Comparative Study of Compressive Sensing Techniques For Image Enhancement

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Abstract

Compressive Sensing is a new way of sampling signals at a sub-Nyquist rate. For many signals, this revolutionary technology strongly relies on the sparsity of the signal and incoherency between sensing basis and representation basis. In this work, compressed sensing method is proposed to reduce the noise of the image signal. Noise reduction and image reconstruction are formulated in the theoretical framework of compressed sensing using Basis Pursuit de-noising (BPDN) and Compressive Sampling Matching Pursuit (CoSaMP) algorithm when random measurement matrix is utilized to acquire the data. Ultimately, it is demonstrated that the proposed methods can't perfectly recover the image signal. Therefore, we have used a complementary approach for enhancing the performance of CS recovery with non-sparse signals. In this work, we have used a new designed CS recovery framework, called De-noising-based Approximate Message Passing (D-AMP). This method uses a de-noising algorithm to recover signals from compressive measurements. For de-noising purpose the Non-Local Means (NLM), Bayesian Least Squares Gaussian Scale Mixtures (BLS-GSM) and Block Matching 3D collaborative have been used. Also, in this work, we have evaluated the performance of our proposed image enhancement methods using the quality measure peak signal-to-noise ratio (PSNR).

Keywords: Compressive Sensing, Basis Pursuit (BP), Compressive Sampling Matching Pursuit (CoSaMP), Approximate Message Passing (D-AMP), Non-local Means (NLM), Bayesian Least Squares Gaussian Scale Mixtures (BLS-GSM), Block Matching 3D collaborative filter (BM3D).

1. INTRODUCTION

Compressed Sensing as a new rapidly growing research field promises to effectively recover a sparse signal at the rate of below Nyquist rate. The Shannon/Nyquist sampling theorem states that sampling a signal at a rate at least twice the highest frequency exist in the signal is known as the Nyquist rate that leads to a perfect signal reconstruction. For many signals including audio or images the Nyquist rate is very high which may cause acquiring a very large number of samples. Besides being time consuming it is also necessary to place a high requirement on the equipment to sample the signal. Compressive Sampling (also referred to as Compressed Sensing or CS) is a newly introduced method that can reduce the number of measurements required. Compressive Sensing is a technique that is able to perfectly reconstruct particular classes of signals if the original signal is sampled at a rate below the Nyquist rate. Generally, Compressive Sensing is based on sparse signals. In many applications the signal of interest is primarily sparse meaning that the signal has a sparse representation in some pre-determined basis in which most of the coefficients are zero. Unlike traditional measurement techniques, Compressive Sensing utilizes linear sampling operators which are a combination of sampling and compression that avoids excessive oversampling.

Compressive Sensing (CS)-based Noise Reduction (CSNR) technique has an elegant mathematical background and theoretical advantages in decreasing the sampling rate [1, 2]. This technique has been achieved extremely success for MRI and image processing [3]. However, on the other hand, CSNR technique feasibility of being applied to other domains requires more investigation.

In this research, Compressive Sensing (CS) will be applied to noisy image signals. Therefore, the goal of this particular work is to study and apply Compressive Sensing techniques for the enhancement of image signal. For this purpose, firstly we apply two CS based methods such as basis pursuit denoising (BPDN) and Compressive Sampling Matched Pursuit (CoSaMP) on an image signal which has contaminated by Additive White Gaussian Noise (AWGN). The results show that these methods are not able to reconstruct the image perfectly. Therefore, we have used another technique called Approximate Message Passing (AMP) to enhance the image signal.

This paper has been organized as follows. Section 2 includes a general review of Compressive Sensing (CS) technique. Section 3 describes the procedure of solving CS problem using BPDN and CoSaMP. The experimental results of BPDN and CoSaMP are shown in section 4. Section 5 presents Approximate Message Passing (AMP) approach. The comparison results between D-AMP, CoSaMP, BPDN and also Wiener filtering are provided in section 6. Finally, section 7 concludes the paper.

2. A REVIEW OF COMPRESSIVE SENSING TECHNIQUE

The area of Compressed Sensing was introduced by two ground breaking papers, namely by Donoho [4] and by Cand`es et. al [5]. Compressed Sensing (CS), also known under the terminology of Compressive Sampling or sparse recovery is a rapidly growing and novel process of reconstructing a signal that promises sampling a sparse signal from a far fewer numbers of measurements than dimension of the signal. Compressive Sensing developed from questions and problems raised about the efficiency of the conventional signal processing pipeline for compression, coding and recovery of signals, including audio, video, image, etc. It can be forecasted that natural signals or images can be reconstructed from what was considered as a highly incomplete measurements or information. In other words, Compressed Sensing is based on the idea that one can sufficiently and efficiently capture all the information in a sparse signal by sensing only part of the signal using a sampling domain that is incoherent to the signal representation domain. For this purpose, CS relies on two principles such as sparsity and incoherency.

2.1 Sparsity

A signal $x = [x[1], \dots, x[N]]$ is called sparse if most of its components are zero, and also a signal is referred as k-sparse meaning that exactly k samples have non-zero values. The support of the signal is defined as [6]

$$\Lambda(\mathrm{x}) = \{1 \le \mathrm{i} \le \mathrm{N} \mid \mathrm{x[i]} \neq 0\} \tag{1}$$

Practically signals are often compressible which means that the sequence of coefficients decays quickly. It means a large percent of small coefficients can be thrown away without much perceptual loss. If a signal is not sparse it can be sparsely presented in an appropriate transform domain. It is mathematically shown as

$$x = \sum_{i=1}^{N} s_i \, \psi_i \tag{2}$$

where s_i is the coefficient sequence of x. The above equation can also be expressed in the matrix form $x = \psi s$, where s is a $N * 1$ column vector, ψ is a $N * N$ matrix which is called basis matrix. In CS the best basis matrix is a matrix that leads to the best sparsest representation. Basis matrices that can be used in CS are Wavelet transforms, Curvelet transform, Discrete Cosine transform, Ridget transform and Fourier transforms [6]. Using sparse signals includes some advantages such as taking less time in calculations involving multiplying a vector by a matrix and also less storage space in a computer is required [7].

2.2 A mathematical introduction to Compressive Sensing

Generally Compressive Sensing (CS) problem is defined as follow:

$$\mathrm{y} = \phi \mathrm{x} \tag{3}$$

where x is the signal vector with size $N * 1$, y is the measurement vector with size $m * 1$ and ϕ is called the measurement matrix with size $m * N$ with $m \ll N$. Measurement matrix is fixed and it does not depend on the signal x. This is a great because if we get a perfect result from a measurement matrix ϕ, it assures us that we can apply this measurement matrix on any type of signals without worrying about the stability. Since $m \ll N$ the equation (3) is said to be an underdetermined system meaning that the number of unknowns is more than the number of equations. Therefore, it just can be solved based on a priory which states the signal x is sparse. As x has a sparse representation in another domain, the general form of Compressive Sensing problem becomes

$$y = \phi\psi s \rightarrow y = As \tag{4}$$

A is a $m * N$ matrix called dictionary and s can be obtained through some CS techniques such as greedy methods or convex optimization approach which will be fully elaborated in the next sections. The number of measurements can easily be obtained by having compression ratio (sampling rate) as follow:

$$CR = \frac{m}{N} \tag{5}$$

where N is the size of the signal.

To reconstruct signal x perfectly two matrix ϕ and ψ must be orthogonal and their mutual coherence should also be less. Mutual-coherence is denoted as $\mu(A)$ and is defined as the maximal inner product between the matrixes A columns which are assumed to be normalized. $\mu(A)$ is defined mathematically as follow [8]

$$\mu(\mathrm{A}) = \max_{1\leq \mathrm{i,j} \leq \mathrm{m}} |\langle \psi_{\mathrm{i}}^{\mathrm{T}} \phi_{\mathrm{j}} \rangle \tag{6}$$

The mutual coherence of such two orthogonal matrices satisfies $1/\sqrt{m} \leq \mu(A) \leq 1$.

2.3 Restricted Isometry Property (RIP)

For a perfect reconstruction, we need to choose a proper basis matrix. It is also necessary to select a good measurement matrix which can sample a signal properly without modifying the structure of the signal. For this purpose, Candes and Tao [9] proposed a condition for the sampling matrix ϕ which is called "Restricted Isometry Property "(RIP). This condition states that for all k-sparse vector x, a $m * N$ matrix ϕ has the k-restricted isometry property if

$$(1 - \delta_k)||x||_2^2 \leq \left||\phi x|\right|_2^2 \leq (1 + \delta_k)||x||_2^2 \tag{7}$$

when δ_k is less than 1, the inequality (7) implying all of the sub-matrices of ϕ with k columns are well-conditioned and close to an isometry. Also, if $\delta_k \ll 1$ then the sampling matrix ϕ has a large probability of reconstructing the $\left(\frac{k}{2}\right)$-sparse signal x. To check whether a measurement matrix follows RIP or not is computationally difficult. But fortunately, there are many types of measurement matrices, such as random matrices which have well-restricted isometry behavior and satisfy RIP condition with high probability. The mostly used measurement matrix is Gaussian measurement matrix which can reconstruct a signal with high probability if [10]

$$m \geq c.k\log(\frac{N}{k}) \tag{8}$$

k also can be obtained from the following inequality

$$k \leq c.\frac{m}{\log(N/m)} \tag{9}$$

where c is a positive constant.

3. COMPRESSIVE SENSING PROBLEM

There are so many algorithms to solve the CS problem but the ideal algorithm is the one that follows the following four concepts [9].

Stability: The algorithm should be able to reconstruct the signal that is contaminated by noise approximately accurately.

Fast: The algorithm should be fast in obtaining the results.

Uniform guarantees: If we could acquire linear measurements through a specific method, it is also possible to apply these linear measurements to all sparse signals.

Efficiency: The algorithm gives perfect results by requiring as few measurements as possible.

In order to solve the CS problem $y = As$, the best solution will be the sparsest vector that has the most zero coefficients. To do this, firstly, we consider the $l_0 - norm$ minimization that counts the number of non-zero entries. In this case, the reconstruction problem (4) becomes:

$$\min ||s||_0 \text{ , subject to } y = As \tag{10}$$

Unfortunately, this problem is NP-hard meaning that it is computationally intractable to solve. Another solution is treating the CS problem as $l_2 - norm$ minimization. This is also called the Euclidean length of a vector which is defined as

$$||s||_2 = \sqrt{(\sum_{i=1}^{N} |x_{i|}{}^2} \tag{11}$$

Therefore, we have:

$$\min ||s||_2^2 \text{ , subject to } y = As \tag{12}$$

This problem is solved by Regularization approach [8] and has a unique solution which is $s_{opt} = A^{+}y$, where A^{+} is pseudo inverse of matrix A and is defined by the formula $A^{+} = (A^{*}A)^{-1}A^{*}$ [9]. However, this solution is not sparse; therefore it can't be the desired solution.

Ultimately, we use the $l_1 - norm$ minimization which is defined as $||s||_1 = \sum_{i=1}^{N} |x_i|$, hence the CS problem is changed to

$$min||s||_1 \text{, subject to } \quad y = As \tag{13}$$

This problem can be solved by a convex optimization approach called basis pursuit [11]. Obviously, this problem has many solutions. Among these solutions the one which has the number of non-zeros fewer than $\frac{spark\,(A)}{2}$, is necessarily the sparsest one possible. Also, any other solution must be denser [8], where spark (A) is defined as the size of the smallest set of linearly dependent vectors of A. For any matrix $A \in R^{m*n}$, the following relationship holds:

$$spark\ (A) \geq 1 + \frac{1}{\mu(A)} \tag{14}$$

As mentioned previously in this paper, we aim to solve the CS problem by using basis pursuit (BP) algorithm and one of the greedy algorithms called Compressive Sampling Matching Pursuit (CoSaMP).

3.1 Basis Pursuit (BP) Algorithm

Chen et. al [11] proposed Basis Pursuit algorithm (BP) which recovers the signal by few number of measurements. Basis Pursuit algorithm is a major approach for solving the underdetermined linear equations by minimizing the l_1 norm of coefficients. In most cases the l_1 norm can exactly recover k –sparse signals and also approximate the compressible signal with high probability using only $m = O(k.log(N/k))$ i.i.d Gaussian measurements. Equation (13) is equivalent to the linear programming [9],

$$\min \sum_{j=1}^{2N} v_j, \text{ subject to } v \geq 0, y = (\phi, -\phi)v \tag{15}$$

where v is a positive real number of size $2N$. The signal x can be obtained from the solution of v^* of (15) via $x = (I, -I)v^*$. Equation (15) can be solved with other methods such as interior-point methods, projected gradient methods, and iterative thresholding [9].

3.1.1 Pursuit Denoising (BPDN) Algorithm

In compressive sensing, in the presence of additive noise, the CS problem turns to:

$$y = \phi x + e \tag{16}$$

where e is an Additive White Gaussian Noise (AWGN) and mathematically is defined as $e \approx N(0, \sigma^2)$. AWGN is a random signal with zero mean and variance σ^2. In order to solve this problem a constraint is added to aforementioned basis pursuit algorithm to design a new algorithm. This new algorithm is called Basis Pursuit De-Noising (BPDN) algorithm. Therefore, we have

$$\min ||s||_1, \text{ subject to } ||y - \phi\psi s||_2 \leq \epsilon \tag{17}$$

where ε is the noise level (power of noise) [12]. So, for any k-sparse signal x and corrupted measurements $y = \phi x + e$ with $||\varepsilon||_2 \leq \varepsilon$, the solution $\hat{x}$ to (17) satisfies [7].

$$\|\hat{x} - x\|_2 \leq C_s \, \varepsilon \tag{18}$$

where C_s depends only on RIP constant δ.

BP presents many advantages over other algorithms in Compressed Sensing. Once a measurement matrix satisfies the restricted isometry property, Basis Pursuit reconstructs all sparse signals. BP is also stable which is necessary for practice. Its ability to handle noise and non-exactness of sparse signals makes the algorithm applicable to real world's problem.

3.2 CoSaMP Algorithm

Needell and Tropp proposed a new algorithm in 2009 based on Orthogonal Matching Pursuit (OMP) which is called Compressive Sampling Matching Pursuit (CoSaMP) [13]. The main goal of CoSaMP as in the case of OMP is identifying the k-largest components in signal x. CoSaMP introduces a signal called the proxy of signal x which is defined as

$$u = \phi^* \phi x \tag{19}$$

where ϕ^* is the conjugate transpose of matrix ϕ.

It has been proven that according to the restricted isometry property, by giving a sampling matrix ϕ with the restricted isometry constant $\delta_k \ll 1$, the $l_2 - norm$ of the k largest entries of vector u is close to the $l_2 - norm$ of the k-largest signal x. That's why the vector u in (19) is considered as the proxy of signal x.

In CoSaMP at each iteration the algorithm first selects the largest $2k$ components of the signal proxy u and then adds the index of these components to the support set. Next by using the least squares method, we can get signal estimation b. Therefore, the sparse signal x can be obtained by keeping only the largest k components of the estimation b to make it sparse, this is called pruning. In the presence of additive noise, CoSaMP produces a $2k$-sparse signal approximation $\hat{x}$ that satisfies [13].

$$\|\hat{x} - x\|_2 \leq C(\|e\|_2 + \frac{\|x - x_k\|_1}{\sqrt{k}}) \tag{20}$$

where x_k is the best k-sparse approximation of x, ϕ is a $M * N$ sampling matrix with restricted isometry constant $\delta_{2k} \ll c$. Also, C refers to positive constants. Experimental results illustrate that the performance of signal recovery by CoSaMP reduces easily if we add noise in samples. To the best of our knowledge, no iteration bound exists for CoSaMP because it is independent of signal structure [14].

4. EXPERIMENTAL RESULTS

In this research, we have applied Basis Pursuit and CoSaMP on two different test images such as “Lena” and “Brain MRI”. For the experimental results in this paper I have used “Lena” because it has a sparse representation in Discrete Cosine Transform (DCT). In this research, the reconstruction error is defined as [12]:

$$Reconstruction\_error = \frac{\|\hat{x} - x\|_2}{||x||_2} \tag{21}$$

where $\hat{x}$ is the reconstructed image and x is the original image.

Read Image
Read image with size $a * b$
Adding Noise to image
Adding White Gaussian noise which is a zero mean signal with variance sigma.
Compressed measurements
$N = a * b$, initializing a value for compression ratio (CR) so that m=CR*N
Defining measurement matrix
Measurement matrix is a Gaussian random matrix with zero mean and variance $1/m$.
Creating compressive measurements
Converting image to vector
$Y = \phi x$, (x is the vector of image signal)
Choosing the proper representation basis ψ
Discrete Cosine Transform (DCT) for "Lena"
Reconstruction algorithm
$Y = \phi\psi s = As$
In order to find sparse coefficients vector (s) we use one of the following algorithms:
CoSaMP : using CoSaMP.m
BP or BPDN: using spgl1 toolbox
Convert vector S to image
Reconstructed image= ψs
Plotting results

TABLE 1: The procedure of implementing compressive sensing in MATLAB.

Table 1 illustrates the procedure that is used for implementing compressive sensing for image reconstruction. As it can be noticed this procedure has been used in MATLAB for image reconstruction using either BP or CoSaMP. In this process, an image with size $N = a * b$ is loaded and then an additive white Gaussian noise (AWGN) with a predetermined variance (sigma) is added to the image to make a noisy image. It should be noted that the value of compression ratio (CR) must be already defined. Because the number of measurements (m) can be easily determined if the value of CR and N are known. In order to apply the Compressive Sensing a measurement matrix and a proper basis matrix are required. For this purpose, we have used the Gaussian random matrix and Discrete Cosine Transform (DCT) as the measurement matrix and basis matrix respectively. Then, the algorithms BPDN and CoSaMP are conducted on the noisy image separately to obtain the de-noised image. The following images illustrate the results of applying CoSaMP and BPDN on "Lena" of size $128 * 128$ which has been contaminated by AWGN with variance 10.

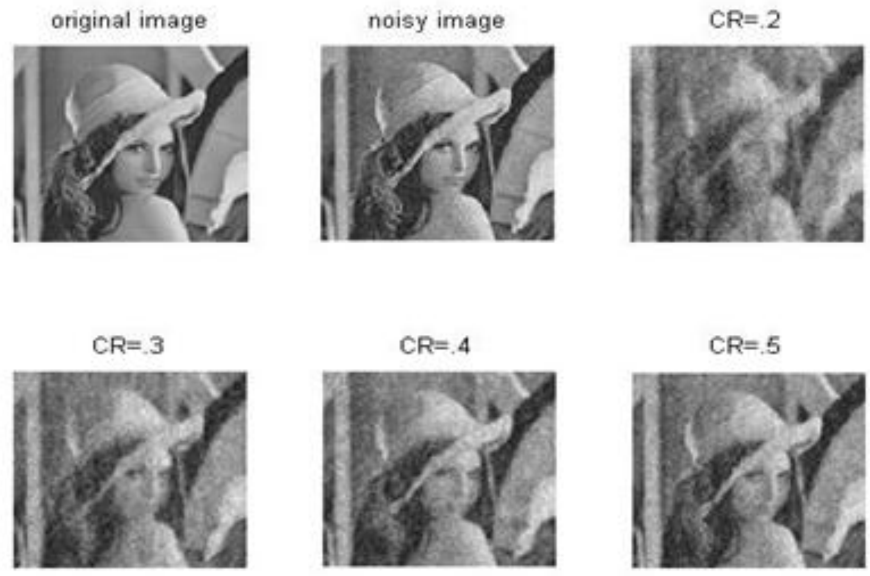

FIGURE 1: The results of de-noising “Lena” by BPDN algorithm using different compression ratios.

Figure 1 illustrates the results of applying BPDN on noisy “Lena”. In this figure, the original image was contaminated by AWGN with variance 10 at different compression ratios. As it is shown the best result has been obtained at $CR = 0.5$. The numerical results of applying BPDN on noisy “Lena” at different compression ratios have been shown in Table 2. This table illustrates the reconstruction error and reconstruction PSNR after applying BPDN algorithm on noisy “Lena” at different compression ratios. As it is shown the best result appears at $CR = 0.5$. The result shows that as compression ratio increases the reconstruction error decreases but the PSNR of de-noised increases.

Compression Ratio	**0.2**	**0.3**	**0.4**	**0.5**
Reconstruction error	0.0494	0.0370	0.0299	**0.0233**
PSNR_noisy (dB)	28.1367	28.1263	28.1278	28.0916
PSNR_de-noised (dB)	20.4468	22.2018	23.3559	**24.3219**

TABLE 2: Numerical results of applying BPDN on noisy “Lena” at different compression ratios.

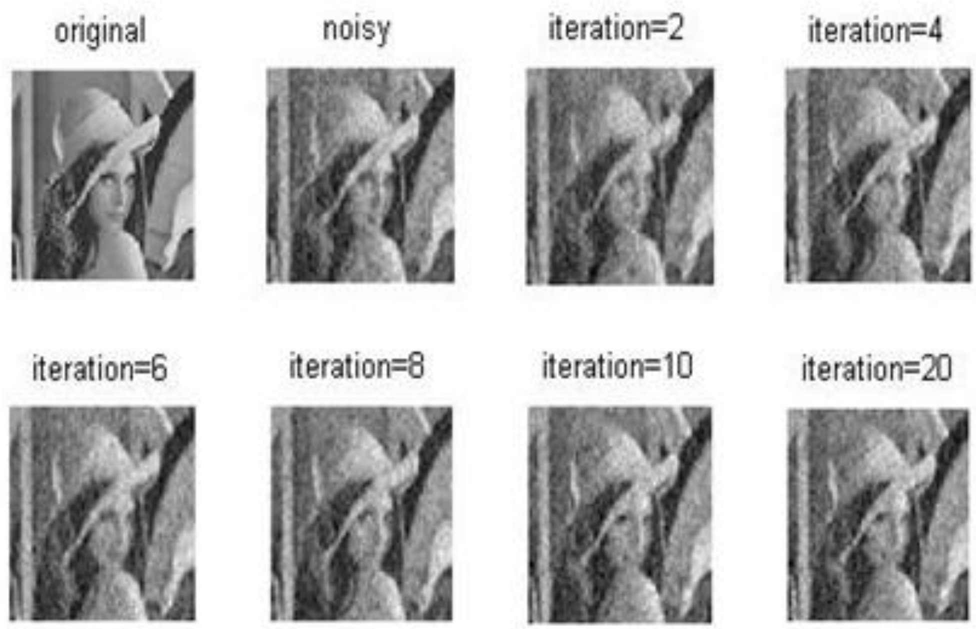

FIGURE 2: The results of de-noising “Lena” by the CoSaMP algorithm using different iterations and $CR = 0.5$.

In Figure 2 shows the results of applying CoSaMP on noisy “Lena” using different iterations. The compression ratio of applying CoSaMP has been set to 0.5. It can be seen from Figure 2 that the result obtained at iteration 10 has better quality compare to other iterations. Also, Table 3 shows the numerical results of applying CoSaMP on noisy “Lena” using different iterations. According to the numerical results, the higher de-noised PSNR has been obtained at iteration 10. It should also be noted that at iterations higher than 10 the quality of the image will be decayed, so the best performance will be occurred at iteration 10. The results also show that the reconstruction error is quite low at iteration 10.

Number of iterations	**2**	**4**	**6**	**8**	**10**	**20**
PSNR_ Noisy (dB)	28.079	28.079	28.079	28.079	28.079	28.079
PSNR_de-noised (dB)	22.93	22.677	22.533	22.757	**22.539**	22.382
Reconstruction error	0.0370	0.0324	0.0325	0.0317	**0.0326**	0.0329

TABLE 3: Numerical results of applying CoSaMP on noisy “Lena” using different iterations.

Figure 3 demonstrates the results of de-noising “Lena” by applying CoSaMP at iteration 10 using different compression ratio. It is clear from Figure 3 that the best result is achieved at compression ratio 0.5. It should also be noted that the CoSaMP can’t perform well at compression ratio less than 0.3. The numerical results of applying CoSaMP on noisy “Lena” using different compression ratios at iteration 10 has been shown in Table 4. It is clear from the result that as compression ratio increases the PSNR of de-noised increases as well but the reconstruction error decreases. Therefore, according to the numerical results of Table 4 it is clear that the algorithm performs better at compression ratio 0.5 comparatively. In Figure 4 and Figure

5, the elapsed running time and performance analysis of BPDN and CoSaMP algorithms on the noisy "Lena" have been compared. The result of Figure 4 and Figure 5 illustrate that as the performance analysis of the algorithms is relevant to the value of compression ratio, it is clear that each approach performs better at higher compression ratios but comparatively the BPDN approach is less time-consuming and also more efficient that the CoSaMP algorithm in the case of image de-noising.

Compression Ratio	0.2	0.3	0.4	0.5
PSNR-noisy (dB)	-	28.15	27.99	28.18
PSNR-de-noised (dB)	-	18.57	21.32	**22.66**
Reconstruction error	-	0.0553	0.0370	**0.0325**

TABLE 4: Numerical results of applying CoSaMP on noisy "Lena" using different compression ratios and at iteration 10.

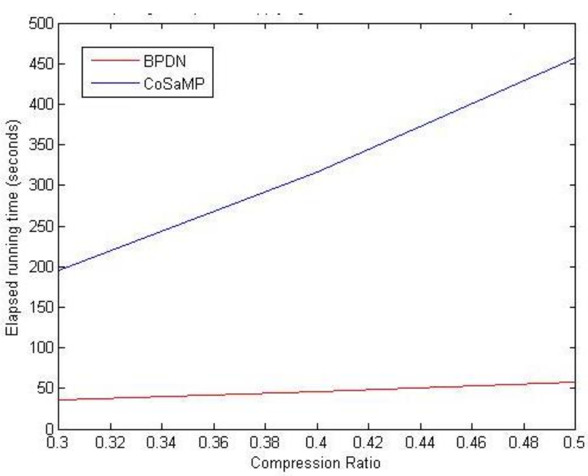

FIGURE 4: illustrates the comparison result of elapsed running time of applying BPDN and CoSaMP on noisy "Lena".

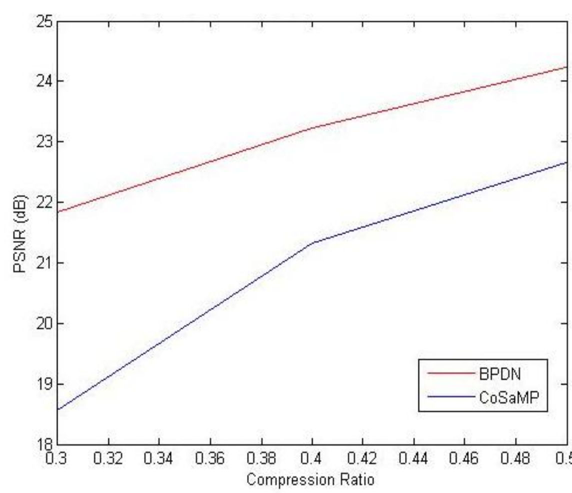

FIGURE 5: illustrates the comparison result of performance analysis of applying BPDN and CoSaMP on noisy "Lena".

From the experimental results it was noted that the Basis Pursuit and Compressive Sampling Matching Pursuit are not able to count the number of non-zero elements and their locations accurately, therefore, these approaches may be failed in exact signal reconstruction. However, there is a new approach called Greedy Matching Pursuit (GMP) [15], which is able to determine the exact sparsity level. In this work, we have used a complimentary approach that is called Approximate Message Passing (AMP) to enhance the noisy image.

5. APPROXIMATE MESSAGE PASSING (AMP) APPROACH

In this section, we intend to present how to employ a generic denoiser in a CS reconstruction algorithm. For this purpose, we need to use a de-noising-based approximate message passing (D-AMP) algorithm which is capable of high-performance reconstruction. We will show that for an appropriate choice of denoiser, D-AMP offers the best CS recovery performance for natural images. There are many image de-noising techniques, but in this research, we consider Non-

Local Means (NLM) [16], Bayesian Least Squares Gaussian Scale Mixtures (BLS-GSM) and Block Match 3D (BM3D) [17] as a denoiser in D-AMP approach. In this section, we will review the Iterative thresholding (IT) algorithm and de-noising-based approximate message passing (D-AMP). We also show the results of implementing D-AMP by using aforementioned denoisers. Finally, the comparison results between this method and CoSaMP, BPDN will be presented.

5.1 A Review of Iterative Thresholding and AMP Algorithms

Thresholding is a simple technique for image reconstruction. When the signal is represented in terms of a suitable basis (for instance a wavelet basis or DCT), then small coefficients are set to zero and larger coefficients above some threshold are possibly shrunk. Thus, thresholding (or shrinkage) usually produces signals that are sparse. So, it works particular well if the original clean signal can be well-approximated by a sparse one. The soft and hard thresholding operators have been widely described in [18].

Given the noisy observation signal as $y = Ax_0 + w$ where x_0 and y are sparse signal and measurement vector respectively, and A, w are considered as measurement matrix and additive noise respectively. For recovering x_0 Iterative thresholding acts as follow:

$$x^{t+1} = \eta_\tau(A^* z^t + x^t)$$

$$z^t = y - Ax^t \tag{22}$$

where x^t is the estimation of x_0 and z^t is an estimation of residual signal $y - Ax_0$ at iteration t. η_τ is a shrinkage or thresholding non-linearity that expresses the soft or hard iterative thresholding. So, when $\eta_\tau(y) = (|y| - \tau) + sign(y)$, the algorithm is known as iterative soft-thresholding (IST). AMP is generally like IT. It just extends IST by adding an extra term to the residual known as Onsager correction term as follow:

$$x^{t+1} = \eta_\tau(A^* z^t + x^t)$$

$$z^t = y - Ax^t + \frac{1}{\delta} z^{t-1} < \eta'_\tau(A^* z^{t-1} + x^{t-1}) > \tag{23}$$

where δ is the compression ratio, η'_τ that represents the derivative of η_τ, is the Onsager correction term. $\langle \quad \rangle$ denotes the average of a vector. Figure 6 compares the quantile-quantile (Q-Q) plot of effective noise which defines as $A^* z^t + x^t - x_0$ for IST and AMP. As it is shown in Figure 6, the Q-Q plot of effective noise in AMP is a straight line which means the noise is approximately Gaussian. Therefore, modeling a Gaussian distribution for noise in all iteration brings some advantages [19] such as the accurate analysis of algorithm, the optimal tuning of the parameters and also leads to the linear convergence of x^t. The most important advantage of this feature is that we can add all denoisers which have been developed for additive Gaussian noise to the AMP. Therefore, this leads to more accurate recovery compared to the usual techniques utilized in CS recovery. As it is mentioned the innovation of the AMP method is that the effective noise has a Gaussian distribution in each iteration. That's because of the Onsager correction term. In order to depict this effect an amount of noise in some iteration per standard deviation of the normal distribution is plotted as Q-Q plot. If the input data, follow a normal distribution then the Q-Q plot will be a straight line as it is clear in Figure 6. Inspired by this philosophy we used a new designed CS recovery framework, called de-noising-based approximate message passing D-AMP that utilizes a de-noising algorithm to recover signals from compressive measurements. Compared to existing CS algorithms D-AMP has several advantages as [19]: (1) it can be applied to various signal classes. (2) It performs better than existing algorithms and is extremely robust for measuring noise. The D-AMP algorithm is defined as follow [20]:

$$\mathrm{x}^{t+1} = \mathrm{D}_{\delta\sigma^t}(\mathrm{A}^* \mathrm{z}^t + \mathrm{x}^t)$$

$$\mathrm{z}^t = \mathrm{y} - \mathrm{A}\mathrm{x}^t + \frac{\mathrm{z}^{t-1} \mathrm{D}'_{\sigma^{t-1}}(\mathrm{A}^* \mathrm{z}^{t-1} + \mathrm{x}^{t-1})}{\mathrm{m}}$$

$$(\sigma^t)^2 = \frac{||z^t||_2^2}{m} \tag{24}$$

where x^t is the estimation of x_0 and z^t is an estimation of the residual at iteration t. According to [20], $A^*z^t + x^t$ can be written as $x_0 + v^t$ where v^t can be considered as i.i.d Gaussian noise. σ^t is an estimation of the standard deviation of noise. The term $\frac{z^{t-1} D'_{\sigma^{t-1}}(A^*z^{t-1} + x^{t-1})}{m}$ is the Onsager correction, and $D'_{\sigma^{t-1}}$ defines the divergence of the denoiser. D-AMP applies an existing de-noising algorithm on vectors that are obtained from compressive measurements.

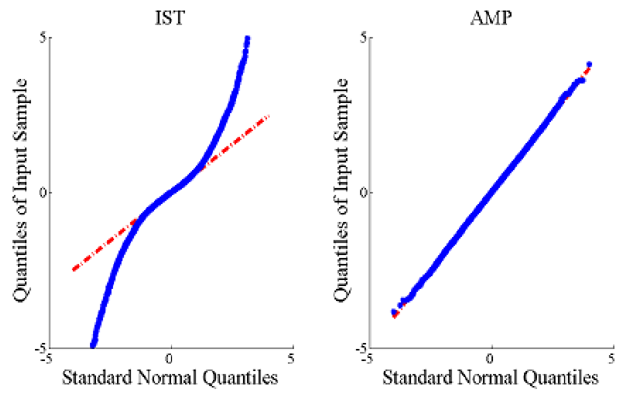

FIGURE 6: Comparing the distribution of effective noise for IST and AMP [20].

6. COMPARISON RESULTS

Figure 7 demonstrates the results of applying D-AMP and CS based algorithms on noisy “Lena” at compression ratio 0.4. As it is cleared from Figure 7, D-AMP based approaches show an outstanding performance even at low Compression Ratios compared to other methods. Table 5 also shows the comparison results of D-AMP method with CoSaMP and BPDN in different Compression Ratios. It is observed from Table 5 that as compression ratio increase the PSNR of de-noised image increases too for all five methods. The numerical results of Table 5 also show that BM3D has a better performance and leads to a perfect image reconstruction compared to other D-AMP algorithms.

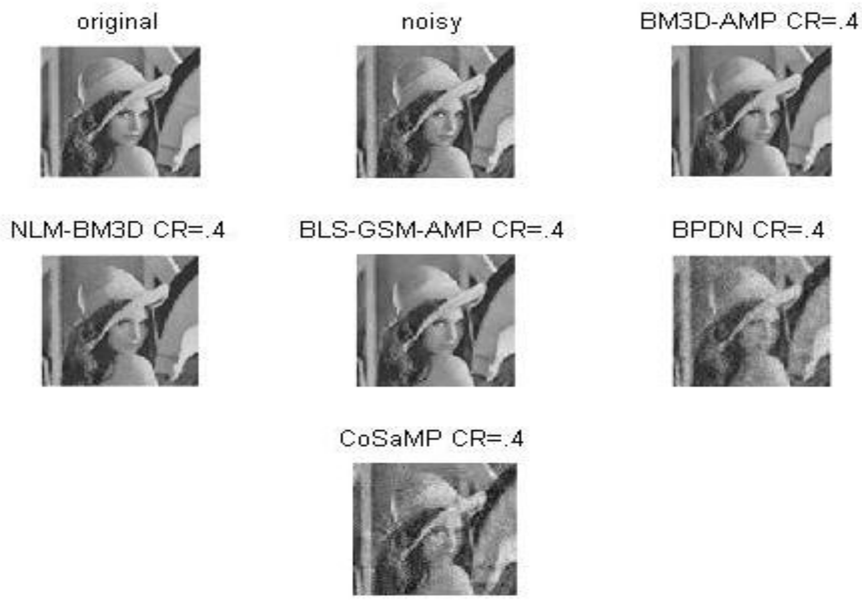

FIGURE 7: Comparison results of D-AMP method, CoSaMP and BPDN algorithms for noisy “Lena “at Compression Ratio 0.4.

Lena	$Cr = 0.2$	$Cr = 0.3$	$Cr = 0.4$	$Cr = 0.5$
PSNR- noisy (dB)	28.12	28.12	28.12	28.12
PSNR- denoised-BM3D (dB)	**27.13**	**28.93**	**29.54**	**29.80**
PSNR- de-noised BLS-GSM (dB)	26.40	28.02	28.74	29.05
PSNR_de-noised BPDN (dB)	20.44	22.20	23.35	24.2319
PSNR-denoised CoSaMP (dB)	-	19.2	22.18	23.8
PSNR- de-noised NLM (dB)	23.88	27.06	28.45	28.50

TABLE 5: The comparison results of D-AMP method with CoSaMP and BPDN in different Compression Ratios.

	Wiener Filtering	**AMP-NLM**	**AMP-BLS-GSM**	**AMP-BM3D**
PSNR-noisy (dB)	28.12	28.12	28.12	28.12
PSNR-denoised (dB)	24.54	28.50	29.05	**29.80**

TABLE 6: The comparison results of D-AMP method at CR=0.5 with wiener filtering on noisy “Lena”.

Table 6 also shows the comparison results of applying Wiener filtering and D-AMP-based approach on noisy “Lena” which was contaminated by AWGN with standard deviation 10. It is clear from Table 6 that the PSNR of de-noised image using D-AMP approaches are higher compared to Wiener filtering [21]. Therefore, the proposed AMP-based approaches perform far more efficient comparatively [22].

7. CONCLUSION

Compressive Sensing (CS) is a novel nonlinear sampling method that compared to Shannon-Nyquist theorem accelerates the acquisition rate without decreasing reconstructed signal quality and improves the image quality without increasing the quantity of required data. Compressive Sampling technique can find a solution for an underdetermined linear algebra equation based on the assumption that the desired solution is sparse.

In this work compressed sensing was applied to clean image signal using BP and CoSaMP algorithms separately. It can be concluded from experimental results of clean and noisy image signals the reconstruction based on both algorithms is roughly equivalent. Moreover, we have investigated a novel noise reduction technique that can be referred to as Compressive Sensing (CS)-based Noise Reduction (CSNR) technique. It is cleared from the results that these methods do not perform well for image de-noising and the recovered image doesn't seem perfect. Therefore, we used the other de-noisers such as BM3D, NLM, and BLS-GSM along with Compressive Sampling method by using a newly designed framework named D-AMP algorithm for image enhancement. Through computer simulations, we have demonstrated that the proposed method is quite effective in image enhancement at compression factor 0.5 or even less. Also, compared to the traditional CSNR techniques the proposed method is faster and utilizes less memory space. We also showed that D-AMP also outperforms Wiener filtering in image de-noising. For further study we will use the Wiener filter in D-AMP approach for image de-noising like other de-noisers that is used in D-AMP approach.

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