Performance Comparison of Hybrid Haar Wavelet Transform with Various Local Transforms in Image Compression using Different Error Metrics

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Abstract

A novel image compression using hybrid Haar wavelet transform has been proposed in this paper. Hybrid wavelet transform is generated using two different orthogonal transforms. Haar transform acts as a base transform and other sinusoidal transforms like DCT, DST, Hartley and Real-DFT are paired with Haar transform to generate hybrid Haar wavelet. Among these four pairs Haar-DCT hybrid wavelet gives lower error as compared to Haar-DST, Haar-Hartley and Haar-Real-DFT. Performance of Haar-DCT hybrid wavelet is further analyzed using multi resolution hybrid wavelet and Haar-DCT hybrid transform. Experimental results show that hybrid wavelet with component size 16-16 gives lower error at higher compression ratios than multi resolution analysis and hybrid transform performance. Performance is measured using RMSE which is traditional parameter to measure error. Lowest RMSE obtained is 9.77 at compression ratio 32 using Haar-DCT Hybrid Wavelet with component size 16-16. Various other error metrics like MAE, AFCPV and SSIM are used to measure error. Lowest MAE and AFCPV are observed at compression ratio 32 are in Haar (16x16) –DCT (16x16) hybrid wavelet having values 6.86 and 0.31 respectively. When blocked SSIM is applied on 16-16 Haar-DCT hybrid wavelet it gives value 0.993 at compression ratio 32 which is closer to one indicating that good quality of compressed image is obtained.

Keywords: Hybrid Transform, Haar Wavelet, SSIM, MAE, Image Compression.

1. INTRODUCTION

Digital images are inevitable part of today's multimedia world. Downloading and transmitting the images using internet consume considerable amount of time and bandwidth. Hence if these images are compressed and used, it will help to save time and bandwidth required for transmission. Image compression deals with reducing the number of bits required to store and transmit the image. It eliminates redundant information in image by retaining important information and intrinsic structure of the original image such that quality of compressed image is acceptable to human visual system. Wavelets have gained immense popularity in image compression during last two-three decades due to their high energy compaction property and multi-resolution analysis [1]. Basically wavelets are mathematical functions that represent time-frequency analysis of the data. Selecting a wavelet prototype function or mother wavelet is an essential step in wavelet based analysis. Other wavelets are produced by translation and contraction of mother wavelet. These generated wavelet functions must be orthogonal which will comprise wavelet transform. Haar transform is a simple, orthonormal transform proposed by Alfred Haar in 1910 [2]. It serves as a prototype for wavelet transform. [3]. Many wavelet based image compression techniques have been used till now including Haar wavelet. Commonly used error metrics to judge the performance of compression methods are Mean Square Error (MSE) and Peak Signal to noise ratio (PSNR). This paper proposes hybrid Haar wavelet transform and compares its performance with its multi-resolution hybrid wavelet transform and hybrid transform using various error metrics like root mean square error (RMSE), mean absolute error (MAE), average fractional change in pixel value (AFCPV) and structural similarity index (SSIM).

2. REVIEW OF LITERATURE

A lot of research has been done on wavelet based image compression. Image compression using sparse Haar wavelet has been proposed by R. Mehala and Kuppusamy [***4***]. In this paper they have used 8x8 Haar matrix by inserting appropriate zeroes and ½ in Haar wavelet and it is applied on blocked gray scale image. Performance is measured using compression ratio and PSNR. But this method is applicable only on low intensity image. Haar wavelet and neural network based image compression is proposed by S. Shridhar et al. [5]. Using Haar wavelet image is decomposed into different frequency sub bands and then scalar quantization and Huffman coding are used for compression of different sub bands***.*** The coefficients in low frequency band are compressed by Differential Pulse Code Modulation (DPCM) and the coefficients in higher frequency bands are compressed using neural networks. Complexity of this method is high and it measures performance using traditional error parameter i.e. MSE. Wavelet based extension of JPEG 2000 standard is proposed by Singh and Sharma [6]. In this paper first level wavelet decomposes the image only in vertical direction and subsequent wavelet levels use full horizontal and vertical splitting for all image components. But performance of this method degrades when images are with low colour depth. Medical image compression using Haar wavelet, Daubechies wavelet and Coeflit wavelet has been proposed by Krishna Kumar et al. [7] Where performance has been measured using PSNR and SSIM. Different wavelets perform better for different kind of medical images and SSIM up to 0.7 is obtained for ECG images. Singular value decomposition combined with linear and quadratic interpolation has been proposed by J Hizadian, A Hosaini and M Jalili [8]. But this method is time consuming. A simple wavelet transform based image compression is proposed in [9] by H. B. Kekre, Tanuja Sarode and Prachi Natu. In this paper performance of full wavelet transform is compared with respective column wavelet and row wavelet transform. To save number of computations, column wavelet transform can be used with slight increase in error in reconstructed image. Hybrid wavelet based compression using DCT with RealDFT has been proposed in [10]. Hybridisation helps to incorporate properties of both transforms used to generate hybrid wavelet transform and hence error reduces drastically as compared to orthogonal transform and orthogonal wavelet transform.

In this paper a simpler method of image compression using Hybrid Haar wavelet has been proposed. Sinusoidal transforms like DCT, DST, Real DFT and Hartley are combined with Haar transform to generate hybrid Haar wavelet. This transformation matrix is generated in three different ways to study the effect of global, local and semi global properties on image compression. Performance of proposed method is measured using various fidelity criteria like RMSE, MAE, AFCPV and SSIM.

3. HAAR TRANSFORM

Haar transform is the simplest and basic transform used in image processing. It is faster to implement and helps to analyze local features of a signal. Haar transform uses Haar function as its basis function which varies in both scale and position. 8x8 Haar transform matrix is given below. It contains only real elements 1,-1 and 0.

$$H[m,n] = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \quad (1)$$

Haar functions for 8x8 Haar matrix are shown in Fig. 1

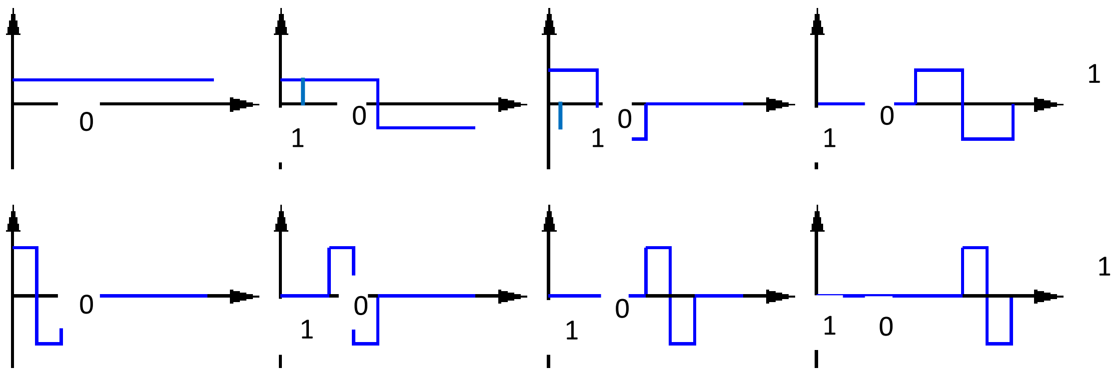


FIGURE 1: Haar functions of 8x8 Haar matrix.

4. FIDELITY CRITERIA

4.1 Root Mean Square Error

It is commonly used fidelity criteria to measure the distortion between original image and reconstructed image. Mathematically it is calculated as:

$$\text{RMSE}=\sqrt{\frac{\sum_{i=1}^{i=p}\sum_{j=1}^{j=q}(x_{ij}-y_{ij})^2}{p*q}} \quad (2)$$

But it is not very well matched to perceived quality [11]. It is calculated with assumption that loss of perceptual quality is directly related to visibility of error signal.

4.2 Mean Absolute Error

It is average of absolute difference in pixel values. It gives better perceptibility than RMSE.

$$\text{MAE}=\frac{\sum_{i=1}^{i=p}\sum_{j=1}^{j=q}(|x_{ij}-y_{ij}|)}{p*q} \quad (3)$$

4.3 Average Fractional Change in Pixel Value

It represents the fractional change in pixel values and hence reflects perceptibility in better way.

$$\text{AFCPV}=\frac{\sum_{i=1}^{i=p}\sum_{j=1}^{j=q}(|x_{ij}-y_{ij}|)/x_{ij}}{p*q} \quad (4)$$

4.4 Structural Similarity Index (SSIM) [11]

Concept of SSIM is based on the assumption that human visual system is highly adapted for extracting structural information from still image and hence measure of structural similarity can provide a good approximation to perceived image quality. SSIM considers image degradation as perceived change in structural information. Structural information is the idea that the pixels have strong inter-dependencies especially when they are spatially close. These dependencies carry important information about the structure of the objects in the visual scene. SSIM is calculated as

$$SSIM\ (x, y) = (2\mu_x\mu_y+c_1)\ (2\sigma_{xy}+c_2)\ /\ (\mu_x^2+\mu_y^2+c_1)\ (\sigma_x^2+\sigma_y^2+c_2) \qquad (5)$$

5. PROPOSED METHOD

5.1 Hybrid Wavelet Transform [12]

In this paper hybrid Haar wavelet has been proposed using Kekre's algorithm to generate hybrid wavelet transform [12]. Haar transform is combined with sinusoidal transforms like DCT, DST, Hartley and Real-DFT. All these sinusoidal transforms are selected as local components of hybrid wavelet transform. Use of hybrid wavelet transform helps to incorporate traits of both component transforms. It is generated using following Kronecker product.

$$T_{AB} = \begin{pmatrix} A_p \otimes B_q(1) \\ I_p \otimes B_q\ (2) \\ I_p \otimes B_q\ (3) \\ . \\ . \\ . \\ . \\ I_p \otimes B_q\ (n) \end{pmatrix} \qquad (6)$$

Here 'A' is pxp 'base' transform and 'B' is qxq 'local' transform used to generate pqxpq hybrid wavelet transform denoted as T_{AB}. Size of component transforms p and q are selected such that size of T_{AB} is same as image size. B_q (1) indicates first row of matrix B whose Kronecker product with A is taken. It generates first p rows of transform matrix representing global features of an image. Kronecker product of Identity matrix 'I' of size pxp and each row of matrix B is used to translate the rows of matrix B which will contribute to local properties.

5.2 Multi-resolution Hybrid Wavelet [*13*]

Above generated hybrid wavelet matrix gives global and local features of an image hence give bi-resolution analysis. It is modified as below to include semi global features of an image.

$$T_{AB} = \begin{pmatrix} A_p \otimes B_q\,(0:i_1) \\ I_{r0} \otimes (A_{p/r0} \otimes B_q\,(i_1+1:i_2)) \\ I_{r1} \otimes (\,A_{p/r1} \otimes B_q\,(i_2+1:i_3)) \\ \vdots \\ I_{rn-1} \otimes (\,A_{p/rn-1} \otimes B_q\,(i_{n-2}+1:i_{n-1})) \\ I_p \otimes (B_q(i_n:q)) \end{pmatrix} \begin{matrix} \text{Global} \\ \text{Semi global 1} \\ \text{Semi global 2} \\ \vdots \\ \text{Semi global n} \\ \text{Local} \end{matrix} \qquad (7)$$

In above Kronecker product A is pxp and B is qxq orthogonal transforms. Bq(i:j) represents i to j rows of transform B. Lower order matrix A of size p/r_n is generated where r_n is divisor of p except 1 and p itself. Lower order matrix A is used for scaling operation and Identity matrix is used for shifting operation in generation of hybrid wavelet transform.

5.3 Hybrid Transform

It is the limiting case of hybrid wavelet transform obtained by full Kronecker product of two component transforms.

It is given as $$A \otimes B = a_{ij}\,[B] \qquad (8)$$

Where, a_{ij} is individual element of matrix A.

Generated hybrid wavelet transform is applied on each plane of color image. Coefficients in transformed plane are sorted in descending order of energy. Low energy coefficients are discarded. Image is reconstructed by applying inverse transform on transformed planes with retained high energy coefficients. Distortion between original and reconstructed image is measured using various fidelity criteria mentioned above. Performance of different hybrid Haar wavelets is compared using experimental database and required graphs are plotted below.

6. EXPERIMENTS AND RESULTS

Dataset used for experimental work is shown in Fig. 2. In contains colour images from different classes. All images are of 256x256x3 bytes.

FIGURE 2: Dataset of Color Images used for Experimental Work.

Fig. 3 shows plot of RMSE vs. compression ratio using hybrid Haar wavelet with different sinusoidal transforms. It shows that Haar-DCT hybrid wavelet gives less RMSE for all compression ratios. RMSE 9.91 is obtained at highest compression ratio 32. Performance of Haar-DCT is followed by Haar-RealDFT. DST combined with Haar gives higher error among all.

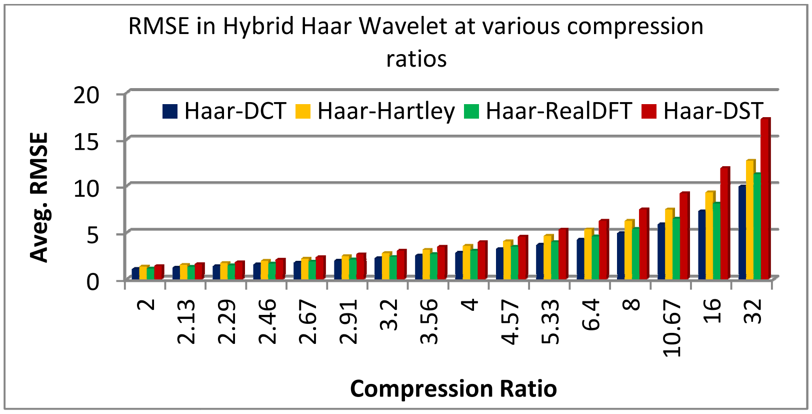


FIGURE 3: Average RMSE vs. Compression Ratio in Hybrid Wavelet using Haar Transform (8x8) with different Sinusoidal transforms (32x32).

As Haar-DCT gives less error, further it is analyzed to obtain the best component size combination of Haar and DCT that gives lowest error. Four combinations of component transforms are selected and RMSE in each of them is compared as shown in Fig. 4. Up to compression ratio 10.67, 16-16 and 32-8 size give nearly equal error. Slight increase in error is observed for size 32-8 at compression ratios 16 and 32. Thus 16-16 becomes better combination at higher compression ratios.

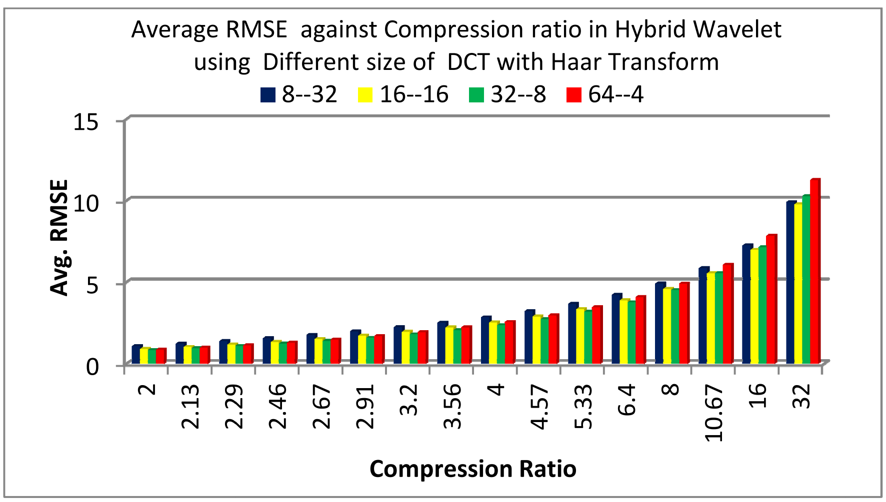


FIGURE 4: Comparison of RMSE in Haar-DCT hybrid wavelet using different component transform size.

Fig. 5 shows comparison of RMSE in multi-resolution hybrid wavelet. Here also Haar-DCT proves to be better giving less error. At compression ratio 32, RMSE 10.13 is obtained with component size 8-32. It is slightly higher than error in Haar-DCT hybrid wavelet shown in Fig. 3

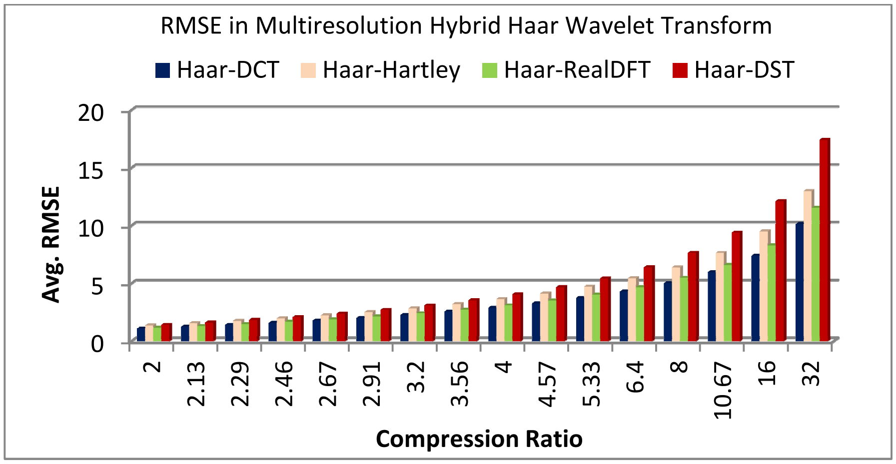


FIGURE 5: Avg. RMSE vs. Compression ratio in Multi-resolution using Haar (8x8) and sinusoidal component transforms of size 32x32.

For multi-resolution analysis, four different sized pairs of Haar-DCT are tried and their error is plotted against compression ratio in Fig. 6. Pair of size 8-32 gives lower RMSE than all other pairs at compression ratio 32. At lower compression ratios, difference in error is negligible.

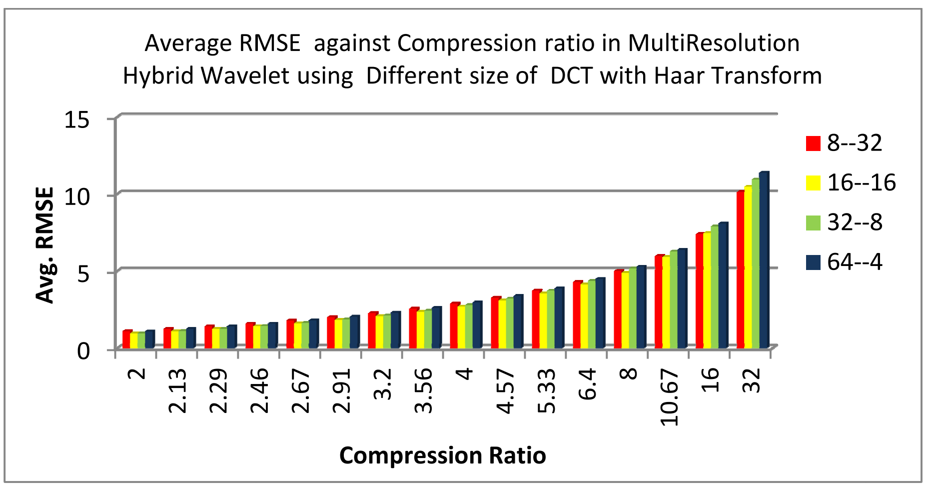


FIGURE 6: RMSE vs. Compression ratio for different component sizes of Haar-DCT in Multi-resolution analysis.

Fig. 7 shows RMSE obtained in Hybrid transform. In hybrid transform, which is obtained by full Kronecker product of two component matrices, all rows of transformation matrix represent global features of an image. Local and semi global features are absent in hybrid transform. In hybrid transform also Haar-DCT gives better performance in terms of RMSE.

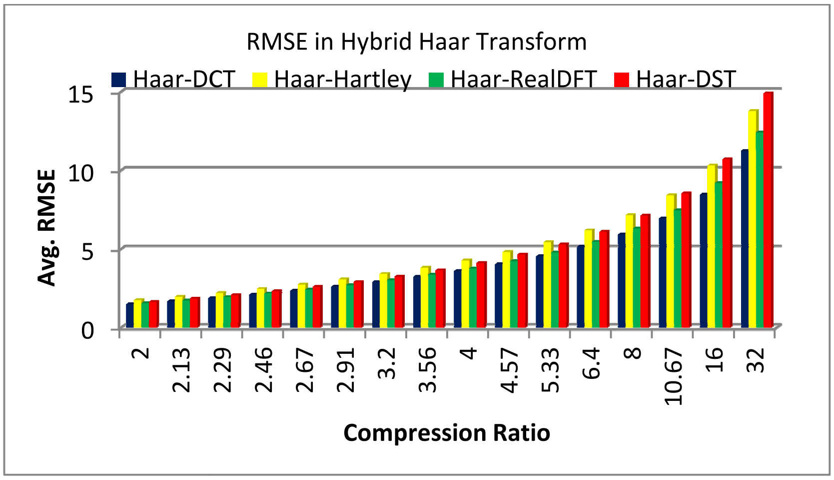


FIGURE 7: Avg. RMSE vs. Compression ratio in Hybrid transform using Haar transform (8x8) with different sinusoidal transforms (32x32).

Different size combinations of Haar-DCT hybrid transform are tried and error is plotted in Fig. 8. It shows that four different combinations give nearly equal error for selected compression ratio. Error increases with increase in compression ratio.

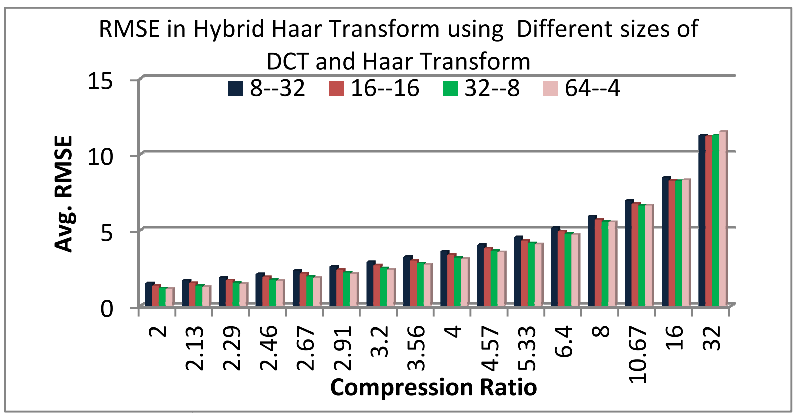


FIGURE 8: Average RMSE against Compression ratio in Hybrid Transform using different size of DCT with Haar Transform.

Fig. 9 shows overall comparison of RMSE in hybrid Haar wavelet, its multi resolution analysis and hybrid transform at various compression ratios. Up to compression ratio 8, hybrid wavelet with components Haar 32x32 and DCT 8x8 gives less error. Onwards, as compression ratio increases, Haar 16x16 and DCT 16x16 give less RMSE.

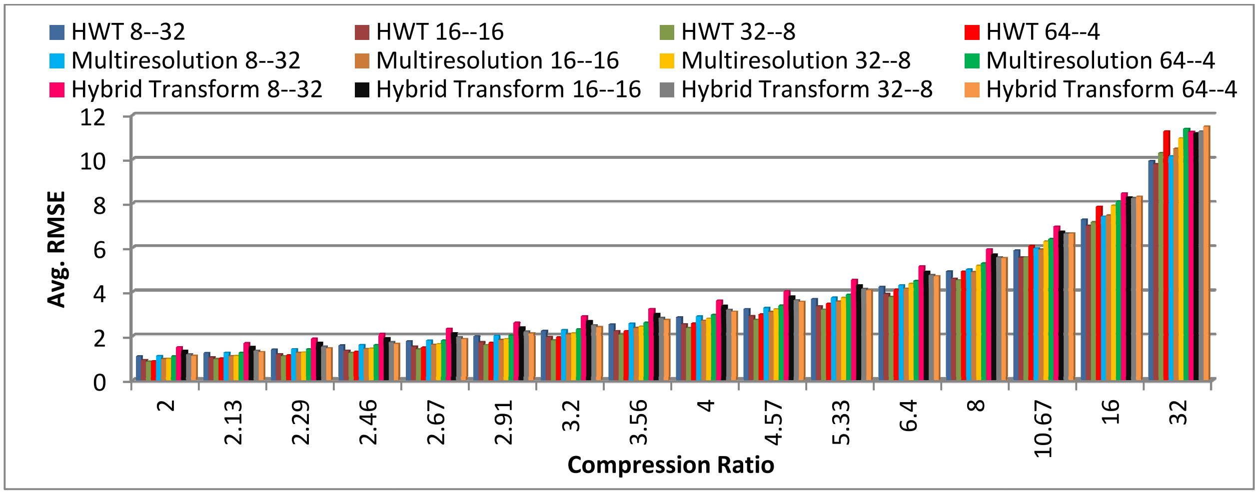


FIGURE 9: Average RMSE against compression ratio using different sizes of component transforms in Haar–DCT Hybrid Wavelet, Multi resolution analysis and Hybrid transform.

Fig. 10 shows performance comparison of Haar hybrid wavelet using MAE as error measurement criterion, Fig. 11 shows this comparison for multi resolution analysis and in Fig. 12 comparison of different hybrid transforms is done. Similar to RMSE, Haar-DCT gives better performance than other sinusoidal transforms in multi resolution as well as in Hybrid transform.

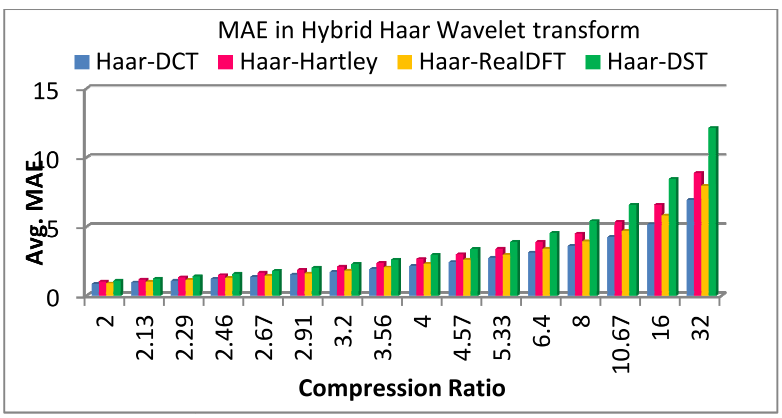


FIGURE 10: Avg. MAE against Compression Ratio in Hybrid Wavelet transform using different Sinusoidal transforms with Haar Transform.

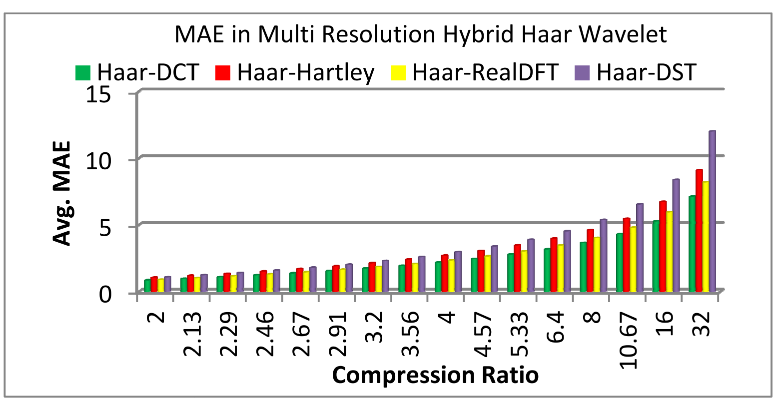


FIGURE 11: Avg. MAE against Compression ratio in Multi Resolution Hybrid Wavelet using different Sinusoidal transforms with Haar Transform.

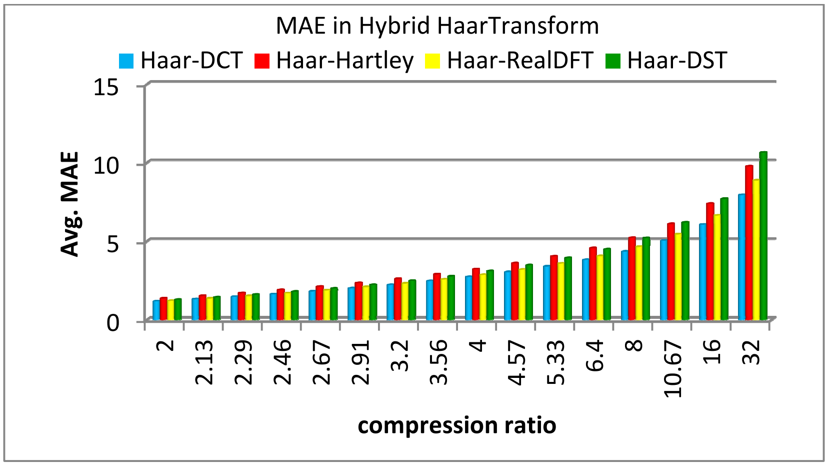


FIGURE 12: Avg. MAE vs. Compression ratio in Hybrid Transform using different Sinusoidal Transforms with Haar Transform.

Fig. 13, 14 and 15 show graph of MAE against compression ratio using various size combinations of Haar-DCT in Hybrid wavelet, its multi resolution and hybrid transform respectively. The best pair obtained in Hybrid wavelet is of size 16-16 and in Multi resolution analysis, it is of 8-32 for higher compression ratio 32.

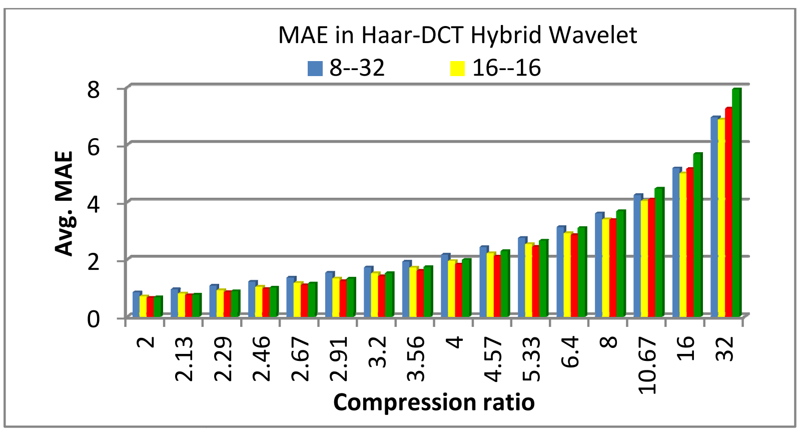


FIGURE 13: Avg. MAE against Compression ratio using Different size of Haar-DCT pairs in Hybrid Wavelet.

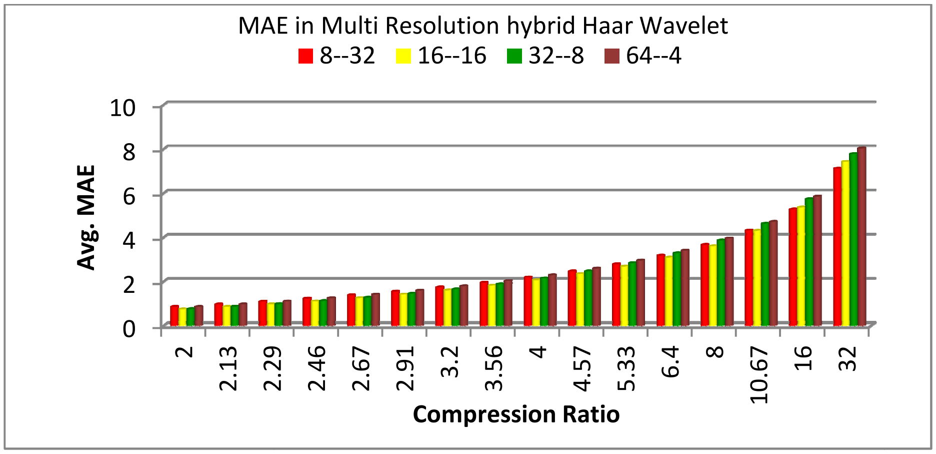


FIGURE 14: Avg. MAE in Multi Resolution hybrid Wavelet using different size variations in Haar-DCT.

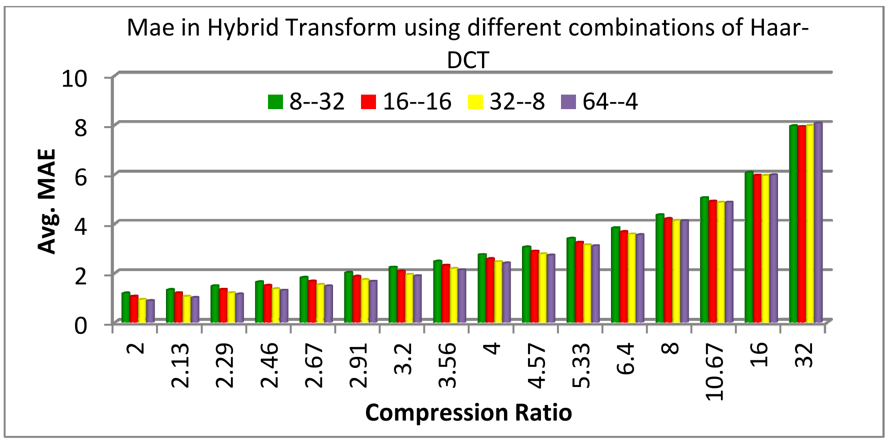


FIGURE 15: Avg. Mae vs. Compression ratio in Hybrid Transform using different combinations of Haar-DCT.

Fig. 16 shows overall comparison of MAE in Haar-DCT Hybrid Wavelet, Multi resolution analysis and hybrid Transform using various size combinations. It has been observed that Haar-DCT hybrid wavelet of size 16-16 gives lower MAE at higher compression ratio 32.

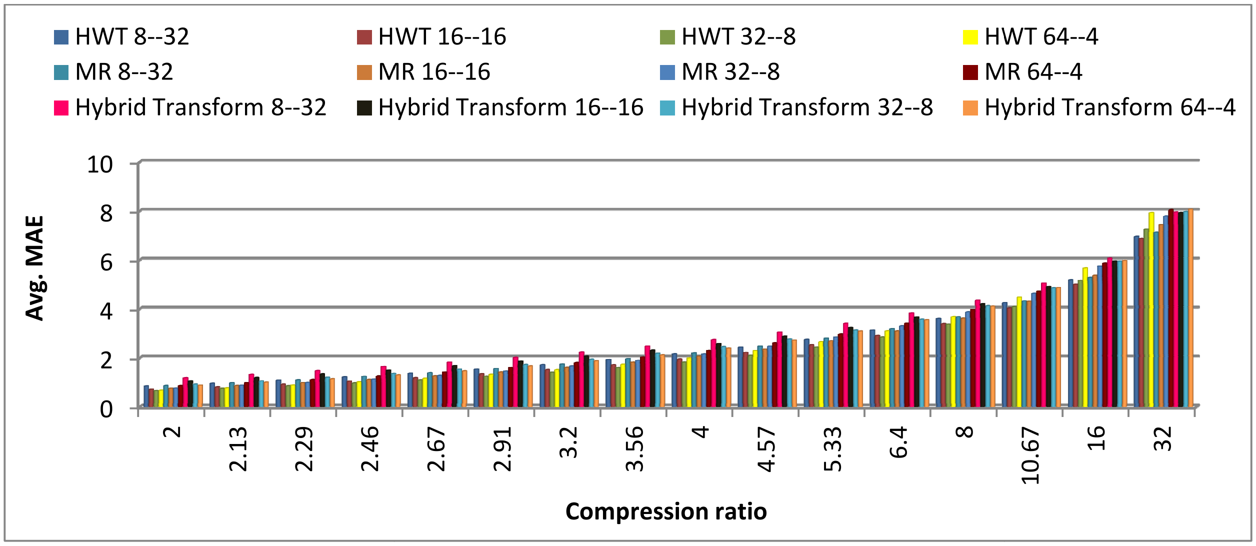


FIGURE 16: Average MAE against compression ratio using all possible sizes of Haar –DCT in Hybrid Wavelet, Multi resolution and hybrid transform.

Fig. 17, 18 and 19 show comparison of AFCPV in Haar hybrid wavelet, its multi resolution hybrid wavelet and hybrid transform respectively. AFCPV gives change in perceived value of a pixel. Lower the AFCPV value better is the image quality.

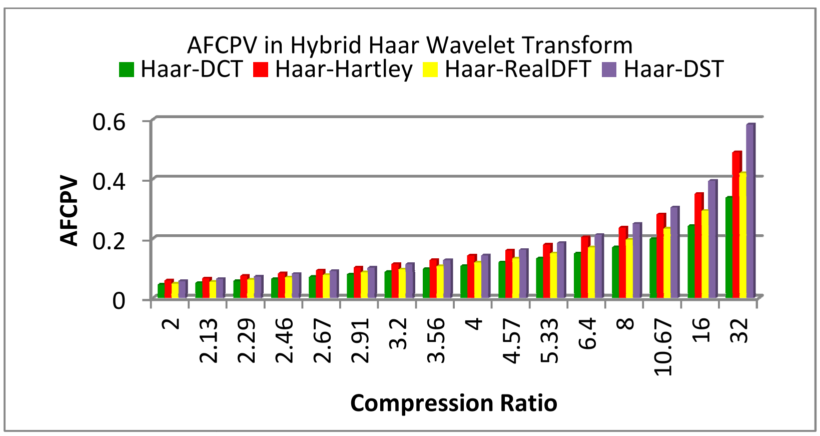


FIGURE 17: AFCPV vs. Compression ratio in Hybrid Wavelet using different Sinusoidal Transforms with Haar Transform.

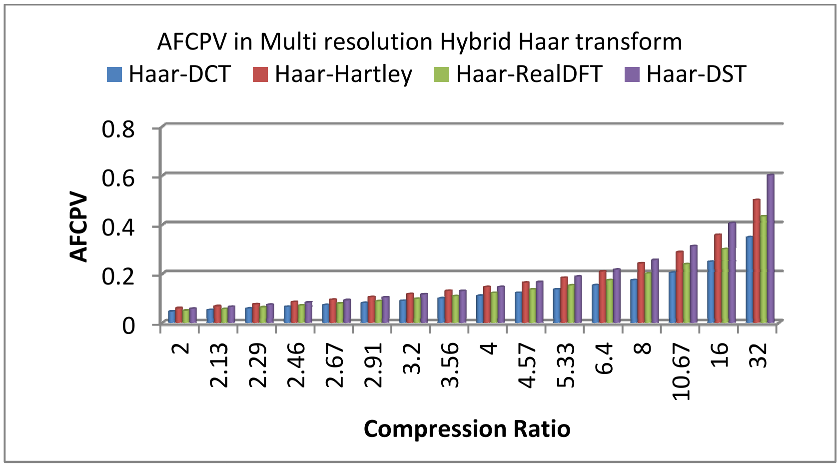


FIGURE 18: AFCPV vs. Compression Ratio in Multi resolution using different sinusoidal transforms with Haar transform.

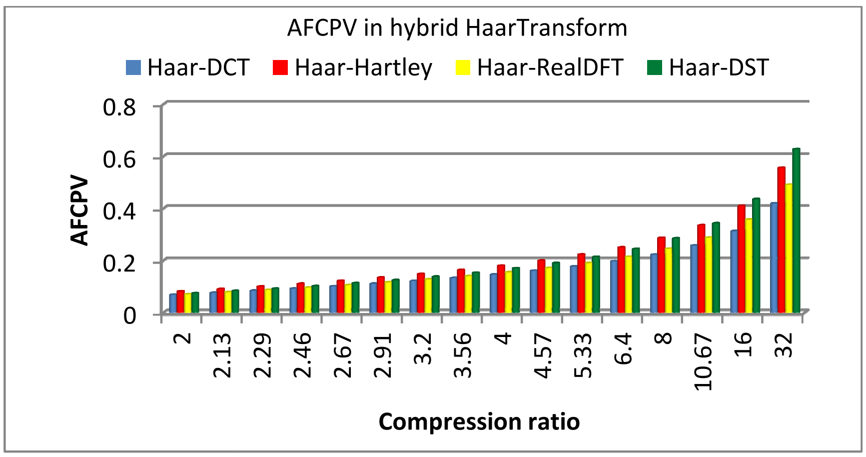


FIGURE 19: AFCPV against Compression ratio in hybrid Transform using different sinusoidal Transforms with Haar Transform.

Fig. 20, 21 and 22 show AFCPV in Haar-DCT using its Hybrid wavelet, Multi resolution analysis and hybrid transform with different component sizes. In hybrid wavelet, 32-8 pair of Haar-DCT gives low AFCPV. At compression ratio 32, Haar (32x32) and DCT (8x8) pair gives low AFCPV. In multiresolution 16-16 size gives better AFCPV up to compression ratio 16. Negligible difference is observed between AFCPV obtained using variations of component size. In Haar-DCT hybrid transform, 64-4 size gives lower value of AFCPV for all compression ratios.

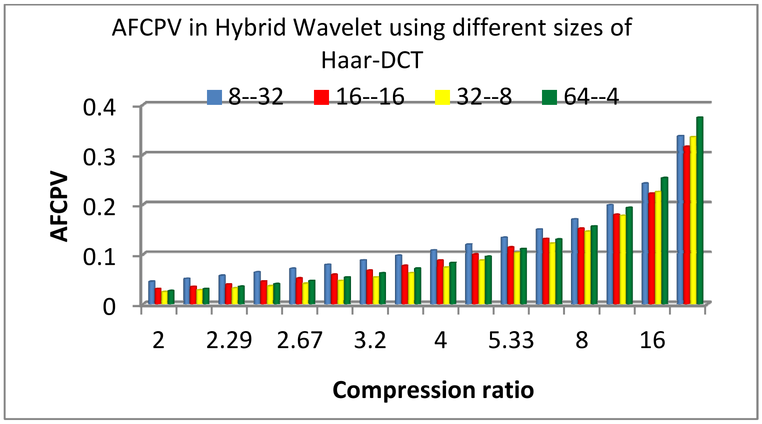


FIGURE 20: AFCPV vs. Compression Ratio in Hybrid Wavelet using different sizes of Haar-DCT.

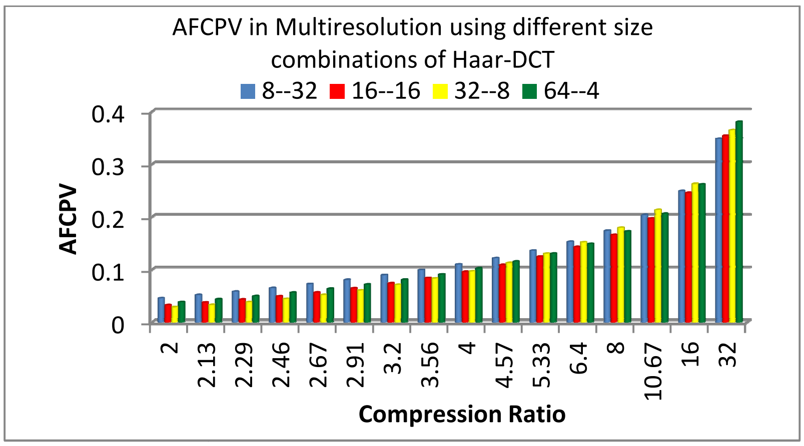


FIGURE 21: AFCPV vs. Compression Ratio in Multi resolution using different size combinations of Haar-DCT.

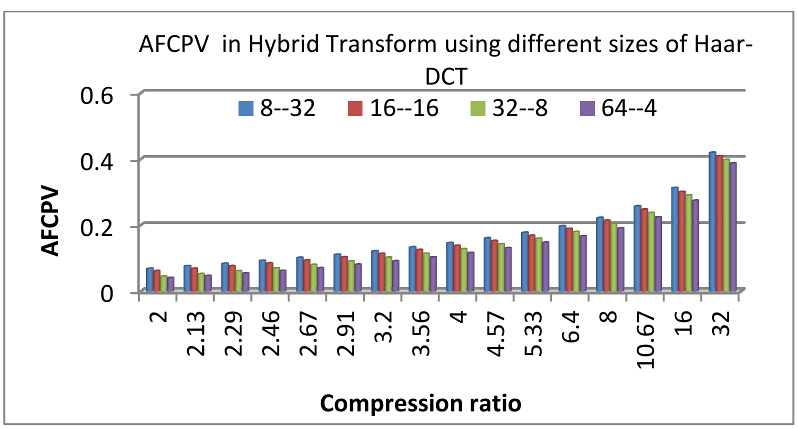


FIGURE 22: AFCPV vs. Compression Ratio in Hybrid Wavelet using different sizes of Haar-DCT.

Overall comparison of AFCPV in Haar-DCT hybrid wavelet, its multi resolution and hybrid transform is done in Fig 23. Hybrid wavelet again proves to be better for Haar 16x16 and DCT 16x16.

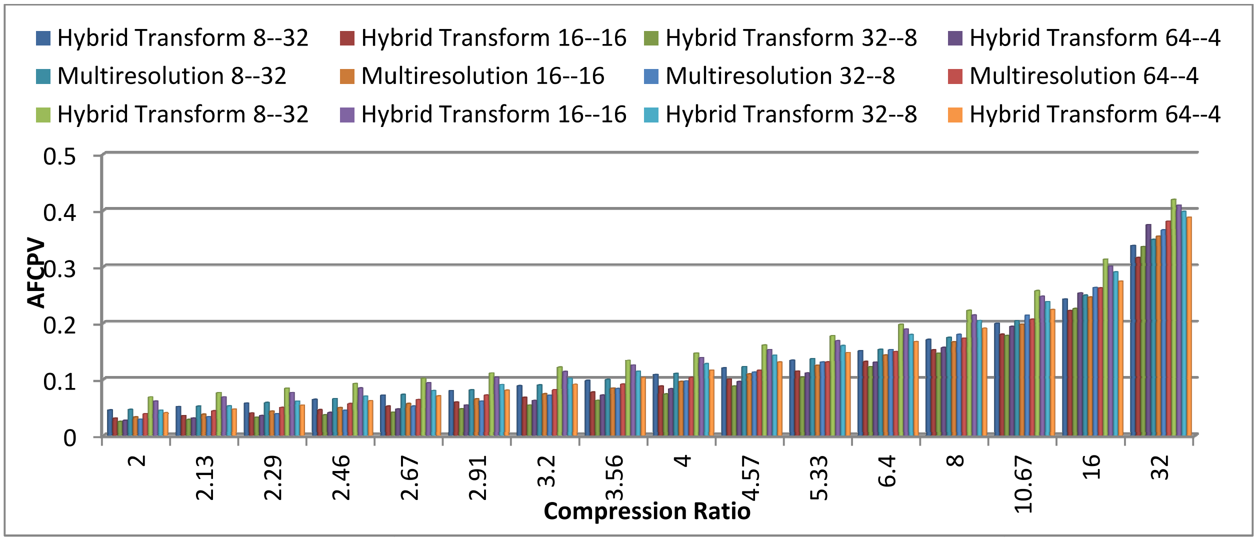


FIGURE 23: Comparison of AFCPV using various component transforms sizes in Haar-DCT Hybrid wavelet, Multi resolution hybrid wavelet and hybrid transform.

Fig. 24 shows Structural Similarity Index in Haar-DCT hybrid wavelet at various compression ratios for sample 'Lena' Image.

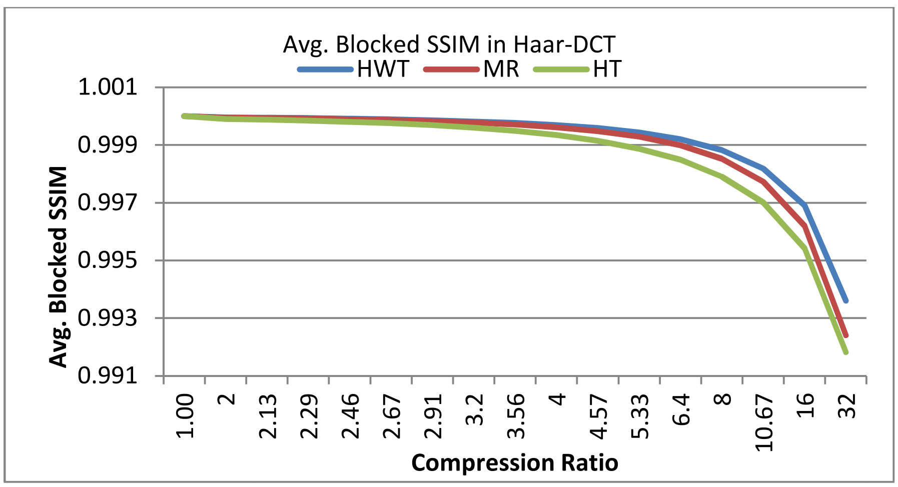


FIGURE 24: Average Blocked SSIM at different Compression Ratios in Haar-DCT hybrid Wavelet Transform, Its Multi resolution and Hybrid Transform with component size16-16 for 'Lena' image.

At compression ratio 1 i.e. when two images are exactly same, SSIM is one. As compression ratio increases, distortion in the reconstructed image increases and hence SSIM reduces. Image is divided into 16x16 blocks and SSIM of individual block is calculated to get more accuracy in perception of image to HVS. In hybrid wavelet SSIM is close to one at lower compression ratios and attains value 0.993 at compression ratio 32, indicating better perceived image quality. This objective measure of perceptibility is not obtained by traditional RMSE parameter. Multi resolution hybrid wavelet and hybrid transform show marginal difference in image quality than Hybrid wavelet in terms of SSIM.

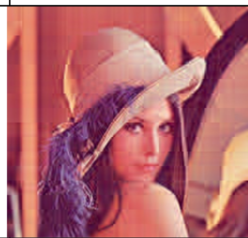
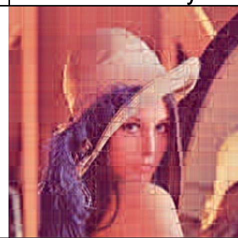
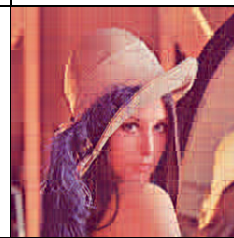
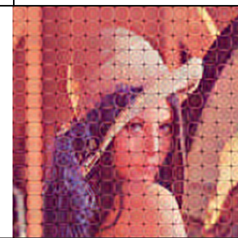
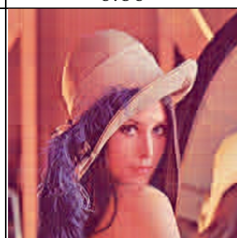
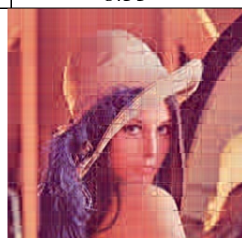
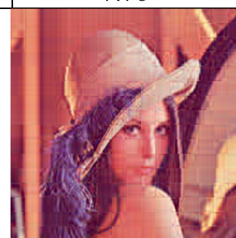
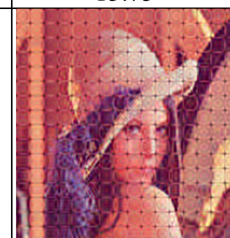
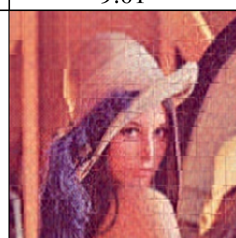
	Haar-DCT	Haar-Hartley	Haar-Real DFT	Haar-DST
Hybrid Wavelet				
MAE	6.86	8.55	7.76	13.75
Multi-resolution Hybrid Wavelet				
MAE	7.53	9.01	8.39	12.62
Hybrid Transform				
MAE	8.10	9.62	8.87	10.46

FIGURE 25: Reconstructed 'Lena' image at Compression ratio 32 using Hybrid Haar Wavelet, its Multi Resolution Analysis and Hybrid Transform with Haar 16x16 and different Local Component Transforms of Size 16x16.

Fig. 25 shows reconstructed Lena images at compression ratio 32 using different hybrid Haar wavelet transforms. Local component transform is varied keeping size of both components as 16x16. It shows that Haar-DCT pair gives lower MAE than other pairs. Also Hybrid wavelet transform gives less MAE than multi resolution analysis and hybrid transform. Haar-DST shows poor performance including grid effect in reconstructed image.

7. CONCLUSION

In this paper Hybrid Haar wavelet with bi resolution analysis, multi resolution analysis and with global features is implemented and compared. Sinusoidal transforms DCT, DST, Hartley and Real DFT are used as local component transforms and combined with Haar transform. Haar-DCT hybrid wavelet gives less error than Haar-DST, Haar-Hartley and Haar-Real DFT wavelet. In multi resolution analysis and hybrid transform also Haar-DCT gives less error. In Haar- DCT hybrid wavelet, size variation of component transforms is done to observe the changes in different fidelity criteria. AFCPV is used to measure fractional change in pixel values. It gives better perceptibility of image than traditional RMSE metric. Lowest AFCPV is obtained for Haar-DCT pair with size 16-16. SSIM is objective fidelity criteria used to approximate the perceived image quality. SSIM equal to 1 indicates that original and reconstructed images are exactly similar. Using hybrid Haar wavelet SSIM 0.993 is obtained at compression ratio 32. In multi resolution analysis and hybrid transform, values of SSIM are 0.992 and 0.991 for same compression ratio. Thus there is marginal difference in image quality when it is compressed using Hybrid wavelet, multi resolution and hybrid transform.

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