Assessing Error Bound For Dominant Point Detection

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Abstract

This paper compares the error bounds of two classes of dominant point detection methods, viz. methods based on reducing a distance metric and methods based on digital straight segments. We highlight using two methods in each class that the error bounds within the same class may vary depending upon the fine details and control parameters of the methods but the essential error bound can be determined analytically for each class. The assessment presented in this paper will enable the users of dominant point detection methods to understand the nature of error in the method of their choice and help them to make better decision about the methods and their control parameters.

Keywords: Dominant point detection, digital straight segment, error bound, comparison, discrete geometry.

1 : INTRODUCTION

Dominant point detection in digital curves is a preliminary but important step in various image processing applications like shape extraction, object detection, etc. [1-13]. Despite being a very old problem of interest, this problem attracts significant attention even today in the research community. Some of the recent PA methods are proposed by Masood [14, 15], Carmona-Poyato [16], Wu [17], Kolesnikov [18, 19], Chung [20], Ngyuen [21], and Bhowmick [22] while few older ones are found in [23-33]. These algorithms can be generally classified based upon the approach taken by them. Often, algorithmic approach is used for classification. For example, some used dynamic programming [18, 19, 23], while others used splitting [27-29], merging [24], tree search [21, 22, 34], suppression of break points [14-16, 35, 36], etc.. However, this is not the focus of the current work.

The focus of the current work is the basic discrete geometry approach used in the methods since the geometric concept used in the method determines the achievable accuracy or the inherent error bound of the dominant point detection methods. In the sense of geometric approach, there are three major categories – methods based on minimization of a distance metric (like maximum deviation, integral square error, etc.) [14-20, 27-33, 37], based on the concept of digital straight segments [21, 22, 38], or based on curvature and convexity [17, 20, 25, 26, 30-32]. We highlight that the methods based on curvature and convexity often use k-cosine term for studying the convexity and curvature changes and the decisive factor in the selection of dominant points is often based on some distance metric or another. Thus, the error analysis of methods based on curvature and convexity is considered redundant with the error analysis of methods based on distance metric presented in this paper.

The outline of the paper is as follows. The error bound for the methods based on distance metrics is presented in section 2. The error bound for the methods based on digital straight segments is presented in section 3. The paper is concluded in section 4.

2 : METHODS BASED ON MINIMIZING THE MAXIMUM DEVIATION

Two most famous and classic methods from this class of dominant point detection methods are considered here, Ramer-Douglas-Peucker method [28, 29] (RDP) and Lowe's method [27].

These methods are classical methods which use splitting the digital curve based on maximum deviation. They have laid foundation for dominant point detection in terms of the algorithm, the idea of using distance measures, and the support region of the dominant points. Thus, they have been the basis of several later works on dominant point detection methods. Notwithstanding the later work, their computational efficiency and effectiveness in representing the digital curves has ensured the popularity and use of these methods in several image processing applications. These methods and their error metrics are discussed in the subsequent sub-sections.

2.1 Ramer-Douglas-Peucker method

Ramer-Douglas Peucker (RDP) method was first proposed in [28, 29]. The method is briefly described as follows. Consider a digital curve $S = P_1 \quad P_2 \quad \ldots \quad P_N$, where P_i is the i th edge pixel in the digital curve e. By default, the start and end points P_1, P_N are included in the list of dominant points. If the digital curve is a close loop, then only one of them is included. For the straight line joining the points P_1, P_N , the deviation d_i of a pixel $P_i(x_i, y_i) \in S$ is computed. Accordingly, the pixel with maximum deviation (MD) can be found. Let it be denoted as $P_{\max}$. Then considering the pairs $P_1, P_{\max}$ and $P_{\max}, P_N$, two new pixels of maximum deviations are found from S. This process is repeated until the condition in inequality eq. (1) is satisfied by all the line segments. The algorithm is terminated when the condition in eq. (1) is satisfied.

$$\max(d_i) < d_{\mathrm{tol}}, \tag{1}$$

where d_{tol} is the chosen threshold and its value is typically a few pixels.

Thus, the error bound of the RDP method is determined by the theoretical value of the maximum deviation $\max(d_i)$ of the pixels from the polygon formed by the dominant points or the control parameter d_{tol}. In general, researchers heuristically choose the value of d_{tol} in the range $1,2$.

It is notable that the maximum deviation is used as optimization goal or termination condition in several methods [14, 15, 20, 36, 37]. Further, several other methods use the integral square error (ISE) as the optimization goal or termination condition [18-20, 32, 33]. Since the maximum value of the integral square error is upper bounded by $N \ \max(d_i)^{\ 2}$ where N is the number of pixels, thus $\max(d_i)$ serves as the indicator of the upper bound for these methods as well.

2.2 Lowe's method

While the algorithmic structure of the Lowe's method proposed in [27] is essentially similar to RDP, its termination condition is different from RDP and more useful than using only the maximum deviation. Lowe considers two distances, first the maximum deviation $\max(d_i)$ of the pixels in the digital curve spanned by two consecutive dominant points (just like RDP method), second the distance between the two dominant points (say s). He defines the significance ratio:

$$r = s/\max(d_i), \tag{2}$$

and uses it as the basis for the decision to retain a dominant point. For three consecutive dominant points DP_{j-1}, DP_j, and DP_{j+1}, if $\max \ r_{j-1,j}, r_{j,j+1} > r_{j-1,j+1}$, then the dominant point DP_j is retained. This is done for all the dominant points except the start and end points. Then the points with $r_j < 4$ are also deleted.

It is notable that Lowe assumes that the maximum deviation is always at least 2 pixels. It was shown in [37, 39] that the maximum deviation is less than 2 pixels in most cases. Further, it is evident that the error bound of Lowe's method is determined by the inverse of the significance ratio $\max(d_i)/s$.

It is also notable that such similar significance ratios were used by several subsequent researchers as well [11, 16, 17, 20, 31-35], sometimes as a constraint and other times as the decision determinant. Most notably, such a ratio has served as the criterion of the support region of the dominant points.

2.3 Comparison of the methods based on error bound of the maximum deviation

It was shown in [37, 39, 40] that if a continuous line segment is digitized, the maximum distance of the pixels of the digital line from the continuous line segment is given by:

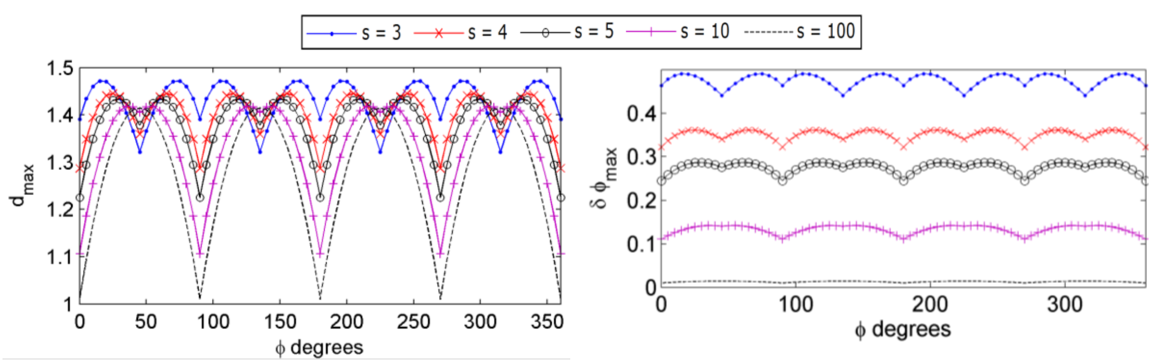
$$d_{\max} \approx \max\left(s \tan^{-1}\left\{ \frac{1}{s} \left|\sin\phi \pm \cos\phi\right| \quad 1 - t_{\max} + t_{\max}^2 \right\} \right) \tag{3}$$

where s is the length of the continuous line segment, $\tan\phi$ is the slope of the line segment and $t_{\max}$ is given by:

$$t_{\max} = \left(\frac{1}{s}\right) \left|\cos\phi\right| + \left|\sin\phi\right| \tag{4}$$

While the error bound $d_{\max}$ directly applies to the methods based on maximum deviation and integral square error, such as discussed in section 2.1, the ratio $d_{\max}/s$ applies to the methods based on significance ratio such as discussed in section 2.2. The expression for this is given by:

$$\delta\phi_{\max} = \frac{d_{\max}}{s} \approx \max\left(\tan^{-1}\left\{ \frac{1}{s} \left|\sin\phi \pm \cos\phi\right| \quad 1 - t_{\max} + t_{\max}^2 \right\} \right) \tag{5}$$



(a) plot of $d_{\max}$ (b) plot of $\delta\phi_{\max}$

Figure 1: Illustration of the error bounds of the distance based methods.

These two errors bounds are plotted in Figure 1. Figure 1(a) shows the theoretical error bound for RDP method. Based on the analysis, it is seen that the typical range of $d_{\text{tol}} \in 1,2$ used by most researchers incorporates the theoretical error bound. This is also the basis of the non-parametric framework presented in [37]. Figure 1(a) also indicates that Lowe's assumption that the maximum deviation is at least two pixels is incorrect and the maximum deviation is typically less than two pixels. Figure 1(b) shows the theoretical error bound for Lowe's method. It is seen that the maximum value of $d_{\max}/s$ is close to 0.5, which indicates that the constraint on the value of the significance ratio can be $r \geq 2$, as opposed to Lowe's criterion $r \geq 4$. Similar analysis can be made regarding the error bounds of other distance based methods also using eqn. (3).

3 : METHODS BASED ON DIGITAL STRAIGHT SEGMENTS

The concept of the digital straight segments (DSS) is a mathematical concept of discrete geometry [41] which specifically discusses a continuous line segment and a digital line segment and its properties. Evidently, it should serve as an important concept for dominant point detection methods. However, the simplicity and effectiveness of already popular distance based methods and the mathematical rigor of the concepts of DSS have restricted the researchers' interest in using DSS for dominant point detection. Nevertheless, DSS based methods are a very important class of dominant point detection methods, especially for extremely noisy digital curves for which the distance based metrics force over-fitting of the dominant points. Here, we consider two recent methods based on DSS, Nguyen's method [21] and Bhowmick's method [22].

3.1 Nguyen's method of blurred digital straight segments

Nguyen [21] uses the concept of maximally blurred segments for determining the dominant points on a noisy digital curve. The concept of the blurred segments in turn is based upon the concept of DSS which is presented here briefly. A digital curve is called a digital straight segment $D\ a,b,\mu,\omega\ ; a,b,\mu,\omega \in \mathbb{Z}^{(+)}$ if the points on the digital curve satisfy the equation below:

$$\mu \leq ax - by \leq \mu + \omega \tag{6}$$

The digital straight segment is called maximal if $\omega = |a| + |b|$ and blurred segment of width v if $\omega - 1 / \max(a,b) < v$ [21]. Thus, the error bound of Nguyen's method is determined by the value of the control parameter v. The value of v used in [21] varies from 0.7 for noiseless digital curves to 9 for noisy digital curves.

3.2 Bhowmick's method of approximate digital straight segments

Bhowmick's method [22] is based on approximate digital straight segments (ADSS), which is also based upon the concept of DSS. However, as compared to several usual works on DSS, Bhowmick uses the properties of the DSS derived from the Freeman chain code in [41]. Out of the four properties of DSS (R1-R4 in [22]), only two (R1 and R2) are used for defining ADSS and two additional conditions (c1 and c2 in [22]) are imposed on the digital curve to be concluded as ADSS. It is highlighted that the isothetic error bound of the ADSS was presented in [22] but is reconsidered for comparison with other methods. According to [22], the isothetic error bound is given by:

$$\varepsilon \leq 1 + \frac{d}{p+1} \tag{7}$$

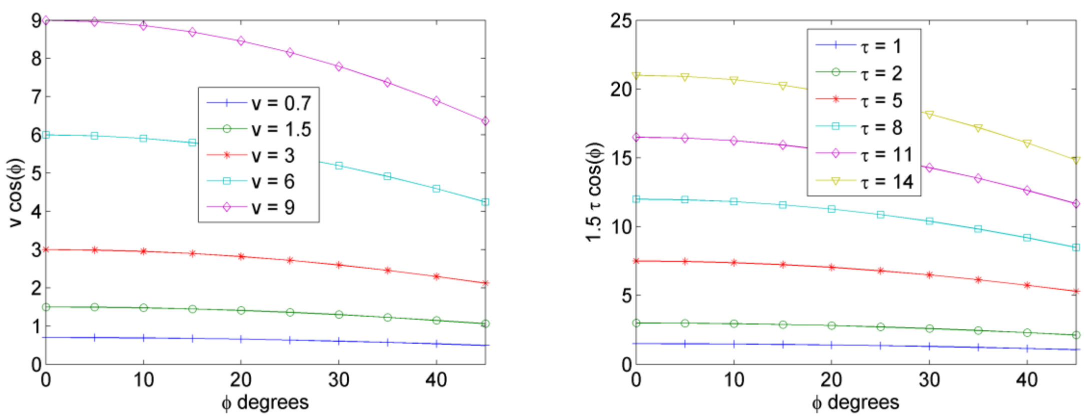
where $d = \left\lfloor \frac{p+1}{2} \right\rfloor$ and p is the minimum intermediate run-lengths of the freeman chain code of the digital curve (see [22] for details). Thus, the maximum value of the isothetic error is $\varepsilon \leq 1.5$. However, the error bound of the polygonal approximation in [22] is the product of both the isothetic error and the control parameter τ selected heuristically. Thus, the net error bound of Bhowmick's method is given by 1.5τ. The value of τ in [22] varies from 1 to 14.

3.3 Comparison of the methods based on digital straight segments

The methods based on DSS can be compared with each other directly based on the maximum isothetic (vertical/maximum distance of the pixels from the continuous line segment) distance of the digital curve from the line segments formed by dominant points. Thus, from the control parameters' values and the error bounds of Nguyen's and Bhowmick's methods, it can be seen that the maximum values of the upper error bounds of these methods are 9 and 21 respectively, while the minimum are 0.7 and 1.5 respectively.

If the DSS based methods should be compared with distance based methods, the maximum perpendicular distance of the pixels from the digital curves should be determined for DSS based methods also. Such distance can be easily computed by taking the projection of the maximum isothetic distance on the direction normal to the line segments. If the isothetic distance is denoted by d_{iso}, then the desired distance for comparison is computed as $\left|d_{\mathrm{iso}} \cos\phi\right|$ where $\tan\phi$ is the slope of the line segment joining the dominant points. It is notable that in both [21] and [22], it is assumed that $0 < a < b$, which implies that $\tan\phi \in 0,1$.

Accordingly, the upper bound of the maximum deviation of Nguyen's method (for width $v \in 0.7,9$) varies as shown in Figure 2(a) and Bhowmick's method (for $\tau \in 1,14$) varies as shown in Figure 2(b). From both the upper bounds, it is seen that DSS based methods allow for a large value of maximum deviation, which is especially suitable for extremely noisy curves. As a trade-off, the quality of fitting in the DSS based methods is severely dependent upon the choice of the control parameters.



(a) Maximum deviation for Nguyen's method for $v \in 0.7,9$

(b) Maximum deviation for Bhowmick's method for $\tau \in 1,14$

Figure 2: Illustration of the error bounds of the maximum deviations of Nguyen's and Bhowmick's methods.

It is also worth considering the error bound when both the blurred segments of Nguyen [21] and the ADSS of Bhowmick [22] are both forced to be the maximal straight segments (which is a well-defined control parameter independent) quantity. In such situation, considering the equation of a line in eqn. (8):

$$ax - by = c \tag{8}$$

where a and b correspond to a maximal digital line segment $D\ a,b,\mu$ while the points $P(x,y)$ belong to the continuous two-dimensional space. For the pixels $P'\ x',y'$ belonging to the digital straight segment a,b,μ, if they are to satisfy eqn. (8), then c has to satisfy inequality (9):

$$\mu < c < \mu + |a| + |b| \tag{9}$$

Using the above, the perpendicular distance (deviation) of the pixels in the maximal DSS from the line given in eqn. (8) satisfies eqn. (10) below:

$$0 < d_i < |\sin\phi| + |\cos\phi| \tag{10}$$

where $\phi = \tan^{-1} -a/b$. This error bound $d_{\mathrm{DSS}} = |\sin\phi| + |\cos\phi|$ is plotted in Figure 3. It can be seen that the maximum deviation for the DSS based methods (assuming no blurring or approximation of DSS) is about $\sqrt{2}$ pixels.

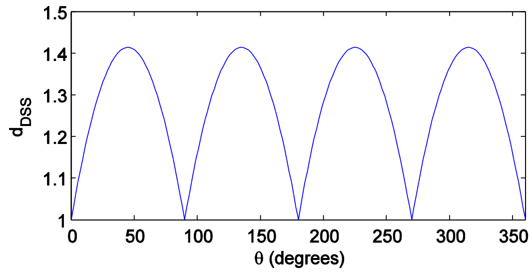


Figure 3: Illustration of the error bounds of the methods based on digital straight segments.

4: CONCLUSION

The error bounds of various methods falling into two categories of dominant point detection methods are assessed and compared in this paper. It is shown that the analytical bound on the maximum deviation can be computed for both distance based and DSS based dominant point detection methods. It is observed in each analysis that the error bound depends upon the orientation of the line segment and the control parameter of the algorithm. The assessment also gives clues on assessing the error bound of other methods as well. Thus, this work shall help researchers on studying the effect of the control parameters and the error bounds of the dominant point detection methods. A well understood choice of dominant point detection method shall in turn result into better performance for their higher level applications as well [1-13, 42, 43].

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