An Efficient Thresholding Neural Network Technique for High Noise Densities Environments

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Abstract

Medical images when infected with high noise densities lose usefulness for diagnosis and early detection purposes. Thresholding neural networks (TNN) with a new class of smooth nonlinear function have been widely used to improve the efficiency of the denoising procedure. This paper introduces better solution for medical images in noisy environments which serves in early detection of breast cancer tumor. The proposed algorithm is based on two consecutive phases. Image denoising, where an adaptive learning TNN with remarkable time improvement and good image quality is introduced. A semi-automatic segmentation to extract suspicious regions or regions of interest (ROIs) is presented as an evaluation for the proposed technique. A set of data is then applied to show algorithm superior image quality and complexity reduction especially in high noisy environments.

Keywords: Thresholding Neural Networks, Image Denoising, High Noise Environments, Wavelet Shrinkage.

1. INTRODUCTION

According to American Cancer Society, breast cancer is the second most common form of cancer in women. The chance of dying with breast-cancer is one in 33 however that number is decreasing as new forms of treatment and early detection are being implemented. Magnetic Resonance Imaging (MRI) is the state-of-the-art medical imaging technology which allows cross sectional view of the body with unprecedented tissue contrast. MRI provides a digital representation of tissue characteristic that can be obtained in any tissue plane. The images produced by an MRI scanner are best described as slices through the breast [1]. There has been considerable effort aimed at developing computer-aided diagnosis (CAD) systems that might provide a consistent and reproducible second opinion to a radiologist. Currently, most CAD systems are designed to prompt suspicious regions [2]. Early detection of breast cancer starts with qualitative analysis of medical imaging data by radiologists. But Diagnosing using MRI is a time consuming task even for highly skilled radiologists because MRI are noisy images. Thus, it is crucial to apply further work in denoising areas. This paper uses a hybrid denoising-semi automatic segmentation algorithm to solve this problem.

2. IMAGE DENOISING

Denoising means suppress or remove noise from image data while preserve the image quality. Imperfect instruments, problems with acquisition process, and transmission media can all

degrade the data of interest. High noise density images might lose information if not applied to a robust denoising preprocessing toolkit before analysis either manually or via CAD systems.

2.1. Statistical Filters

Early methods introduced for image denoising were based on noise statistical in spatial domain [3]. Among are the histogram modification, mean filters, Gaussian filters, unsharp masking, median filters, and morphological filters [4].

2.2. Denoising Problem and Objective

The general denoising problem can be formulated as follows.

$$y = x + n \tag{1}$$

Where

y: is a noisy image in wavelet domain, which can be represented as finite data sample:

$$y = [y_0, y_1,, y_{N-1}]^T$$
 (2)

N: is the number of wavelet coefficients;

x: is the noise free image in wavelet domain can be represented as finite data sample:

$$\mathbf{x} = [\mathbf{x}_0, \mathbf{x}_1, \dots, \mathbf{x}_{N-1}]^{\mathrm{T}}$$
 (3)

n: is a noise data vector. Gaussian white noise with distribution **N** $(0, \sigma^2)$.

And the objective of noise reduction is to reduce the noise in y and make the denoised image x as close to the original image x as possible. The measurement of the closeness is the mean square error (MSE) risk which is calculated:

$$J_{MSE} = \frac{1}{2} E \left\| \hat{x} - x \right\|^2 = \frac{1}{2N} \sum_{i=0}^{N-1} (\hat{x}_i - x_i)^2$$
 (4)

Where $\hat{x} = [\hat{x}_0, \hat{x}_1, \dots, \hat{x}_{N-1}]^T$ is denoising image or the output of thresholding function in wavelet domain.

2.3. Wavelet Transform and Wavelet Shrinkage

Wavelet transform (WT) has become an important tool to suppress noise [5-9]. In WT decomposition, coefficients could be categorized as those that contain the main energy of the signal and others that can be ignored [10]. Moreover, noise is spread among all the coefficients in the wavelet domain, and the WT of a noisy signal could be considered as a linear combination of the WT of the signal and noise. Research in this area aims to suppress noise significantly while preserving the original data. Noise reduction methods using WT, or wavelet shrinkage, can be summarized as follows:

First, a noisy image is decomposed in the wavelet domain. Selecting a proper threshold value to filter the coefficients, according to thresholding function, is to be applied. Finally, using the inverse wavelet transform (IWT), the reconstructed image is obtained.

2.4. Thresholding Function

Donoho and Johnstone [11-14] had proposed a denoising method based on thresholding in the wavelet domain. The wavelet coefficients of a noisy image usually divided into important coefficients "keeping (shrinking)" and non-important coefficients "killing". These groups are modified according to certain rules. There are two thresholding rules, namely soft and hard thresholding.

2.4.1 Soft Thresholding Function

If an amplitude is smaller than a predefined threshold, it will be set to zero (kill); otherwise it will shrunk in the absolute value by an amount of the threshold, i.e.

$$\eta_{S}(y,t) \stackrel{\Delta}{=} sgn(y)(|y|-t)_{+} = \begin{cases} y+t, & y<-t \\ 0, & |y| \le t \\ y-t, & y>t \end{cases}$$
 (5)

Where $t \ge 0$ is the threshold value.

2.4.2 Hard Thresholding Function

Same as soft thresholding, if its amplitude is smaller than a predefined threshold, it will be set to zero (kill); otherwise it will be kept unchanged

$$\eta_{h}(y,t) \stackrel{\Delta}{=} \begin{cases} y, & |y| > t \\ 0, & |y| \le t \end{cases}$$
(6)

Soft and hard functions are shown in figure (1).

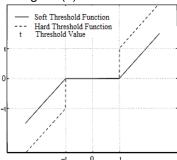


FIGURE (1): Stander soft and hard thresholding functions

2.5. Threshold Value

The key decision in the denoising using thresholding technique is to select an appropriate threshold. If this value is too small, the recovered image will remain noisy. On the other hand, if the value is too large, important image details will be smoothed out. Threshold values are divided into three main groups. The first group is universal-threshold in which the threshold value is the same for all wavelet detail subbands of the noisy image. The main method of this group (Visu Shrink) is presented with the hard and soft thresholding function in [11-14]. The second group is SureShrink, where the threshold value is selected differently for each detail subband [16 and 17]. In the third group each detail wavelet coefficient has its own threshold value [18].

2.5.1 Universal Thresholding

Donoho proposed the universal threshold [15], given by

$$t_{universal} = \sigma \sqrt{2\log(M)}$$
 (7)

Where: M: is the sample size.

 σ : is the noise standard deviation of noisy image. And if it is not known, a robust median estimator is used from the finest scale wavelet coefficients [12]:

$$\overset{\Lambda}{\sigma} = \frac{\text{median } (\mid y_{ij} \mid)}{0.6745} \qquad , y_{ij} \in \text{ subband } \quad \text{HH }_{1} \qquad (8)$$

2.6. Thresholding Neural Network

Artificial neural networks (ANN) are composed of simple elements operating in parallel. One can train a neural network to perform a particular function by activation function and adjusting the values of the connections (weights) between internal elements. The network is adjusted or trained, based on a comparison of the output and the target essentially to minimizing the mean square error between the output and the target, until the network output matches the target. Iteration through the process of providing the network with an input and updating the network's weights is an epoch. Typically many epochs (iteration) are required to train the neural network. Zhang [19, 20] constructed a **new type of neural network** known as **thresholding neural network** (TNN) to seek the optimal threshold value and to perform the thresholding in wavelet

domain for better image denoising. A TNN is different from the conventional ANN. In TNNs, a fixed linear transform is used and the nonlinear activation function is adaptive, while in conventional multilayer neural networks the activation function is fixed and the weights of the linear connection of input are adaptive [21]. However, a TNN has some basic elements similar to ANN, i.e., interconnection of input, nonlinear activation functions, epochs, learning (threshold value), training and finding optimal MSE etc. Most ANN learning algorithms employ gradients and higher derivatives of the activation function. High-order differentiable activation functions make a neural network have better numerical properties. However, the standard soft-thresholding function is only differentiated and does not have high order derivatives. The standard hard-thresholding function is a discontinuous function and cannot be differentiated at all.

We will present new types of smooth soft thresholding and hard-thresholding functions which are infinitely differentiable. They make many gradient-based learning algorithms feasible. The input of the TNN is noisy image samples in wavelet domain y=x+n, where x is the noise free image and n is additive noise and the output of the TNN is the denoised image \hat{x} .

2.7. High-Order Differentiable Thresholding Functions

2.7.1 Zhang Thresholding Function

Two functions were proposed to overcome the discontinuously derivative of soft thresholding function. The first function [22] is a type of soft-thresholding function which has second order weak derivatives and proved to be useful, where k is a positive integer. Note that the limit of $\eta_k(y,t)$ when $k{\to}\infty$ is just the commonly used soft-thresholding function

$$\eta_{Zhang (1998)}(y,t,k) = \begin{cases}
y+t-\frac{t}{2k+1} & y < -t \\
\frac{1}{(2k+1)t^{2k}}y^{2k+1} & |y| \le t \\
y-t+\frac{t}{2k+1} & y > t
\end{cases}$$
(9)

The second [19] is smooth soft-thresholding function which is infinitely differentiable. Where λ is positive value when λ =0, $\eta_k(x,t)$ is just the standard soft thresholding function

$$\eta_{\text{Zhang}(2001)}(y,t,\lambda) = y + 0.5(\sqrt{(y-t)^2 + \lambda} - \sqrt{(y+t)^2 + \lambda})$$
 (10)

It can be seen that the Zhang shrinkage functions perform the similar operations to the standard soft-thresholding function, as shown in figure (2). Therefore, the similar smoothness property of the estimate using this thresholding functions can be expected.

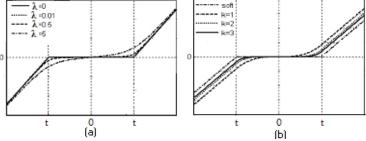


FIGURE (2): Zhang thresholding function with different values of shape tuning factor (a) Zhang2001 (b) Zhang1998.

2.7.2 Nasri and Nezamabadi Thresholding Function

In [24], a new nonlinear thresholding function with a high capability has been presented by Nasri and Nezamabadi based on adding shape tuning factors. The differentiability property is valid through these functions and has high-order derivatives. The main difference between this function and other thresholding functions is in the non-important coefficients. Classic functions set the coefficients below the threshold value to zero, but in this proposed method these coefficients are tuned by a polynomial function.

$$\eta(y,t,m,k) = \begin{cases} y + (k-1)t - 0.5 \frac{k * t^{m}}{y^{m-1}} & y > t \\ 0.5 \frac{k * |y|^{m + \lfloor (2-2k)/k \rfloor}}{t^{m + \lfloor (2-2k)/k \rfloor}} sign(y) & |y| \le t \\ y - (k-1)t - 0.5 \frac{k * (-t)^{m}}{y^{m-1}} & y < -t \end{cases}$$
(11)*

Where: m, k are shape tuning factors.

Parameter **m** determines the shape of the function for coefficients that are less and bigger than absolute threshold value. By tuning the parameter k, the thresholding function can be somewhere between hard and soft functions. In other words, for k=1, the function tends to hard thresholding function and when $k\rightarrow 0$ it tends to soft thresholding. Figures (3and 4) show the function for several values of the tuning parameters.

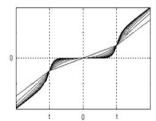


FIGURE (3): Nasri and Nezamabadi thresholding function for K=1 and several values of \mathbf{m} in the range [2, 10].

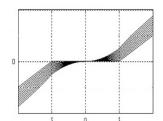


FIGURE (4): The Nasri and Nezamabadi class of thresholding function for m=2 and k=[0,1]. Note that for $k\rightarrow 0$ the function tends to soft thresholding function.

2.8. TNN Learning Algorithms

One of the most practical and efficient methods in neural networks learning is least mean squares (LMS) algorithm [23]. Using LMS algorithm and thresholding function presented in [24], in each step the threshold value is adjusted along with gradient descent of the MSE risk. The equations of the algorism as follows:

$$t_{p}(j+1) = t_{p}(j) + \Delta t_{p}(j)$$
 (12) $x_{i} = \eta(x_{i})|_{t=t}$ (15)

$$\Delta t_{p}(\mathbf{j}) = -\alpha \frac{\partial J_{MSE}}{\partial t} \Big|_{t=t_{p(\mathbf{j})}}$$

$$= -\alpha \sum_{i=0}^{M-1} \varepsilon_{i} * \frac{\partial \dot{x}}{\partial t} \Big|_{t=t_{p(\mathbf{j})}}$$

$$(13) \qquad \frac{\partial \dot{x}}{\partial t} \Big|_{t=t_{p(\mathbf{j})}}$$

$$= \frac{\partial \eta(x_{i})}{\partial t} \Big|_{t=t_{p(\mathbf{j})}}$$

$$\varepsilon_{i} = x_{i} - x_{i}$$

$$(17)$$

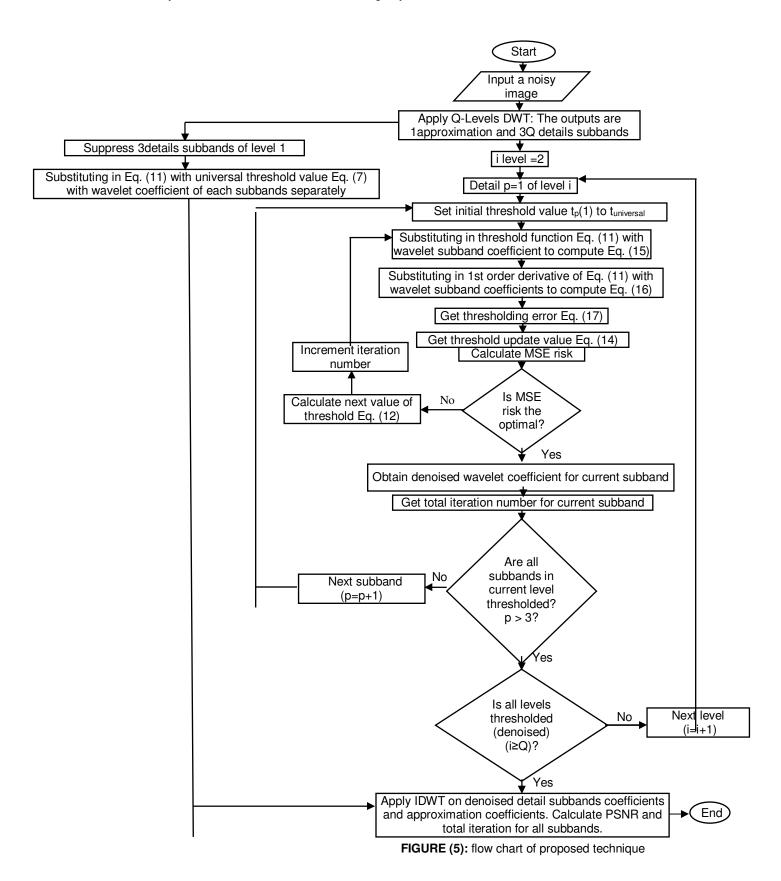
$$= -\alpha \sum_{i=0}^{M-1} \varepsilon_{i} * \frac{\partial x}{\partial t}$$

$$\varepsilon_{i} = x_{i} - x_{i}$$

$$(14)$$

Where α is Learning rate, j is Learning step count M is Length of subband p, and $\epsilon_{:}$ is Thresholding error or the difference between denoising and original wavelet coefficients.

^{*} We have made a limited correction in this equation



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2.9. The Proposed TNN With Modified Thresholding Concept

After studying and implementing remarkable existing TNN algorithms, it could be observed that the threshold in the highest frequency subbands when obtained by TNN does not differ from universal threshold equation (7). The work presented in this paper assumes universal threshold value in the wavelet decomposition level (1). Time saving and iterations reduction could both be obtained as consequences of this proposed idea. Figure (5) shows the flow chart of image denoising by proposed technique. The use of peak signal to noise ratio (PSNR) is very common in image processing, probably because it gives better-sounding numbers than other measures. The PSNR is given in decibel units (dB)

PSNR =
$$20 \text{Log}_{10} \left[\frac{255}{\sqrt{\frac{1}{N} \sum_{i=0}^{N-1} (x - x)^2}} \right]$$
 (18)

3. SEGMENTATION ALGORITHM

Segmentation is to distinguish important regions of interest (ROI) from the background after the denoising phase. The main idea for applying the segmentation technique is to measure the performance of the proposed denoising algorithm at different noise densities. Segmentation techniques can be classified into two main categories: edge-based segmentation (locating object's boundary using image gradient) and region-based segmentation (identifying all pixels that belong to the object based on the intensity of pixels) techniques. Edge- based techniques such as Roberts, Prewitt, Robinson, Kirsch, Laplacian and Frei-Chen [25] have been well studied. Region-based techniques such as Region growing [26], Watershed algorithm [27], and Thresholding [28] are more suitable for breast tumors extraction since suspicious regions are belonging to the same texture class, while surrounding tissues are belonging to others.

3.1. Proposed Algorithm

In this paper semi-automatic region based thresholding method is used. The word 'Semi-automatic' means squaring the expected suspicious regions to remove the top body edges (for example, the shoulders, pectoral muscle and breastbone). Breast tissues (for example, ducts, fat, lobules, lymph nodes and lymph vessels) are also excluded. The infected tissues are then selected automatically by selecting a threshold based on the image histogram or local statistics such as mean value, standard deviation and the local gradient [29]. For a given image, the binarization can be done using the pixel intensity values and is given

$$bin(x,y) = \begin{cases} 1 & I(x,y) \ge T \\ 0 & I(x,y) < T \end{cases}$$
 (19)

Where bin(x, y) is the resulting binary image and T is the threshold value.

The threshold value can be estimated based on the mean of the pixel intensity (M) and the standard deviation (σ) for each region of interest. It is given by

$$T = M + \alpha . \sigma + M.K$$

Where α and K are the constants.

4. RESULTS AND DISCUSSION

In image denoising, six selected algorithms: soft, hard, Zhang (1998), Zhang (2001), Nasri-Nezamabadi with universal threshold values, and Nasri-Nezamabadi with TNN are all implemented in MATLAB. Then, the proposed TNN method is coded. Finally, a comparison is held between these seven different algorithms. To compare accurately, we have to deal with Lena image, the reference chosen in their papers. Furthermore, these algorithms are applied to the target medical images, where the WT is applied using db8 four levels of decompositions.

4.1. Image Denoising

4.1.1. Universal Case

While implementing the Nasri-Nezamabadi thresholding function [24], it is used in universal Visu Shrink method, and compared with conventional functions such as hard, soft, Zhang(1998) and Zhang(2001) functions classes with their optimized values (k=3 and $\lambda=0.01$).

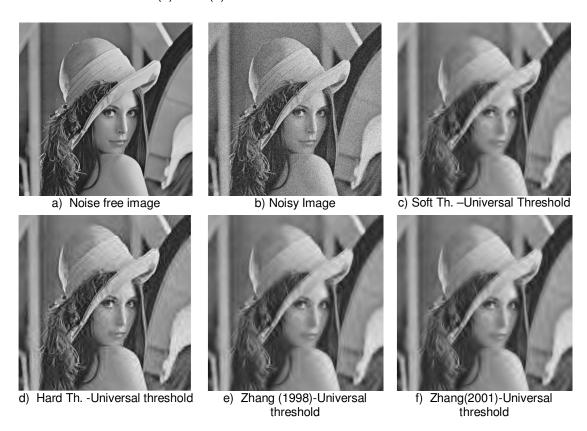
4.1.2 TNN with Threshold Learning Method

In the second step, the Nasri and Nezamabadi thresholding function [24] is used in the subband-adaptive threshold with TNN method. In this method, the threshold value is learnt and shape tuning factors of function are k=1 and m=2. The learning step is set to $\alpha=1e-6$ and the convergence criteria is considered as Δthr (i)/thr (i) <1e-6.

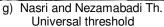
4.1.3 Proposed Modified TNN Technique

As mentioned before, one of our main concerns when introducing our innovative TNN is to save time. This will be achieved by reducing the total number of epochs with preserving the image quality.

Figure (6) and table (1) show the results of denoised Lena image with PSNR = ± 23.98 dB using the seven different algorithms. From figures (6-h) and (6-i), we can see how the last two algorithms offer good visualization success. Applying the proposed TNN, we have obtained approximately similar PSNR $\approx \pm 31$ but we have reduced the number of iterations significantly as will shown next. Selecting one medical image from the used data set in five different noise densities, results are shown in table (2) and figure(7). Figure (8) is histogram summary for one of the cases shown in table (2) case (5).









h) Nasri and Nezamabadi Th. with TNN

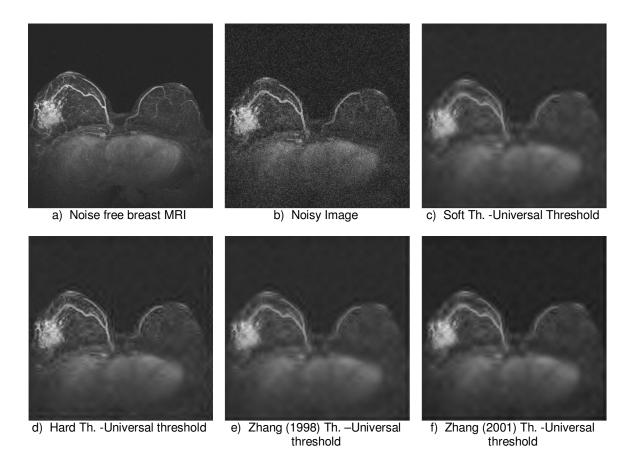


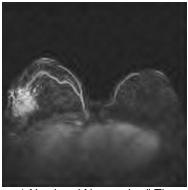
i) The Proposed modified technique

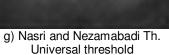
FIGURE (6): The results of denoised Lena image with PSNR=+23.98 dB

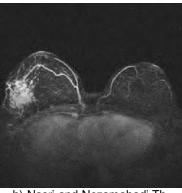
Ī			Ur	niversal T	TNN			
	Noisy image	Soft Hard Th.		Zhang Zhang (1998) (2001) Th. Th.		Nasri and Nezamabadi Th.	Nasri and Nezamabadi Th.	Proposed Technique
	+23.98	+26.24	+28.15	+26.83	+26.73	+29.34	+31.56	+31.50

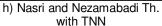
TABLE (1): The PSNR (in dB) results for Lena test image with Gaussian noise PSNR=+23.98, using different thresholding functions

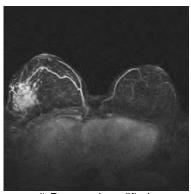












i) Proposed modified Technique

FIGURE (7): The results of denoised breast MRI test image for case (5) (PSNR=+21.82 dB)

			Ur	niversal T	TNN			
	Noisy image	Soft Th.	Hard Th.	Zhang (1998) Th.	Zhang (2001) Th.	Nasri and Nezamabadi Th.	Nasri and Nezamabadi Th.	Proposed Technique
Case (1)	+30.59	+30.99	+33.12	+32.21	+31.61	+34.25	+36.18	+35.96
Case (2)	+27.35	+29.67	+31.56	+30.45	+30.24	+32.56	+33.98	+33.87
Case (3)	+25.04	+28.65	+30.35	+29.17	+29.17	+31.18	+32.32	+32.22
Case (4)	+23.27	+27.79	+29.33	+28.27	+28.18	+30.02	+30.95	+30.90
Case (5)	+21.82	+27.03	+28.39	+27.47	+27.33	+29.00	+29.79	+29.75

TABLE (2): The PSNR (in dB) results for breast MRI test image with different Gaussian noise densities, using different thresholding functions

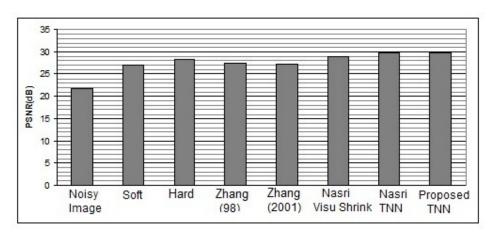


FIGURE (8): The PSNR histogram of different thresholing function to MRI breast image Case (5) (PSNR=+21.82 dB)

From table (3), the efficiency of both the proposed modification technique, and Nasri-Nezamabadi function with TNN method [24] is demonstrated. When comparing total number of iteration, a decreasing percentage is remarkable while preserving the PSNR. In figure (9) the numbers of iterations (execution time) are compared in two algorithms: Nasri-Nezamabadi [24] and the proposed TNN method for all MRI breast noisy cases.

From figure (10) it is clearly seen that the reduction in number of iterations is 22.29244 at PSNR= +30.59 (Case (1)); however it is increased to 38.75152 at PSNR= +21.82 (case (5)). This means that as the noise density increases, the proposed TNN gives better results compared to [24]. The proposed algorithm surpassed [24] in execution time by about 40%. Resultant image has a negligible difference output PSNR in all cases.

	PSNR of	Nasri and Nezamabadi function with TNN			Proposed Modification Technique				
	Noisy image (dB)	Total iteration	Level 1 iteration	PSNR (dB)	Total iteration	Decreasing iteration%	PSNR (dB)	Decreasing PSNR%	
Case (1)	+30.59	182,452	40,673	+36.18	141,779	22.29244	+35.96	0.608071	
Case (2)	+27.35	229,912	68,626	+33.98	161,286	29.84881	+33.87	0.32372	
Case (3)	+25.04	259,538	83,815	+32.32	175,723	32.29392	+32.22	0.309406	
Case (4)	+23.27	295,000	95,095	+30.95	199,905	32.23559	+30.9	0.161551	
Case (5)	+21.82	360,293	139,619	+29.79	220,674	38.75152	+29.75	0.134273	

TABLE (3): Comparison between Nasri and Nezamabadi function with TNN and Proposed Modification Technique performance.

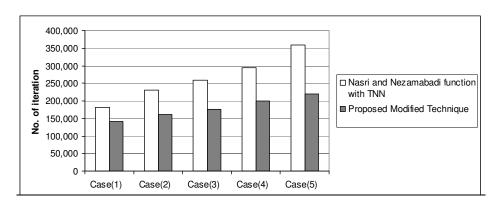


FIGURE (9): Number of iteration comparison between Nasri and Nezamabadi function with TNN method and proposed modified technique for MRI breast image.

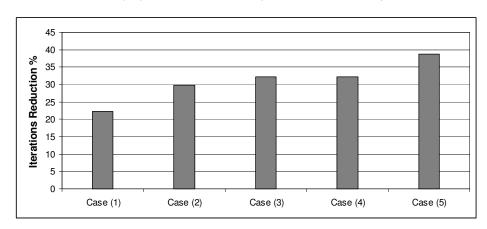


FIGURE (10): Iteration reduction percentages in proposed technique for MRI breast image

4.2 Segmentation Phase

The processed images enter a segmentation procedure to extract ROI. To evaluate the performance of the proposed TNN algorithm, a comparison between extracting ROIs from noisy and denoising image is presented. Figures (11) and (12) show the semi automatic segmentation procedures for case (5).

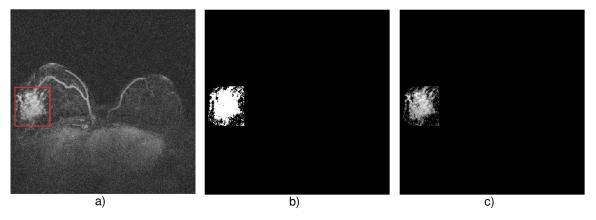


FIGURE (11): Segmentation noisy breast MRI case (5).

- a) Square the bright region to remove the top body edges of noisy image.
- b) Binary segmentation mask of noisy image.
- c) ROI noisy image. Original noisy image after applying the segmentation mask.

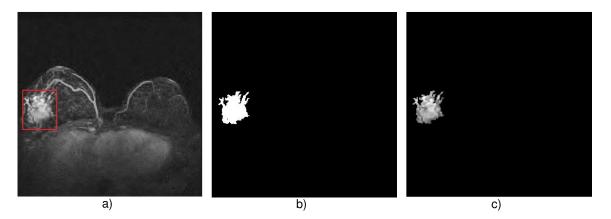


FIGURE (12): Segmentation denoised breast MRI case (5).

- Square the bright region to remove the top body edges of denoised image by proposed technique.
- b) Binary image segmentation mask image of denoised image by proposed technique.
- c) ROI image by proposed both denoising and segmented technique. i.e. Original denoised image after applying the segmentation mask.

5. DISCUSSION

The Nasri-Nezamabadi function [24] has produced better results in noise reduction compared to classic functions such as soft and hard and improved functions such as Zhang's class of functions in the universal threshold case. The hybrid method presented in this paper saves time and complexity without significant reduction in PSNR. Normally in wavelet subbands, as the level increases the coefficients of the subband becomes smoother. For example, horizontal- Level2 is smoother (less noise) than the corresponding subband in the first level, i.e., horizontal- Level1. So the threshold value for of horizontal- Level2 should be smaller than for horizontal- Level1. The

relation between levels and PSNR is summarized as follows: As the level increases, PSNR increases, noise is reduced, number of iterations increases, threshold value decreases. The wavelets act as filter of noise. We have found that the value of threshold by learning is near of universal threshold value only in first level; this is because first level is highest noise and fine tuning is needed in higher wavelet levels due to lower noise. The proposed TNN has shown remarkable execution time reduction especially in higher noisy environments, where the PSNR is low.

By means of the semi-automatic segmentation phase a clearer ROI is obtained. Very small tumor structure becomes more visible and features of the calcifications become more evident. Thus, CAD systems, radiologists, or physicians are towards better classifications and better decisions.

6. CONCLUSION

In this paper, an efficient detection of breast cancer tumor algorithm from medical images in noisy environment has been introduced. Based on a novel thresholding neural network in wavelet domain, and a semi automatic segmentation technique breast images are classified. The proposed method combines the Visu Shrink and the subband-adaptive cases with thresholding neural network. Comparison to other well-known existing algorithms shows that the proposed algorithm yields significantly superior image quality besides a reduced neural network learning time. This means a total reduction in complexity and execution time; while maintaining good image quality.

7. REFERENCES

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