Robust Block-Matching Motion Estimation of Flotation Froth Using Mutual Information

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Abstract

In this paper, we propose a new method for the motion estimation of flotation froth using mutual information with a bin size of two as the block matching similarity metric. We also use three-step search and new-three-step-search as a search strategy. Mean sum of absolute difference (MAD) is widely considered in blocked based motion estimation. The minimum bin size selection of the proposed similarity metric also makes the computational cost of mutual information similar to MAD. Experimental results show that the proposed motion estimation technique improves the motion estimation accuracy in terms of peak signal-to-noise ratio of the reconstructed frame. The computational cost of the proposed method is almost the same as the standard machine vision methods used for the motion estimation of flotation froth.

Keywords: Mutual information, Mean sum of absolute difference, Motion Estimation, Froth Flotation.

1. INTRODUCTION

Froth flotation is one of the most widely used separation methods in the mineral processing industries. Despite being in use for more than a century, it is still not fully understood and remains comparatively inefficient. Conventionally, the flotation process is controlled by trained operators who visually inspect the froth and adjust the process control parameters based on their training. A problematic situation is that different operators tend to interpret the visual characteristics of the froth differently.

However, during the last couple of decades, froth image analysis has been studied widely as a means to obviate these problems, as summarized in a recent review by Aldrich et al. [1]. Froth images are a potentially rich source of information with regard to the state of the flotation process and can be used as a basis for the development of advanced online control systems. Apart from the appearance of the froth, the motion of flotation froth is also an important variable related to the optimal recovery of minerals on industrial plants and has been used in industry among other as a basis for mass pull control strategies [2].

The estimation of motion of froth using image processing techniques is not easy, since bubbles collapse and merge, resulting in bubble deformations that may be difficult to track. Moreover, although the froth as a whole has an average motion, it moves rapidly at the edges of the cell at the launder overflow, while remaining almost stagnant at the centre of the cell [1].

Several approaches to froth motion estimation have been investigated, including bubble tracking [3], cluster matching [4], and pixel tracing [5], but the most popular motion estimation technique is block matching [6-7] using the mean absolute difference (MAD) between successive images of parts thereof [2]. In the block matching algorithm, the image frame is typically divided into nonoverlapping rectangular blocks [8]. The best match to a given block of pixels is subsequently searched in the previous frame of the video sequence within a search area about the location of the current block. In principle, the best solution can be obtained with a full search, which exhaustively searches for the best matched block within all locations of the search window. However, this approach is computationally expensive and several fast search techniques have been proposed to reduce the high computational cost of full search methods. These fast search techniques select a subgroup of possible search candidate locations and thus reduce the number of matching calculations per block image.

Examples of such fast search techniques are three-step-search (*TSS*) [9], new-three-step-search (NTSS) [10], four-step-search [11], 2-D logarithmic search [12] and the diamond search [13]. The mean square error (*MSE*) or mean absolute difference (*MAD*) are considered to be among the best similarity metrics for motion estimation. In this paper, a new method is proposed for estimating the motion of flotation froth using mutual information. As will be shown, estimation of mutual information with a bin size of 2 as the block matching criterion improves the accuracy of motion estimation in terms of the peak signal-to-noise ratio of the reconstructed frame, while the computational complexity of the proposed similarity metric is also similar to MAD.

2. SIMILARITY METRICS

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2.1 Mutual Information with Bin Size of Two

If *X* and *Y* are two image blocks to be matched, the mutual information (*MI*) can defined by

$$MI(X,Y) = H(X) + H(Y) - H(X,Y) \tag{1}$$

where *H(X)* and *H(Y)* are the Shannon entropies [14] of *X* and *Y* respectively, and *H (X,Y)* is the Shannon entropy of the joint distribution of *X* and Y. If $p_X(x)$ and $p_Y(y)$ are the marginal probabilities, and $p_{XY}(xy)$ is defined as the joint probability density of *X* and *Y*. Then *MI* is defined as

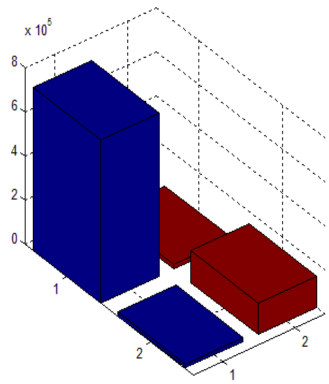
$$MI(X,Y) = \sum_x \sum_y p_{XY}(x,y) * \log\left(\frac{p_{XY}(x,y)}{p_X(x) * p_Y(y)}\right) \quad .. \qquad (2).$$

In this work, we use the histogram method to estimate *MI*. Let $H_X(x)$ and $H_Y(y)$ be the histograms of *X* and *Y* respectively and $H_{XY}(x,y)$ their joint histogram. then *MI* can be defined as

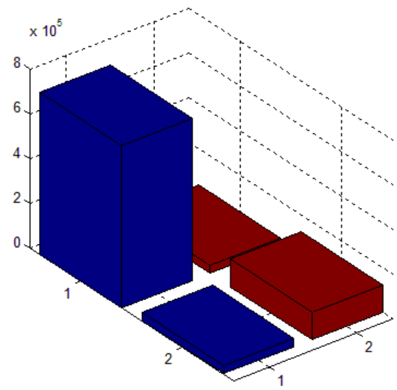
$$MI(X,Y) = \frac{1}{E} \sum_x \sum_y H(x,y) * \log\left(\frac{EH_{XY}(x,y)}{H_X(x) * H_Y(y)}\right) \tag{3}$$

where *E* is the number of entries [15-17].

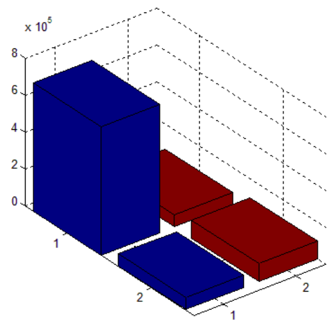
Figure 1 shows the joint grey value histograms and corresponding joint entropies of an image block with values shifted (a) 0 pixels, (b) 5 pixels, (c) 10 pixels and (d) 100 pixels. since the image pairs are identical, all corresponding grey values lie on the diagonal in Figure 1(a). From the joint histograms it can be deduced that i) diagonal features diminish with increasing dissimilarity. ii) the spatial dispersion pattern increases in areas with increasing dissimilarity, and iii) joint entropy increases with increasing similarity.



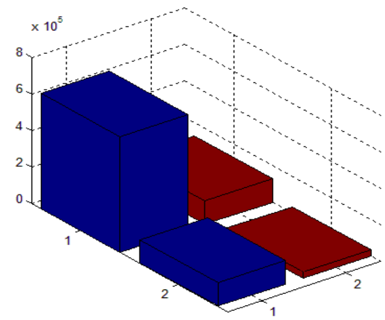
(a) H(X,Y) = 0.53



(b) H(X,Y) = 0.98

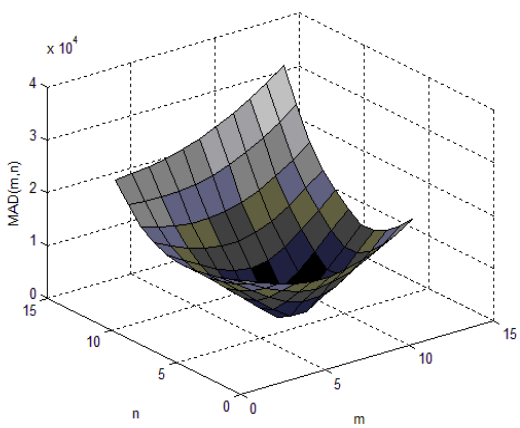


(c) H(X,Y) = 1.05

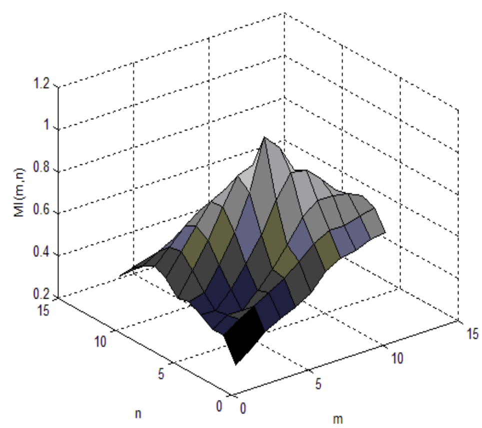


(d) H(X,Y) = 1.20

Figure 1. Joint grey values of a block image of a flotation froth and a shifted version of the image.



(a)



(b)

Figure 2. Example of (a) MAD surface of a block for a search area of 5 pixels (b) MI surface of the same block over a search area of 5 pixels

Mutual information contains the term *–H (A,B),* which means that the minimization of the joint entropy will maximize the mutual information. Since generally joint entropy increases with increasing image block mismatching, the mutual information decreases with increased mismatching, i.e. similarity corresponds with the maximization of the mutual information. That is, if images are aligned the amount of information they contain about each other is maximal.

The computational complexity between two block images depends not only on the number of pixels in each block image (*M*) but also the number of bins used the estimate *MI*. The computational cost of the histogram is $O(M)$. The computational cost relative to the number of histogram bins is $O(R^2)$, where *R* is the number of bins. In this work only two bins were used (R = 2), with a block size of 40 x 40 pixels, so that the computational cost of *MAD* and *MI* were approximately the same. The principle of maximization of mutual information is applied to the motion vector of blocks. The motion vector is obtained by equation

$$mv = \arg \max_{-p \le m,n \le p-1} MI(m,n) \tag{4}$$

Figures 2(a) and 2(b) respectively show the *MAD* surface and *MI2* surface of same block over a search area of 5 pixels in each of the vertical and horizontal directions. The *MI2* surface is sharper than the *MAD* surface, suggesting that *MI2* can be more robust than MAD, since both surfaces are smooth.

2.2 Mean Absolute Difference (MAD)

The full *MAD* search of an image frame divided into $N \times N$ blocks, can be expressed as

$$MAD(m,n) = \sum_{i=0}^{i=N-1} \sum_{j=0}^{N-1} \left| F^t(i,j) - F^{t-1}(i+m, j+n) \right|,$$
$$-p \le m,n \quad \le p \tag{5}$$

where *MAD(m,n)* is the *MAD* computed for the displacement of *(m,n)* pixels. $F^t(i,j)$ and $F^{t-1}(i,j)$ are the grey level values of the current and previous frame respectively. The search area or the maximum displacement, *p*, allowed in the vertical and horizontal directions. The motion vector for a block is obtained by the equation

$$mv = \arg \min_{-p \le m,n \le p-1} MAD(m,n) \tag{6}$$

where *mv* is the motion vector of the vertical and horizontal displacements. The computational complexity of *MAD* is $O(M)$ additions, where M is the number of pixels in each frame

3. SEARCH METHODS

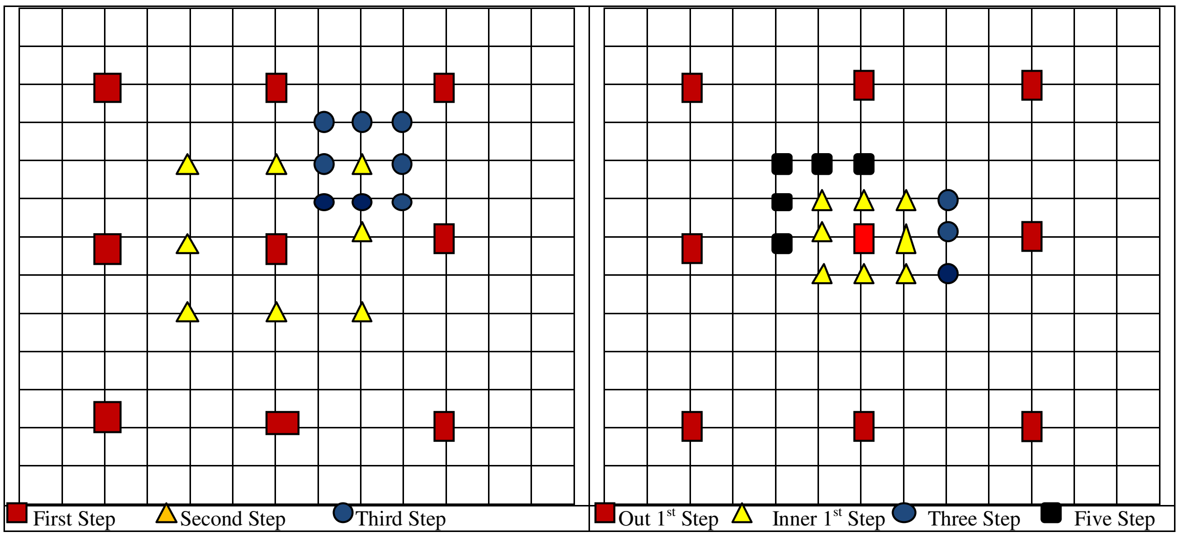
The three-step-search and the new-three-step-search algorithms were used in this investigation.

3.1 Three-Step-Search (TSS)

The TSS algorithm begins with search location considered as the centre and sets a step size depending on the search window. It searches eight locations around the centre. From the nine locations searched, the one with the best similarity metric is selected as the new centre. A new step size is selected, which is half of the original step size and a similar search is repeated for two more iterations. Figure 3(a) illustrates the TSS algorithm.

3.2 New-Three-Step-Search (NTSS)

The NTSS is a modified version of TSS and a centre biased search method. In the first step, 16 points are checked in addition to the search origin for the lowest or highest weight of the cost function. If the optimal cost is at the origin, the search is stopped and the motion vector is (0,0). If the optimal cost is at any of the 8 locations nearest to the origin, then the origin of the search is changed to that point and points adjacent to it are searched. Depending on which point it is, 5 points or 3 points night be checked, as shown in Figure 3(b). The location that gives the optimal cost is the best match and the motion vector is set to that location. On the other hand, if the optimal cost is associated with one of the 8 locations further away from the origin, then the TSS search method is followed. Thus although the NTSS method might need a minimum of 17 points for every macroblock in the image, it has a worst case scenario of 33 points that have to be searched.



(a) (b)

Figure 3(a). The Three Step Search algorithm with rectangles, triangles and circles representing first step search, second step search and third step search respectively, and (b) NTSS with rectangles as first outer step, triangle as inner first step, circles and squares as three step and five step respectively.

4. EXPERIMENTS AND RESULTS

The performance of the proposed similarity metric and MAD were evaluated using two video sequences “Flot1” and “Flot2” from the Anglo Platinum Centre of Excellence laboratory in Stellenbosch. The TSS and NTSS search strategies were used. The frame rate was 30 Hz and the resolution of both sequences was 720 x 1280 pixels. A 4 dB Gaussian was added to Flot2 to test the robustness of the algorithms, while a block size of 40 x 40 and a search area of 5 pixels were used in the vertical and horizontal directions. The peak signal to noise ratio (*PSNR*) of the image given in equation 6 is used to evaluate the performance of the algorithms

$$PSNR = 10\log_{10}\left(\frac{255^2}{MSE}\right) \quad . (7)$$

Figures 4(a) and 4(b) show the motion vector estimate of frame 6 in the *Flot1* sequence using *MAD* with *TSS* and *MI2* with *TSS* respectively. Figures 4(c) and 4(d) also show the motion vector estimate of frame 7 in the *Flot2* sequence using *MAD* with *TSS* and *MI2* with TSS respectively. Figure 5(a) and 5(b) show the comparisons of the *PSNR* of MI2*-with-NTSS*, *MI2-with-TSS*, *MAD-with-NTSS*, and *MAD-with-TSS* of the Flot1 and Flot2 sequences respectively. The performance of the algorithms in terms of the PSNR of the reconstructed frame for the Flot1 sequence is almost the same. The performance of the *MI2* algorithms (MI2-with-NTSS and MI2-with-TSS*)* is better than the MAD algorithms (MAD-with-NTSS and MAD-with-TSS). The overall average PSNR of the experiments for the MI2 algorithms and the MAD algorithms are 66.6 dB and 66.0 dB respectively.

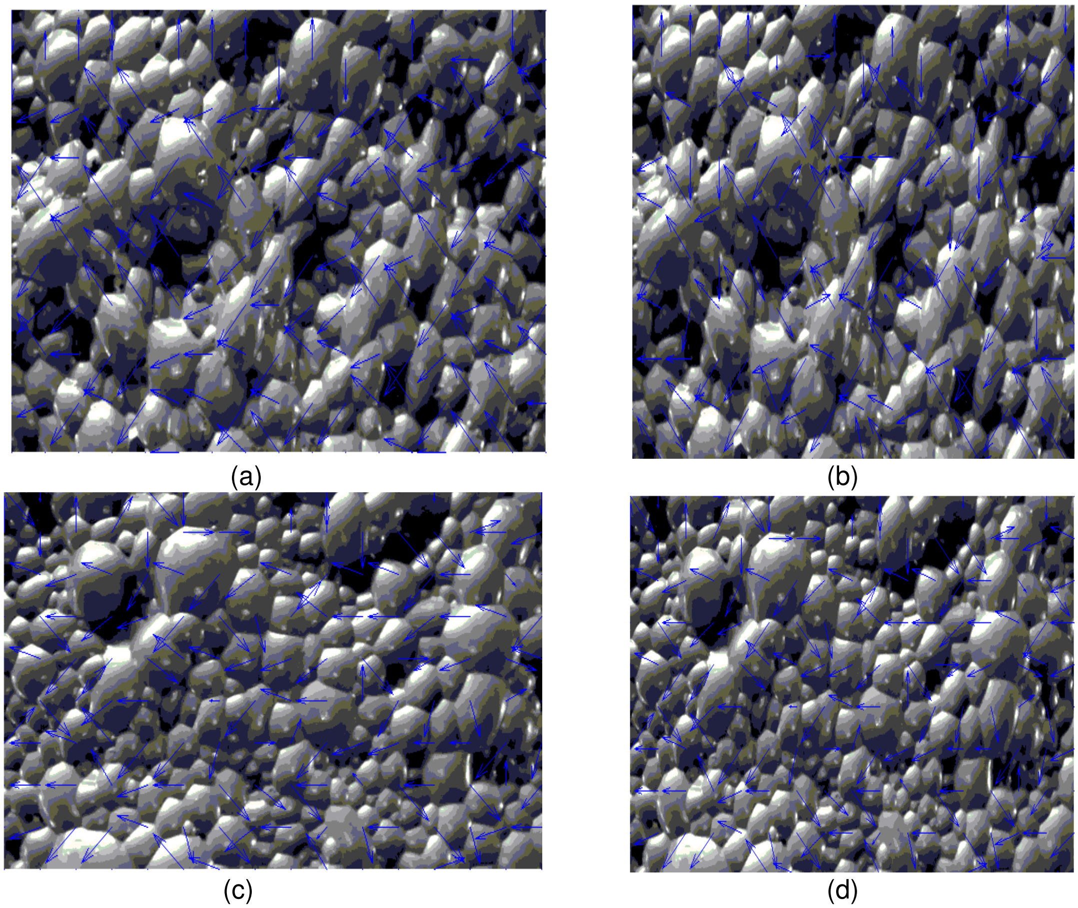
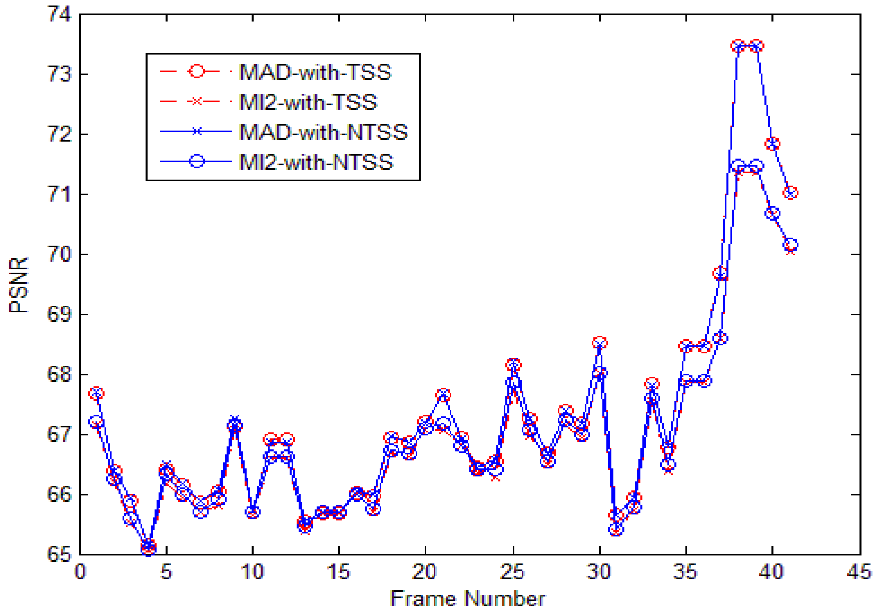


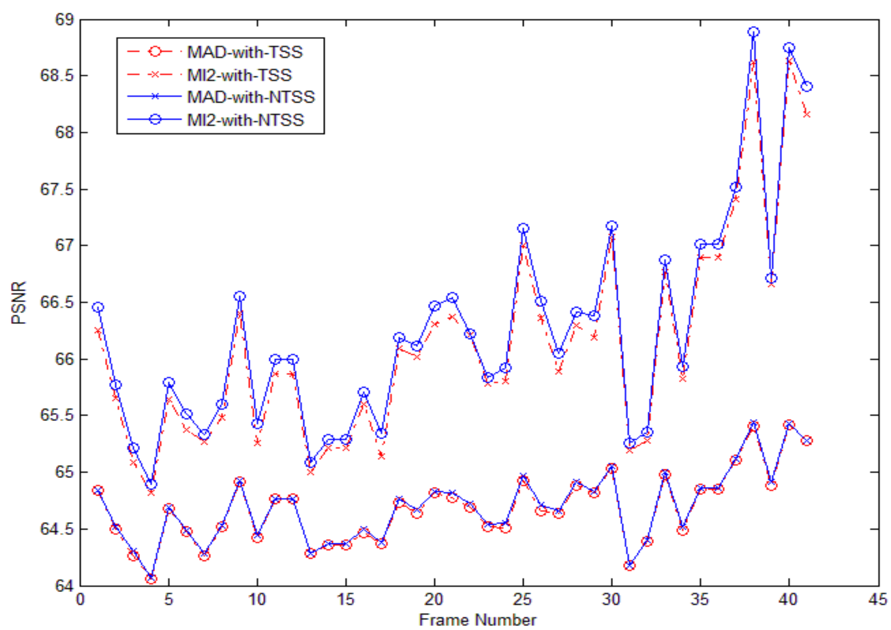
Figure 4(a). Motion vector of frame 6 in the *Flot1* sequence estimated by conventional *MAD* using TSS, (b) Motion vector of frame 6 in the *Flot1* sequence estimated by conventional MI2 using TSS (c) Motion vector of frame 7 in the *Flot2* sequence estimated by the conventional *MAD* using *TSS, and*. (d) Motion vector of frame 2 in the *Flot2* sequence estimated by the conventional *MI2* using *TSS.*

Tabel.1 Average peak to signal noise ratio (PSNR) (dB) of the Flot1 and Flot2 seqeunces reconstructed by various motion estimation algorithms.

Method	**Flot1**	**Flot2**
MAD-with-TSS	67.34	64.67
MAD-with-NTSS	67.35	64.67
MI2-with-TSS	66.94	66.12
MI2-with-NTSS	67.00	66.24



(a)



(b)

Figure 5(a). Comparisons of the peak signal to noise ratio (PSNR) of sequences reconstructed by various motion estimation algorithms. Using the Flot1 sequence with a resolution and frame rate of 720 x 1280 and 30Hz respectively, and (b) using the Flot2 sequence with a resolution and frame rate of 720 x 1280 and 30Hz respectively.

5. CONCLUSIONS

In this paper, a new motion estimation method for flotation froth is proposed by using mutual information with a bin size of two (*MI2*) as a similarity criterion. The computational cost is similar to the popular *MAD* approach. The three-step-search and the new-three-step-search algorithms were used to search the images. *MI2* yielded better results in terms of the PSNR of the reconstructed image frames than the classical minimum absolute difference (MAD). It reduces the amount of bad motion vectors especially for noisy images compared the MAD approach. In the future, the algorithm will tested with more video sequences in combination with different search strategies. We will also investigate optimization algorithms such as the Spall's algorithm [18] to reduce computational cost and the lightening systems for the experimental data.

6. REFERENCES

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