristics. This process in turn results in the formation of a single mask. As the size of individual masks is 7x7, the size of resultant mask will be 13x13. The image with noise is convoluted with the resultant mask. The usage of single mask reduces the computations. This method also reduces the computation time by eliminating the need for multiple convolution operation and works well for the fractional orders of range 0.0001 to 0.18. The de-noising performance of the method is high when compared with that of individual mask convolution method and much higher than traditional noise removal techniques.

The different non reference image performance evaluation criterion like the Equivalent Number of Looks (ENL), Speckle Suppression Index (SSI), Coefficient of Variation (COV) and Correlation Coefficient (CC) are used to evaluate the proposed method and compare with conventional speckle filters. The numerical results substantiates the visual comparisons.

Table 1 shows the experimental results of the fractional integral mask of size 13x13 convoluted with the sonar image of ENL 6.8510. Figure 1 shows the visual comparison of the denoising for varying fractional orders. Performance of the algorithm is compared with that of traditional noise removal method. Experiments show that the mask size required depends on the fractional order. Mask size can be reduced for lower fractional orders thus ensuring the computation complexity reduction for lower orders. Denoising performance is measured based on visual perception and other non reference measurement methods.

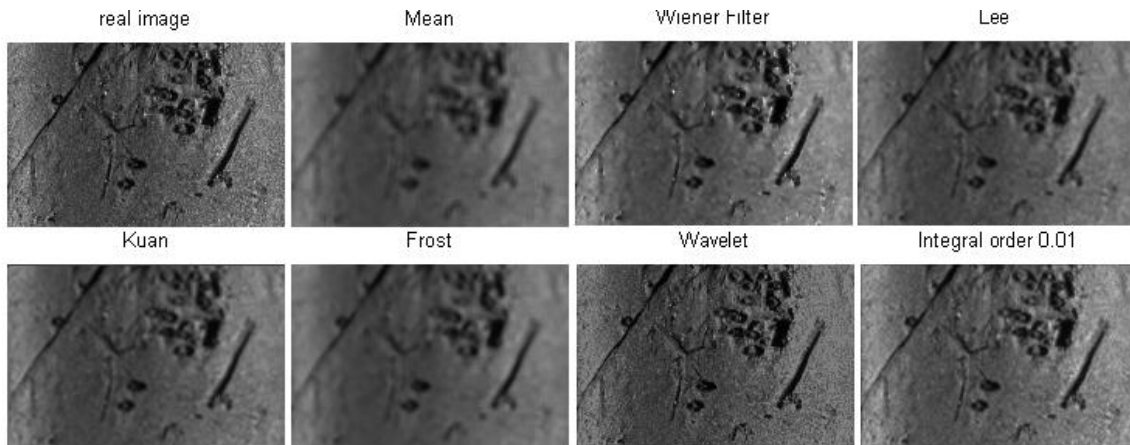


**FIGURE 1:** Visual comparison of the results for varying fractional order of the mask.

| Order | 0.001  | 0.003   | 0.005   | 0.007   | 0.01    | 0.03    | 0.05   | 0.07    | 0.1     |
|-------|--------|---------|---------|---------|---------|---------|--------|---------|---------|
| ENL   | 8.6614 | 8.6821  | 8.703   | 8.7195  | 8.7517  | 8.946   | 9.1347 | 9.3206  | 9.5941  |
| SSI   | 0.8894 | 0.8883  | 0.8872  | 0.8864  | 0.8848  | 0.8751  | 0.866  | 0.8573  | 0.845   |
| CC    | 0.9662 | 0.9661  | 0.9658  | 0.9655  | 0.9652  | 0.9627  | 0.9599 | 0.957   | 0.9523  |
| COV   | 0.3398 | 0.3394  | 0.339   | 0.3387  | 0.338   | 0.3343  | 0.3309 | 0.3275  | 0.3228  |
| SMPI  | 11.771 | 12.2767 | 12.7913 | 13.3061 | 14.0937 | 19.6381 | 25.746 | 32.4686 | 43.7816 |

**TABLE 1:** Metric value comparison of the despeckling for varying fractional order of the mask.

For lower orders the mask size after mask combining can be reduced by extracting the central portion where significant coefficients are actually available. By further reducing the mask size, instead of going with the 13x13 mask, computation complexity can be reduced. The reduction in the mask size of the combined mask to 11x11, 9x9, 7x7, or 5x5 can be based on the significant coefficients which depends on the fractional order.

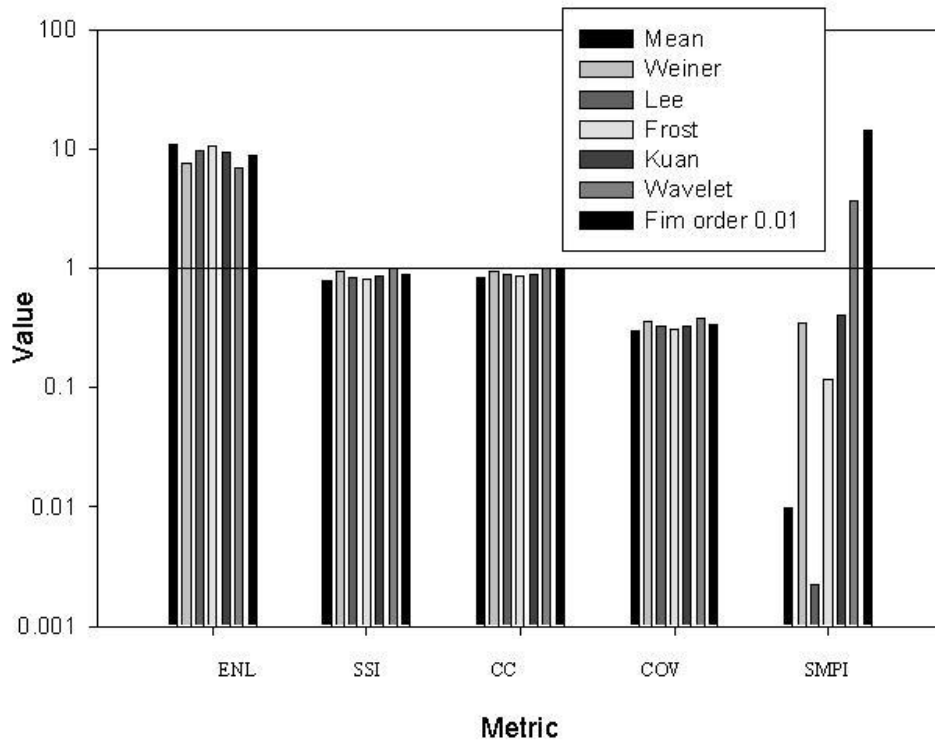


**FIGURE 2:** Visual comparison of the results of the proposed method with the existing methods.

| Filter | Mean     | Wiener | Lee      | Frost   | Kuan   | Wavelet | FIM order 0.01 |
|--------|----------|--------|----------|---------|--------|---------|----------------|
| ENL    | 11.018   | 7.6657 | 9.5421   | 10.5452 | 9.4286 | 6.9838  | 8.7517         |
| SSI    | 0.78853  | 0.9454 | 0.84734  | 0.806   | 0.8524 | 0.9904  | 0.8848         |
| CC     | 0.84147  | 0.9356 | 0.89514  | 0.8636  | 0.8745 | 0.9906  | 0.9652         |
| COV    | 0.30126  | 0.3612 | 0.32373  | 0.3079  | 0.3257 | 0.3784  | 0.338          |
| SMPI   | 0.009928 | 0.3489 | 0.002197 | 0.1163  | 0.4067 | 3.7105  | 14.0937        |

**TABLE 2:** Comparison of the metric value of the proposed method with the existing despeckling methods.

The demonstrated experiment uses a sonar image with ENL 6.8510 and the enhancement in ENL is shown in the quantitative comparison Table 2 and visual comparison of the proposed method in Figure 2. The graphical comparative results is shown in Figure 3.



**FIGURE 3:** Different metric value graphical comparison of the fractional order mask with the existing methods.

### 2.2 Patch Despeckling using Combined Fractional Integral Mask

Most adaptive classical speckle filters like the Lee, the Kuan and the Frost filters are based on the local coefficient of variation, which serves to measure the heterogeneity of sonar images. The sensitivity of the measurements to speckle noise variance of side scan sonar images would greatly deteriorate the speckle reduction. In [19], a texture strength metric which is based on the local image gradient matrix and its statistical properties to select low-rank patches. It is reported that image structure can be measured effectively by the gradient covariance matrix. The dominant direction and its energy can be measured using the eigenvectors and eigenvalues of the gradient covariance matrix [20]. Most of the despeckling algorithms smooth excessively the contours when they are not sufficiently contrasted and remove the edges, resulting in drastic loss of information.

The noise level of the single noisy sonar image is estimated using a patch based algorithm [21] and then non blind denoising is done. Noisy image is decomposed into number of patches in a raster scan form. The patch with the smallest standard deviation among decomposed patches has the least change of intensity. Each patch is assumed to be corrupted by noise.

The paper [22] presented uses a novel parameter for the heterogeneity measurement as a general index to quantitate the sonar image heterogeneity. The method uses a texture strength and patch size related theoretic heterogeneity measurements as a criterion to classify the side scan sonar images as belonging to homogeneous or heterogeneous regions. Then a new speckle reduction algorithm based on the quantitative heterogeneity measurements is proposed. The texture strength  $\xi_i$  is defined as

$$\xi_i = \text{tr}(C_{y_i})$$

The distribution of  $\xi(n)$  is approximated by the gamma distribution to analyze the statistical properties of texture strength.

If the texture strength of that patch is less than the threshold  $\tau$ , then that patch can be regarded as the weak textured patch. The threshold  $\tau$  can be expressed as a function of the given significant level  $\delta$  and noise level  $\sigma_n$ ,

$$\tau = \sigma_n^2 F^{-1} \left( \delta, \frac{N^2}{2}, \frac{2}{N^2} \sigma_n^2 \text{tr}(D_h^T D_h + D_v^T D_v) \right)$$

In equation,  $F^{-1}(\delta, \alpha, \beta)$  is the inverse Gamma cumulative distribution function with the shape parameter  $\alpha$  and scale parameter  $\beta$ .  $N^2$  represents the number of pixels in the patch, and  $D_h$ ,  $D_v$  are the matrices of horizontal and vertical derivative operators derived from the gradient filter.

For denoising, the heterogeneity of image patches is exploited. The small patches are extracted by sliding square windows. If the threshold value is less than  $\tau$ , it is classified as homogeneous patch and the patches with threshold value greater than  $\tau\sqrt{3}$  is classified as belonging to the target patches. Patches of size 7x7 is used to find the noise level of the given side scan sonar image. The confidence level of the natural image patches assumed to be closed to unity as they usually contain some weak textures and for a flat patch the value is taken as 1E- 6.

For calculating the index value of the selected patch, obtain the texture strength matrix based on the local image gradient matrix. From the gradient covariance matrix of the particular patch, obtain the trace of the square root of the eigen values of the covariance matrix. Then the index value of the selected patch is the product of the above value with the number of image patches. Based on this index value classify the patch. If the index value is less than  $\tau$ , it is classified as homogeneous patch and a normal averaging filter is applied to this patch. The patches with index value greater than  $\tau\sqrt{3}$  is classified as belonging to the target patches and are left unprocessed. The patches having an index value lying between these two ranges are considered as the intermediate patches which are assumed to be speckle affected and need to be despeckled before further processing.

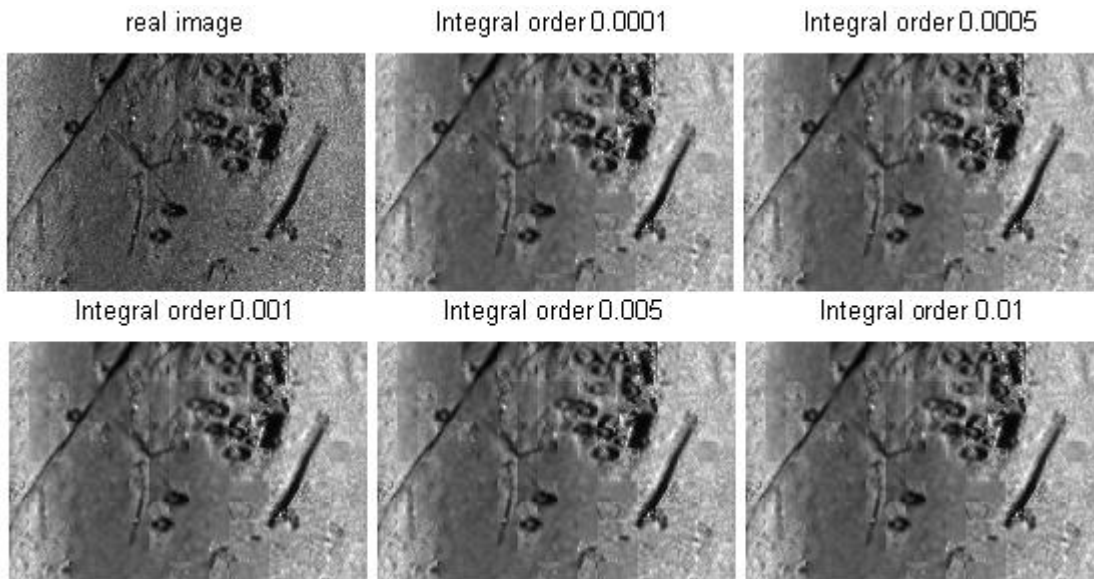
In the proposed method the sonar image heterogeneous patch classification is done based on homogeneity index defined with a patch size of 10 and the convolution with the 13x13 mask is employed on the relevant patches.

First class with a spatially constant reflectivity has the plain averaging of intensity pixel values in a neighborhood as the best estimator. Pixel belonging to the third class should be detected and left unprocessed, as they are intrinsically noise-free. The intermediate class is affected by the speckle noise and denoising is done on these classes using the fractional integral mask convolution method.

| Order | 0.0001 | 0.0005 | 0.001  | 0.005  | 0.007  | 0.01   | 0.05   |
|-------|--------|--------|--------|--------|--------|--------|--------|
| ENL   | 6.8936 | 6.8872 | 6.8792 | 6.8147 | 6.7824 | 6.7338 | 6.0876 |
| SSI   | 0.9969 | 0.9974 | 0.9979 | 1.0027 | 1.005  | 1.0087 | 1.0609 |
| CC    | 0.9486 | 0.9484 | 0.9481 | 0.9459 | 0.9448 | 0.943  | 0.9165 |
| COV   | 0.3809 | 0.381  | 0.3813 | 0.3831 | 0.384  | 0.3854 | 0.4053 |
| SMPI  | 0.4639 | 0.4645 | 0.4651 | 0.4697 | 0.4706 | 0.4719 | 0.4903 |

**TABLE 3:** Metric value comparison for the heterogeneous patch despeckling for varying fractional order of the mask for a patch of size 10.

The computational efficiency of the method lies in the fact that here also the combined fractional integral mask of size 13x13 is used, but the convolution of the mask with only the selected patch is sufficient. Table 3 depicts that as the fractional order increases there is a depreciation seen on the despeckling performance. This shows that the despeckling can be effective with further low fractional orders. When the fractional orders are reduced instead of going with a 13x13 mask, a lower size combined mask for example 5x5 may be sufficient to give the same performance. This is due to the fact that lower fractional order directional mask in eight directions have less significant coefficients themselves and so is the combined mask. Visual comparisons are seen in Figure 4.



**FIGURE 4:** Visual comparison for the heterogeneous patch despeckling for varying fractional order of the mask.

Experimental results substantiate the effectiveness of using the new homogeneity index instead of the classical one. The performance is evaluated for varying patch sizes and the optimum patch size for despeckling is found to be image dependent and has a direct dependence on the homogeneity index for patch classification. This type of patch based denoising helps the sonar image interpreters in defining their areas of interest to be despeckled without doing the convolution operation for the entire image pixels. The ENL enhancement for the lower fractional order as seen in the Table3 encourages to use the lower fractional orders for despeckling and in turn helps in reducing the mask size from 13x13 to lower ones as the significant coefficients in the periphery of the masks becomes less significant. Order adaptive fractional order mask based denoising can be employed to have improved performance.



### 3. CONCLUSION

The popular Riemann-Liouville definition of fractional calculus is used for the construction of eight directional masks. The denoising algorithm was applied on different sonar images affected with speckle noise. The results of the experiments prove that, noise removal using Fractional integral mask is better than traditional despeckling method. According to the observations, mask extraction is possible for lower fractional orders and this helps in reducing the complexity of computation without affecting the de-noising performance adversely. The second proposed technique not only produces smoother images in homogenous areas but also preserve edges. The denoising filter is applied only on to the speckle affected class of sonar image patches. This makes the algorithm computationally effective. The computations can be further reduced by using the reduced mask size.

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