Local Phase Oriented Structure Tensor To Segment Texture Images With Intensity Inhomogeneity

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Abstract

This paper proposed the active contour based texture image segmentation scheme using the linear structure tensor and tensor oriented steerable Quadrature filter. Linear Structure tensor (LST) is a popular method for the unsupervised texture image segmentation where LST contains only horizontal and vertical orientation information but lake in other orientation information and also in the image intensity information on which active contour is dependent. Therefore in this paper, LST is modified by adding intensity information from tensor oriented structure tensor to enhance the orientation information. In the proposed model, these phases oriented features are utilized as an external force in the region based active contour model (ACM) to segment the texture images having intensity inhomogeneity and noisy images. To validate the results of the proposed model, quantitative analysis is also shown in terms of accuracy using a Berkeley image database.

Keywords: Linear Structure Tensor, Quadrature filter, Active contour, Image Segmentation

1. INTRODUCTION

Image segmentation is primary steps for the point of interpretation. This segmentation process relies on the extraction of required features from the image that discriminates the region of interest from the given image. The proposed model is toward the segmentation of texture images having intensity inhomogeneity. Many algorithms have been suggested in various literatures for texture image segmentation some of them are [10, 18, 11, 9]. These segmentation models include Gabor filter based approach, Markov random field based approach and multi-resolution approach. One of the most familiar technique is a Gabor filter to extract the features of the texture image. Another approach is the use of active contour model [16], which is utilized in this paper. The proposed model in this paper utilized the region based geometric active contour model for the texture image segmentation.

It is well known that region based CV model [16] failed to segment the images having either intensity inhomogeneity and texture patterns. Therefore to handle the intensity inhomogeneity, Lee et al [8] presented local kernel based active contour model. But limitation is still not able to segment the texture images and again it is highly sensitive to the size of the kernel. To segment texture images, Wang et al [17] modified the CV model [16] by guiding the active contour with histogram based information. Similarly Bhattachrya distance [12, 20] also proposed as the texture image segmentation model using histogram calculation. Then Kangyu Ni et. al. [19] proposed the similar histogram based level set function where Wasserstein distance was utilized as a distance vector between the histogram in comparison to the Bhattacharya distance. The most common disadvantage of all these models is the need of histogram calculation which incorporates computational complexity. To extract the orientation information, earlier gradient based local structure tensor (LST) method was suggested in [1]. Gradient based LST is a simple method and gives good local orientation localization in two directions. Therefore LST is largely utilized to get the orientation of texture pattern. Another method to extract the orientation features of images is an estimation method based on Quadrature filter set after representation by Knutsson and Andersson [7]. This method has more advantage in terms of orientation localization because Quadrature filter based method can estimate more orientation information as compared to gradient based LST. But filter based approach is utilized to segment the medical images having intensity inhomogeneity [4] and not applied to texture images. Therefore the proposed model combines the advantages/features of a filter based structure into the LST and this information is utilized to guide the active contour model (ACM) for the image segmentation. This makes LST model to segment the texture images having intensity inhomogeneity.

The outline of the paper is as follows: Section 2 introduces the tensor oriented Quadrature filter based structure and the gradient based LST for texture image segmentation. Section 3 demonstrates the proposed model based on section 2 and corresponding results are discussed along with quantitative analysis. Finally in section 4 results are concluded.

2. BACKGROUND

2.1 Tensor oriented Quadrature filter

It is well known that image intensity is highly sensitive to the lighting condition. This variation in contrast introduced by lighting conditions cause difficulties in segmentation. The information carried by the image local phases is invariant to intensity variation. At the same time the phase information allows us to detect the lines and edges simultaneously; which make us to utilize this local phase information as an image feature for the image segmentation. One of the techniques is the use of quadrature filters to extract the phase information. To obtain the Quadrature filter, one radial function R and one directional function D are defined in Fourier domain. Hence,

$$F(u\hat{x}, v\hat{y}) = R(\rho)D(u\hat{x}, v\hat{y}) \tag{1}$$

Were $\rho = \sqrt{u^2 + v^2}$ and u, v are dummy variables in frequency domain and $\hat{x}, \hat{y}$ represents the orientation vector. Component in R is symmetrical in nature, and hence in order to obtain the Quadrature property of F, D has to be Quadrature. i.e.

$$D(u\hat{x}, v\hat{y}) = 0 \quad \text{if } u\hat{x} \le 0, v\hat{y} \le 0 \tag{2}$$

Both R and D are assumed to be real valued. Radial function (R) utilized in the paper is given by

$$R(\rho) = \exp\left(\frac{-4}{B^2 \ln 2}\right) \ln^2(\rho / \rho_i) \tag{3}$$

Where B is the bandwidth and ρ_i is the center frequency.

If this filter is applied to the signal, the filter output is calculated by the product of the Fourier transform of the filter and the Fourier transform of that signal around the point. Let the filter output is denoted by q' which gives,

$$q'(u, v) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(u, v) F(u\hat{x}, v\hat{y}) du dv \tag{4}$$

Where $S(u,v)$ is the Fourier transform of an arbitrary 2-D $s(x,y)$ in spatial domain. In above equation 4, q' represents the filtered signal oriented by directional component D. In this paper, three Quadrature filters are utilized.

Utilization of these Quadrature filters eliminates the negative frequency components. Hans Knutsson and Mats Andersson [7] introduce the orientation tensor T to represent the signal orientation. To generate the orientation tensor, following preference axis is utilized [6].

$$\hat{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix} \tag{5}$$

Where α depends on chosen coordinate systems. Three orientations 0, 2pi/3 and -2pi/3 are utilized to calculate the tensor in this paper.

Based on these orientation corresponding orientation tensor is given by

$$T = \begin{pmatrix} T_{\alpha,11} & T_{\alpha,12} \\ T_{\alpha,21} & T\alpha,_{22} \end{pmatrix} = \begin{pmatrix} {x_1}^2 & x_1x_2 \\ x_1x_2 & {x_2}^2 \end{pmatrix} = \begin{pmatrix} \cos^2\alpha & \cos\alpha\sin\alpha \\ \cos\alpha\sin\alpha & \sin^2\alpha \end{pmatrix} \tag{6}$$

To obtain the phase map of filtered image, linear combination of filter output and tensor z (from equation 6) is utilized. Therefore resulting phase map in respective direction can be written as

$$Z_\alpha = T_{\alpha,11}q_1' + T_{\alpha,12}q_2' + T_{\alpha,13}q_3' \tag{7}$$

The texture images contain repeated pattern within the region. And the task of tensor based Quadrature filter is to extract the local phase information. Therefore, it is not suitable to apply these filter pairs to the texture images. Therefore proposed model in this paper combines this orientation information to LST to obtain the boundary of objects.

2.2 Local structure tensor

Covariance matrix based LST is strongly utilized as a texture descriptor. The advantage of LST is that its dimensionality is small and simplicity in the calculation. Therefore it is widely utilized in corner detection, texture analysis and texture segmentation [2, 5]. Consider a grayscale image . The matrix field of the structure tensor is [2, 3] represented by

$$J_0 = \nabla h \nabla h^T = \begin{bmatrix} h_x^2 & h_x h_y \\ h_x h_y & h_y^2 \end{bmatrix} \tag{8}$$

Where $\nabla h = \begin{bmatrix} \frac{\partial h}{\partial x} & \frac{\partial h}{\partial y} \end{bmatrix}^T$, $h_x = \frac{\partial h}{\partial x}$, $h_y = \frac{\partial h}{\partial y}$ and T = Matrix transpose.

In order to avoid cancellation of the opposite signed gradient when direct integration is performed, the gradient is considered as the form of its outer product. In order to make the matrix field more robust to noise and other artifacts, smoothing is usually performed by convolution of the of the matrix components with a Gaussian kernel K_ρ with standard deviation ρ,

$$J_\rho = K_\rho * \left(\nabla h \nabla h^T\right) \tag{9}$$

Where * indicate convolution operation.

Above LST model contains the orientation and magnitude of texture features and therefore it can be utilized to segment the texture images. However as it lacks the intensity information and thus failed to segment the texture images having intensity inhomogeneity. Again problem with this model is that it can not be combined with active contour model (ACM) due to the dependency of its energy minimization criterion on image intensity rather than texture pattern. Therefore this paper suggests the solution to this problem with a combination of filter based tensor values to the LST. This proposed model is explained in following section.

2.3 Region based Active Contour model

The CV Model is based on piecewise constant approximation of the Mumford Shah energy function [16]. For an image $I(x,y)$ in the image domain Ω, they proposed the energy minimization formula as

$$E(c_1,c_2,C)=\mu Length(C)+vArea(C)+\lambda_1\int_{inside(C)}\left|I(x,y)-c_1\right|^2 dxdy+\lambda_2\int_{outside(C)}\left|I(x,y)-c_2\right|^2 dxdy \quad (10)$$

where c_1 and c_2 are the average intensities inside and outside of the contour, respectively. λ_1 and λ_2 are positive constants. Generally we take λ_1 and λ_2 equal to one. As the last two terms in equation (10) is the global fitting energy, Chan-Vese model is global region based model. Here the contour is represented by the zero level set function ϕ [14]. Keeping c_1 and c_2 constant during the evolution of contour, the energy minimization equation was obtained as

$$\frac{\partial\phi}{\partial t}=\delta_\varepsilon(\phi)\left[\mu k-v-\lambda_1(I-c_1)^2+\lambda_2(I-c_2)^2\right] \quad (11)$$

where k is the Euclidean curvature of the contour C, δ is the smoothed Dirac delta function given by

$$\delta(x)=\frac{\varepsilon}{\pi(\varepsilon^2+x^2)} \quad (12)$$

The limitation of the CV model is not able to segment the images having either intensity inhomogeneity or texture patterns. Therefore this paper proposed the solution using the advantage of LST and the steerable Quadrature filter which is discussed in following section 3.

3. PROPOSED MODEL FOR TEXTURE IMAGE SEGMENTATION

As the local phase map obtained by the tensor oriented Quadrature filter is invariant to intensity inhomogeneity , in the proposed model, local phase map is obtained from texture image using tensor oriented Quadrature filter. Here, three Quadrature filters are utilized having a bandwidth of 3 dB and center frequency of pi/5. Each Quadrature filter has a different orientation and consists of real and imaginary part where the real part is able to detect the line in image and imaginary part detect the edge in image. To enhance local phase, it is required to have a rotation invariant phase map. Therefore we integrated all phase maps. Now during the integration of features from all directions, there may be chance of cancellation of features due to negation of phase at opposite direction. This may cause a discontinuity or flickering in phase map generated by the filter. Therefore we flip the phase along the real axis for a filter with direction opposing the other direction. Thus we obtain the phase maps in three filter orientation. These flipped phase map from all directions are added to obtain the final phase map as shown in equation 13.

$$q=\sum_{k=1}^{3}\left\|q_k'\right\| \quad (13)$$

This flipped phase map is driven by the tensor to form the dual tensor structure using equation 6 as follow.

$$Z_k = T_{k,11}q \quad + T_{k,12}q \quad + T_{k,22}q \qquad k = 1,2,3 \tag{14}$$

From this Eigenvalues are calculated using following two equations:

$$l_1 = \frac{Z_1 + Z_2}{2} + \sqrt{\left(\frac{Z_1 - Z_2}{2}\right)^2 + Z_3{}^2} \; l_2 = \frac{Z_1 + Z_2}{2} - \sqrt{\left(\frac{Z_1 - Z_2}{2}\right)^2 + Z_3{}^2} \tag{15}$$

These Eigenvalues can be considered as equivalent to the horizontal and vertical edge information with enhancement. To extract the perceptual visualized based shape information on the texture object from the given image, here this tensor oriented Quadrature filter output is combined with LST. For that, in LST's equation 8, the horizontal and vertical gradients are replaced by the two Eigenvalues calculated from the equation 15 and then after the local tensor is calculated using equation 9. Thus corresponding LST is defined as

$$L_{QF} = \begin{bmatrix} l_1^2 & l_1 l_2 \\ l_1 l_2 & l_2^2 \end{bmatrix} \tag{16}$$

Localized structure calculated using above equation 16 smooth out the foreground and background texture patterns while keeping enhanced edge or boundary of the object. Thus the present approach enhances the weak edges and solves the intensity inhomogeneity problem with smoothing of texture pattern in the background and foreground pattern as shown in figure 1 (d).

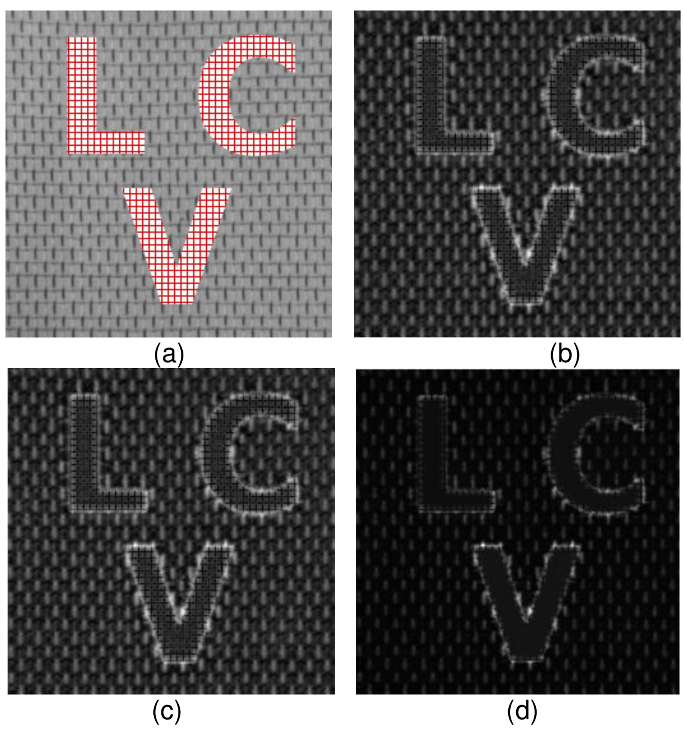


FIGURE 1: (a) Original Image, (b) and (c) Eigen image after tensor oriented Quadrature filter (d) LST based output using equation 16.

Figure 1 shows that use of the phase map helps in identifying the edge maps even in the inhomogeneous region, but it is not applicable to a texture object. Therefore to get the boundary of the texture object, eigenvalues of that phase map are calculated. These eigenvalues remove the texture pattern while keeping the object boundaries. Thus the proposed model is able to segment the texture images having intensity inhomogeneity.

To obtain the segmentation of a given image, this LST based output is utilized as the original image in active region based model (see equation 17). Thus the final energy minimization equation is given by

$$\frac{\partial \phi}{\partial t} = \delta_{\varepsilon}(\phi)\left[\mu k - v - \lambda_1 (L_{QF} - c_1)^2 + \lambda_2 (L_{QF} - c_2)^2\right] \tag{17}$$

4. EXPERIMENT RESULTS

To validate the model, the proposed model is applied to a number of texture images involving different types of complexity. From the point of complexity, here specific images are considered as follows: Like in the first row of figure 7, the head in left middle part and the legs of leopard has similar intensity range as the background. This makes difficult to segment the images, and an individual approach of either LST (due to intensity inhomogeneity) or tensor oriented Quadrature filter (due to texture region) or active contour based model (due to texture region) failed to segment the image (see figure 2), while the proposed model is able to segment it successfully as inhomogeneity is handled by the phase map obtained from the Quadrature filter and texture pattern is smoothen out using the eigen based structure tensor.

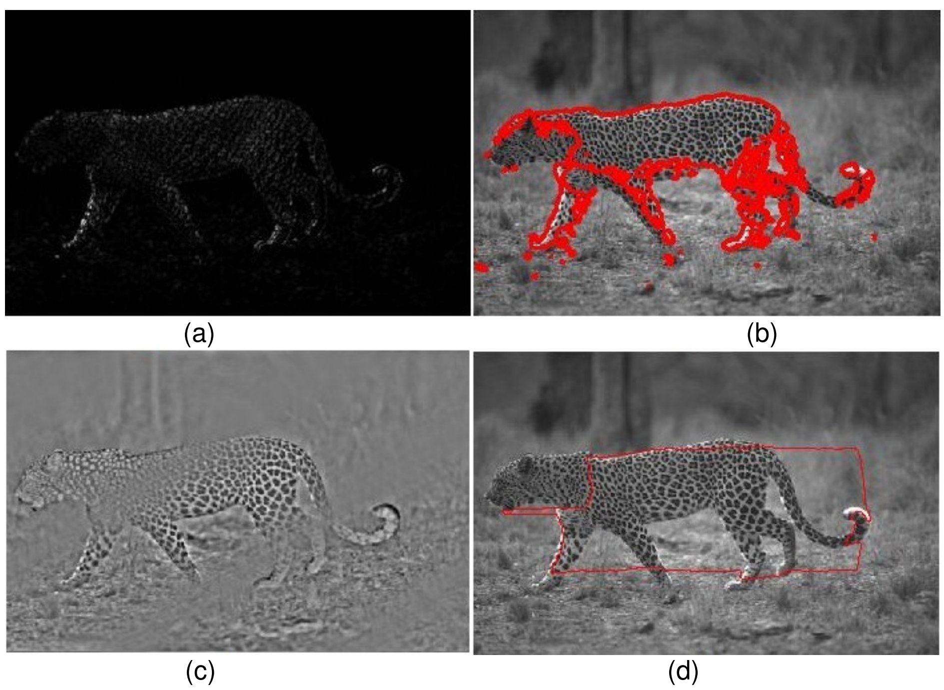


FIGURE 2: (a) LST of leopard image (b) Corresponding CV based segmentation (c) Phase map from Quadrature filter (d) Corresponding CV based segmentation.

Similarly in the tiger image (figure 3) due to inhomogeneity, the boundary is not easily distinguishable, while using tensor oriented Quadrature filter it can be enhanced but the presence of texture region cause failure in segmentation using the CV model. At the same time, LST removes the texture region but it is not able to enhance the boundary due to intensity inhomogeneity (figure 3 b). Using combine approach this part got enhanced (figure 3 c) and make proposed model to segment it accurately (see figure 7).

(a) (b) (c)

FIGURE 3: (a) Tensor oriented Quadrature filter output (b) LST oriented output (c) using combine approach as suggested in the proposed model.

In monkey image (figure 4), lower left (white shaded region) part is also segmented by region based model. But combined approach segments it perfectly. In last synthetic images are shown to highlight the capability of the proposed model to segment images with multiple texture patterns and/or images with multiple objects with similar texture patterns.

(a) (b)

FIGURE 4: CV Model based Segmentation (a) Original Image (b) Segmented Image.

There are so many algorithms are proposed to segment the texture images using region based active contour model. Some of them are [10, 18, 11, 9, 15, and 12]. All these models utilized either Gabor based approach or histogram method to extract the texture region and accordingly the CV model is applied to segment the images. However the Gabor based approach [15] requires too many features channels and induces lots of redundancies. Another utilized method is based on histogram of an image is Bhattacharya distance based ACM [12]. In [12], probability density inside the contour and outside the contour are estimated. The distance between this two probability densities is calculated using the Bhattacharya coefficients given by

$$B(\phi(x)) = \int \sqrt{P_{in}(z \mid \phi(x)) P_{out}(z) \mid \phi(x)} dz \tag{18}$$

In this model, the authors utilized the color histogram which is computationally hard. Therefore an independent component analysis (ICA) was utilized to reduce the feature space. Finally isotropic diffusion function was utilized to minimize the energy function in the active contour model. Some of successful results obtained using this model is shown in figure 5 below.

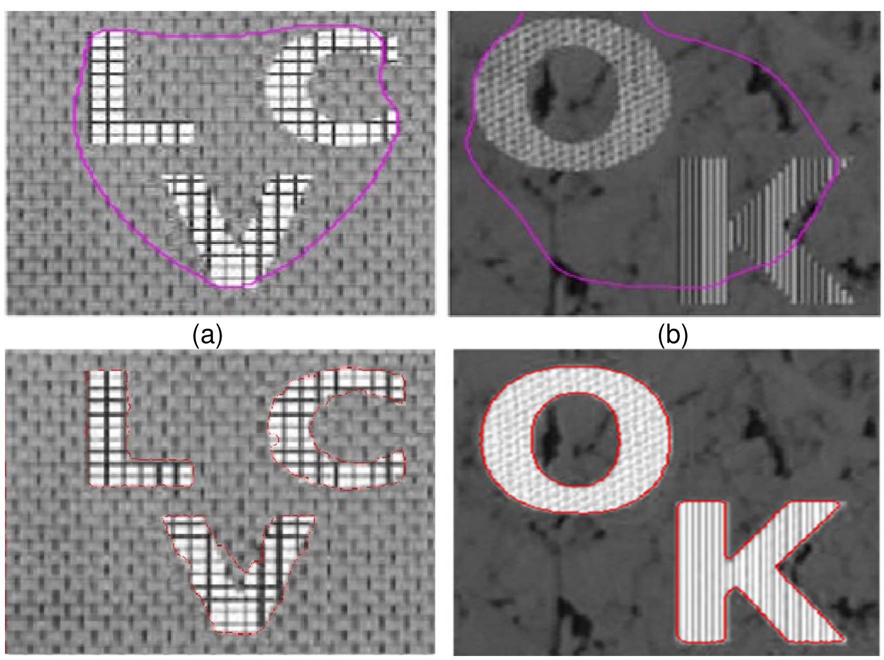
However the Bhattacharya model has some limitations. First one is not able to segment complex geometries as it utilizes the anisotropic diffusion function. In figure 5 (b), it can be seen that the tail part of tiger image is not segmented properly due to its complex geometries. A second limitation is not able to segment images with intensity inhomogeneity. Thus the Bhattacharya based ACM fails to segment the image shown in the first row of figure 7.

(a) (b) (c)

FIGURE 5: Segmentation Results Obtained using Bhattacharya Gradient Flow [12].

Third limitation is not able to segment multiple objects (figure 6) as splitting of contour is not supported in this model. Figure 6 shows the images having multiple object. In figure 6 “LCV” is the image containing a similar texture pattern inside the object and in the background with different scale while in the image “OK” both the foreground and the background have different texture pattern. Thus figure 6 shows that in comparison to the Bhattacharya model [12] , the proposed model is able to segment multiple objects too.

Figure 7 shows the segmentation results obtained while applying the proposed model on the texture images having above said complexities. The ground truth images are also shown to compare the obtained segmentation results visually.



(a) (b)

(c) (d)

FIGURE 6: (a) and (b) Failer of the Bhattacharya based Gradient flow model [12] to segment multiple object (c) and (d) successful segmentation using the proposed model

4.1 Quantitative Analysis

In this subsection we also prove the accuracy and efficiency of the proposed model analytically. We generated ground truth using ImageJ tool for the images not available in the Berkely image database. Then we computed the Dice similarity coefficient (DSC) which represents the spatial

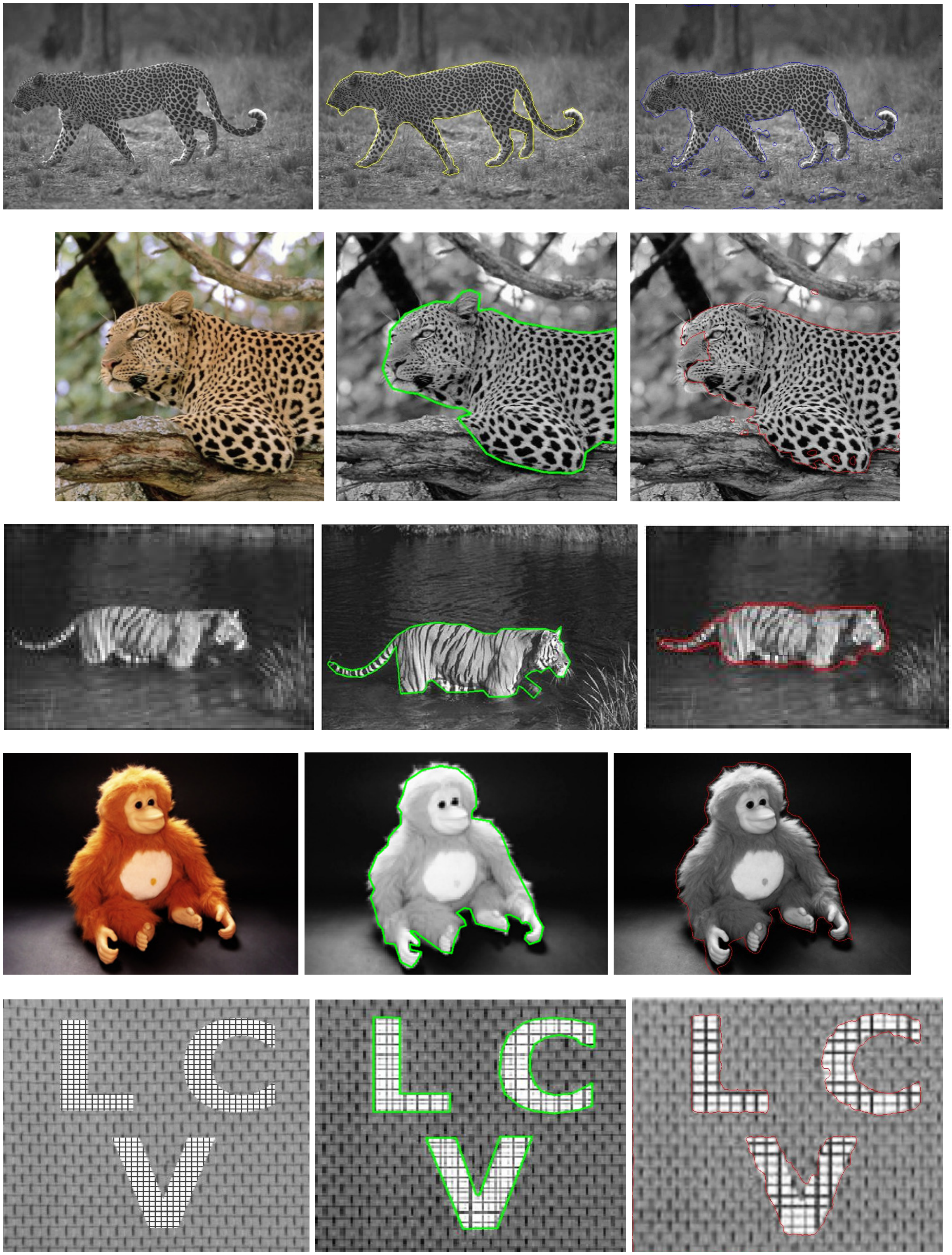


FIGURE 7: Left: Original Image, Middle: Ground truth image and Right: Segmented output using the proposed model.

overlap between the segmented image and ground truth image. Figure 8 shows the two sets: Resultant set (segmented image) and truth set (ground truth image). Where true positive represents the number of pixels detected correctly. False positive represents the number of pixels in segmented image which are actually not a part of the ground truth image, and false negative represents the number of pixels which are actually part of the ground truth image but not detected in segmentation result using the proposed model.

Using these, DSC can be defined as

$$DSC = \frac{2TP}{(FP+TP)+(TP+FN)} \tag{19}$$

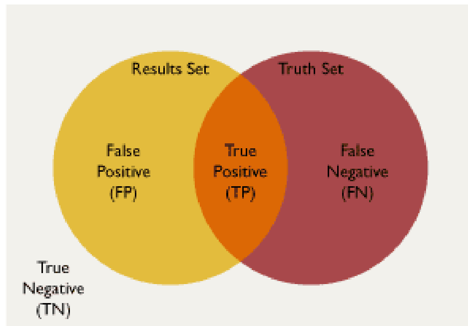


FIGURE 8: Dice Similarity Coefficients Calculation Parameter.

Thus for accurate segmentation (more similarity between the ground truth image and segmented image using the proposed model), DSC should be towards to one. The DSC for figure 7 are presented in table 1. In table 1, NA represents that the model is not able to segment image and therefore corresponding DSC value is not available. For the image 'tiger_bhatta" (second row of figure 7), both the models have approximately equal accuracy, but visually Bhattacharya based model have better segmentation in comparison to the proposed model.

Image	DSC using the proposed model	DSC using Bhattacharya model [12]
Leopard	0.95	NA
Tiger	0.96	0.92
Leopard_bhatta	0.96	0.97
Monkey	0.99	0.90
LCV	0.96	NA
OK	0.99	NA

TABLE 1 TPR (%) Values of the Proposed Model and [12] On Above Shown Test Images.

5. CONCLUSION AND FUTURE WORK

The major limitation of region based active contour model is not able to segment images having texture region and intensity inhomogeneity. To obtain the texture features LST is widely utilized method having the advantage of discrimination of texture pattern. The limitation of LST is containing horizontal and vertical direction information only. At the same time, to tackle the problem of intensity inhomogeneity, tensor oriented Quadrature filter is the one of the best solution as it utilized the intensity invariant local phase features. Therefore they are widely utilized for medical images and limitation is not able to segment the texture image.

Therefore this paper proposed an approach to integrate the advantages of both LST and tensor oriented Quadrature filter which overcome the limitation of each individual method. This integrated approach is utilized to overcome the limitation of the region based CV model. In the proposed model, Eigen values are calculated from the complex output of the tensor oriented quadrature filter. The Eigenvalues represent the intensity invariant local phase map features.

Then a horizontal and vertical gradients in LST are replaced by these Eigenvalues, which contain all directional information. Thus the integration of Eigenvalues in LST overcomes the limitation of two directions information of texture pattern only. Finally this modified local structure tensor based image is utilized in the region based active contour model for the segmentation purpose. The promising results shown in paper conclude that the proposed model removes the limitation of CV model and make it able to segment the images having intensity inhomogeneity and the texture images too. The results are tested for a number of images and comparison is established with the Bhattacharya based Gradient decent flow model. The quantitative analysis also concludes that the proposed model achieves approximately 100% accuracy.

Still model is sensitive to the initialization of the contour of the image domain and different initialization may lead to different results due to non convex nature of active contour. Thus the model can be improved in which is insensitive to the initialization. Again in the proposed model, instead of Quadrature filter, the Cauchy filter [3] can be utilized to improve the accuracy of segmentation.

6. REFERENCES

[1] J. Big Aijn, G. H. Granlund, and J. Wiklund. “Multidimensional orientation estimation with applications to texture analysis and optical flow” IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol.13(8), pp.775 -790, 1991.

[2] T. Brox. “From pixels to regions: Partial differential equations in image analysis”, PhD Thesis, Mathematical Image Analysis Group, Department of Mathematics and Computer Science Saarland University, Germany, 2005.

[3] T. Brox, J. Weickert, B. Burgeth, and P. MrAazek. “Nonlinear structure tensors. Image and Vision Computing”, Vol.24(1), pp.41-55, 2006.

[4] R. Estepar. “Local structure tensor for multidimensional signal processing: Application to medical image analysis”, Ph D Thesis, universitaires de Louvain, 2007.

[5] C. Feddern, J. Weickert, and B. Burgeth. “Level-set methods for tensor valued images”, Proc. Second IEEE Workshop on Variational, Geometric and Level Set Methods in Computer Vision,pp. 65-72, 2003.

[6] G. H. Granlund. “In search of a general picture processing operator”. Computer Graphics and Image Processing, Vol.8(2), pp.155-173, 1978.

[7] H. Knutsson and M. Andersson. “Loglets:Generalized Quadrature and phase for local spatio-temporal structure estimation”. 13th Scandinavian Conference, SCIA-2003 Halmstad, Sweden, July 2003, pp.741 -748.

[8] C. Li, C. Kao, J. Gore, and Z. Ding. “Implicit active contour driven by local binary fitting energy”. IEEE Conf on Computer Vision and Pattern Recognition,2007, pp. 1-7.

[9] S. Li, J. T. Kwok, H. Zhu, and Y. Wang. Texture classification using the support vector machines. Pattern Recognition, Vol.36(12), pp.2883 - 2893, 2003.

[10] A. Lorette, X. Descombes, and J. Zerubia. “Texture analysis through a markovian modelling and fuzzy classification: application to urban area extraction from satellite images”. International Journal of Computer Vision, Vol. 36(5), pp. 221-236, 2002.

[11] H. Lu, Y. Liu, Z. Sun, and Y. Chen. “An active contours method based on intensity and reduced Gabor features for texture segmentation”. Intelligent Control and Information Processing (ICICIP),pp. 1369 -137, Nov 2009.

[12] O. Michailovich, Y. Rathi and Tannenbaum. “Image segmentation using active contours driven by the Bhattacharyya gradient flow”. IEEE Transactions On Image Processing, Vol.16(11), pp.2787 -2801, 2007.

[13] J. Ning, L. Zhang, D. Zhang, and C. Wu. “Interactive image segmentation by maximal similarity based region merging”. Journal of Pattern Recognition, Vol.43(11), pp. 445-456, 2010.

[14] S. Osher and J. Sethian. “Fronts propagating with curvature-dependent speed: algorithms based on hamilton-jacobi formulations”. Journal of Computational Physics,Vol. 79, pp. 12-49, 1988.

[15] B. Sandberg, T. Chan, and L. Vese. “A level-set and Gabor-based active contour algorithm for segmenting textured images”. Technical Report 39, Mathematical Department, UCLA, Los Angeles, 2002.

[16] T.Chan and L. Vese. “Active contour without edges”. IEEE Transactions on Image Processing, Vol.10(2), pp. 266 -277, 2001.

[17] Y. Wang, Y. Xiong, L. Lv, H. Zhang, Z. Cao, and D. Zhang. “Vector-valued chan-vese model driven by local histogram for texture segmentation”. 17th IEEE International Conference on Image Processing (ICIP), , Sept 2010,pp.645 -648.

[18] D. Yang, T. Deng, C. Yang, and J. Bian. “Interactive graph cut method based on improved Gabor features for image segmentation”. Intelligent Control and Information Processing (ICICIP), Vol.1(2), pp.267 - 270, July 2011.

[19] Kangyu Ni, Xavier Bresson, Tony Chan and Selim Esedoglu, “Local histogram based segmentation using the Wasserstein distance”, Internation Journal of Computer Vision, Vol. 84, pp. 97-111, April 2009.

[20] C.C. Reyes-Aldasoroa,A. Bhalerao, “The Bhattacharyya space for feature selection and its application to texture segmentation” Internation Jounral of Pattern Recognition Vol .39 pp. 812 – 826, 2006