Simultaneous State and Actuator Fault Estimation With Fuzzy Descriptor PMID and PD Observers for Satellite Control Systems

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Abstract

In this paper, Takagi-Sugeno (T-S) fuzzy descriptor proportional multiple-integral derivative (PMID) and proportional derivative (PD) observer methods that can estimate the system states and actuator faults simultaneously are proposed. T-S fuzzy model is obtained by linearsing satellite/spacecraft attitude dynamics at suitable operating points. For fault estimation, actuator fault is introduced as state vector to develop augmented descriptor system and robust fuzzy PMID and PD observers are developed. Stability analysis is performed using Lyapunov direct method. The convergence conditions of state estimation error are formulated in the form of LMI (linear matrix inequality). Derivative gain, obtained using singular value decomposition of descriptor state matrix (E), gives more design degrees of freedom together with proportional and integral gains obtained from LMI. Simulation study is performed for our proposed methods.

Keywords: Fault, Descriptor Systems, Estimation, Fuzzy Model, Observers, Robustness, Linear Matrix Inequality, Quadratic Stability.

1. INTRODUCTION

A fault is termed as an unexpected change in the system's behavior that deteriorates the normal functioning of the system. The process of estimating the magnitude of the fault occurring in the system is coined as fault estimation [3]. In order to accomplish successful space missions the safety of satellite/spacecraft attitude control systems is crucial. Actuators and sensors are essential components of satellite control systems. If they get faulty, fault diagnosis must be carried out in order to avert the danger involved in space missions. Using sliding mode observers [6, 7] and adaptive observers [9], fault diagnosis is carried out extensively.

Integral actions are helpful to achieve steady-state accuracy in control systems. The design of proportional-integral (PI) observers were established [10] with the introduction of integral action in observer design. Till now, such observers have attracted many researchers.

The problem of constructing the observers for descriptor linear systems has been studied by many researchers parallel to the standard linear control systems. The design of full-order observers and reduced-order observers for descriptor linear systems can be found in fault diagnosis literature.

In real sense of words, the satellite attitude dynamics show non-linearity. So, Takagi-Sugeno (T-S) fuzzy model [11] can be used to linearise the satellite attitude dynamics at suitable operating points. [4] introduced fuzzy proportional multiple-integral observer method for robust actuator fault estimation. The idea of generalized proportional integral derivative (GPID) and proportional multiple-integral derivative (PMID) observers is proposed by [1, 2].

In this paper, we propose fuzzy PMID and PD observer based methods for robust actuator fault estimation. In our design, derivative gain gives more design degrees of freedom as compared to fuzzy PMI observers. The design constraints in our *fuzzy* PMID method are not strict for observer gains as compared to PMID or PID observers mentioned above.

In Section 2, we formulate the problem for actuator fault estimation of satellite control systems. The T-S fuzzy PMID & PD descriptor observers are designed in Section 3 and Section 4 respectively. Simulation studies are performed in order to validate the proposed methods in Section 5.

2. PROBLEM FORMULATION

2.1 ATTITUDE DYNAMICS

The equation of rotational motion of rigid satellite/spacecraft body is:

$$M = J\frac{\partial \omega}{\partial t} + \omega \times (J\omega + h_w) \tag{1}$$

$$M = J\frac{\partial \omega}{\partial t} + S(\omega)(J\omega + h_w) \tag{2}$$

In (2) we define as follows:

$$\frac{\partial \omega}{\partial t} = \begin{Bmatrix} \dfrac{d\omega_x}{dt} \\ \dfrac{d\omega_y}{dt} \\ \dfrac{d\omega_z}{dt} \end{Bmatrix} = \begin{Bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{Bmatrix}, \quad S(\omega) = \begin{pmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{pmatrix} \text{ and } M = T_u + T_d .$$

Therefore,

$$J\dot{\omega} + S(\omega)(J\omega + h_w) = T_u + T_d \tag{3}$$

where inertia matrix of satellite is J, ω is angular velocity with respect to inertial frame, T_u is output torque of the flywheels, T_d is the disturbance from environment and h_w is the angular momentum of three flywheels [4].

With ω as the state variable and actuator fault f_a, the state space system for (3) is represented as:

$$\begin{aligned} \dot{x} &= -J^{-1}S(\omega)(J\omega + h_w) + J^{-1}T_u + J^{-1}T_d + J^{-1}f_a, \\ y &= Cx. \end{aligned} \tag{4}$$

Now (4) can be written as [4]:

$$\begin{aligned} \dot{x} &= f(x) + Bu + Dd + Ff_a, \\ y &= Cx \end{aligned} \tag{5}$$

where $f(x) = -J^{-1}S(\omega)(J\omega + h_w)$, $B = D = F = J^{-1}$ and $C = I_{3\times 3}$

2.2 T-S Fuzzy Model

The T-S fuzzy model consists of an *if-then* rule base. The antecedent of each rule [8] comprises of scheduling variables and fuzzy sets. The consequent of each rule is a simple functional expression.

The *i*-th rule is described as

Model rule *i*:
If z_1 is Z_1^i and … and z_p is Z_p^i then $y = F_i(z)$

where the vector *z* has *p* components, z_j *j* =1, 2*… p,* and stands for the vector of scheduling variables and their values determine the degree to which rules are active. The sets, Z_j^i , *j*=1, 2, … , *p; i =* 1, 2, …, *m,* where *m* is the number of the rules, are the antecedents fuzzy sets. The values of a scheduling variable z_j belong to a fuzzy set Z_j^i with a truth value given by the membership function $\lambda_{ij} : \mathbb{R} \rightarrow [0, 1]$. The truth value for an entire rule is determined based on the individual premise variables, using a conjunction operator as:

$$\varphi_i(z) = \prod_{j=1}^{p} \lambda_{ij}(z_j) \tag{6}$$

The fuzzy weights are determined as

$$w_i(z) = \varphi_i(z) \Big/ \sum_{j=1}^{m} \varphi_j(z)$$

$$w_i(z) \geq 0 \text{ and } \sum\nolimits_i^p w_i(z) = 1 \tag{7}$$

The T-S fuzzy system for (5) can be written as:

$$\begin{aligned} \dot{x} &= \sum\nolimits_{i=1}^{p} w_i(z)(A_i x + B_i u + Dd + F_i f_a), \\ y &= \sum\nolimits_{i=1}^{p} w_i(z) C_i x, \end{aligned} \tag{8}$$

where $F_i \in R^{n\times k}$.

Each linearised model for satellite can be obtained as [4]:

$$\begin{aligned} \dot{x} &= A(\omega_0)x + Bu + Dd + Ff_a, \\ y &= Cx. \end{aligned} \tag{9}$$

where $A(\omega_0) = -J^{-1}S(\omega_0)J, B = D = F = J^{-1}$, $C = I_{3\times 3}$ and ω_0 is the operating point.

So we [4] have

$$\begin{aligned} \dot{x} &= \sum\nolimits_{i=1}^{p} w_i(z)(A_i x + Bu + Dd + Ff_a) \\ y &= \sum\nolimits_{i=1}^{p} w_i(z) C_i x \end{aligned} \tag{10}$$

3. T-S FUZZY DESCRIPTOR PMID OBSERVER

Consider the following fuzzy descriptor system

$$\begin{aligned} E\dot{x} &= \sum\nolimits_{i=1}^{p} w_i(z)(A_i x + Bu + Dd + Ff_a) \\ y &= \sum\nolimits_{i=1}^{p} w_i(z) C_i x. = Cx \end{aligned} \tag{11}$$

where $x \in R^n$ is the descriptor state vector, $u \in R^m$ and $y \in R^p$ are, respectively, the control input and output vectors, $f_a \in R^k$ is unknown actuator fault. The matrix *E* may be singular.

The q-th derivative of the fault is assumed to be bounded. The fault considered in this paper allows $q \geq 1$ as the first derivatives of faults with time are bounded.

Consider the following system with proportional, multiple integral and derivative weights of the output estimation error

$$E\dot{\hat{x}} = \sum_{i=1}^{p} w_i(z)\left(A_i\hat{x} + Bu + L_p(y - C\hat{x}) + L_d\left(\dot{y} - C\dot{\hat{x}}\right)\right) + F\hat{f}_a,$$
$$\hat{y} = C\hat{x},$$
$$\dot{\hat{y}} = C\dot{\hat{x}}, \tag{12}$$
$$\dot{\hat{f}}_a^{\,q} = \sum_{i=1}^{p} w_i(z)L_i^q(y - C\hat{x}) + \hat{f}_a^{\,q-1}$$
$$\dot{\hat{f}}_a^{\,q-1} = \sum_{i=1}^{p} w_i(z)L_i^{q-1}(y - C\hat{x}) + \hat{f}_a^{\,q-2}$$
$$\vdots$$
$$\vdots$$
$$\vdots$$
$$\dot{\hat{f}}_a^{\,2} = \sum_{i=1}^{p} w_i(z)L_i^2(y - C\hat{x}) + \hat{f}_a^{\,1}$$
$$\dot{\hat{f}}_a^{\,1} = \sum_{i=1}^{p} w_i(z)L_i^1(y - C\hat{x}). \tag{13}$$

Here, $\hat{x} \in R^n$ is an estimation of the descriptor state vector x, and $\hat{f}_a^{\,i} \in R^k\,(i = 1,2,\ldots,q)$ is an estimation of $(q-i)$-th derivative of the fault; the proportional gain $L_p \in R^{n\times p}$, the derivative gain $L_d \in R^{n\times p}$ and the integral gain $L_i \in R^{k\times p}$ are design matrices.

In order to estimate the actuator fault, we construct an augmented descriptor system as follows:

Let $\xi_i = f^{(q-i)}, (i = 1,2,\ldots,q)$ (14)

Using (12), (13) and (14), we get:

$$\overline{E}\dot{\overline{x}} = \sum_{i=1}^{p} w_i(z)\overline{A}_i\overline{x} + \overline{B}u + \overline{D}d + Gf_a^{\,q},$$
$$y = \overline{C}\overline{x} \tag{15}$$

where,

$$\overline{x} = \left[x^T, \xi_1^T, \ldots.., \xi_q^T\right]^T, B = \left[B^T, 0,0\ldots,0\right]^T, \overline{D} = \left[D^T, 0,0..0\right]^T$$

$$G = [0, I_k, 0,..,0]^T, \overline{C} = [C, 0,0..0],$$

$$\overline{A}_i = \begin{bmatrix} A_i & 0 & \cdots & 0 & F \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \end{bmatrix} \quad \overline{E} = \begin{bmatrix} E & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I & 0 \\ 0 & 0 & \cdots & 0 & I \end{bmatrix}$$

Now we develop observer for augmented system (15) as:

$$\begin{aligned}
\dot{\Phi} &= \sum_{i=1}^{p} w_i(z)\left(\overline{A}_i - \overline{L}_p\overline{C}\right)\hat{\overline{x}} + \overline{B}u + \overline{L}_p y, \\
\hat{\overline{x}} &= \left(\overline{E} + \overline{L}_d\overline{C}\right)^{-1}\left(\Phi + \overline{L}_d y\right), \\
\dot{\hat{\overline{x}}} &= \left(\overline{E} + \overline{L}_d\overline{C}\right)^{-1}\left(\dot{\Phi} + \overline{L}_d \dot{y}\right), \\
&= \left(\overline{E} + \overline{L}_d\overline{C}\right)^{-1} \cdot \left(\sum_{i=1}^{p} w_i(z)\left(\left(\overline{A}_i - \overline{L}_p\overline{C}\right)\hat{\overline{x}} + \overline{B}u + \overline{L}_p y + \overline{L}_d \dot{y}\right)\right)
\end{aligned} \tag{16}$$

where,

$$\overline{L}_p = \left[L_p^T, \left(L_i^1\right)^T, \ldots \left(L_i^q\right)^T\right]^T,$$

$$\overline{L}_d = \left[L_d^T, 0, 0 \ldots 0\right]^T$$

Add $\overline{L}_d\,\dot{y}$ to the both sides of (15), and then we have

$$\dot{\overline{x}} = \left(\overline{E} + \overline{L}_d\overline{C}\right)^{-1} \cdot \left(\sum_{i=1}^{p} w_i(z)\left(\left(\overline{A}_i - \overline{L}_p\overline{C}\right)\overline{x} + \overline{B}u + \overline{D}d + Gf_a^q + \overline{L}_p y + \overline{L}_d\dot{y}\right)\right) \tag{17}$$

The dynamic state error equation is:

$$\begin{aligned}
\dot{e} &= \dot{\hat{\overline{x}}} - \dot{\overline{x}} \\
&= \left(\overline{E} + \overline{L}_d\overline{C}\right)^{-1}\left(\sum_{i=1}^{p} w_i(z)\left(\left(\overline{A}_i - \overline{L}_p\overline{C}\right)e - \overline{D}d - Gf_a^q\right)\right) \\
&= \left(\overline{E} + \overline{L}_d\overline{C}\right)^{-1}\left(\sum_{i=1}^{p} w_i(z)\left(\left(\overline{A}_i - \overline{L}_p\overline{C}\right)e - M\overline{d}\right)\right)
\end{aligned} \tag{18}$$

where $M\overline{d} = \begin{bmatrix}\overline{D} & G\end{bmatrix}\begin{bmatrix} d \\ f_a^q\end{bmatrix}$

Condition 1: The trio (E, A_i, C) is completely observable if

$$rank\begin{bmatrix} E \\ C\end{bmatrix} = n, \qquad rank\begin{bmatrix} sE - A_i \\ C\end{bmatrix} = n \tag{19}$$

Condition 2: $rank\begin{bmatrix} A_i & F \\ C & 0\end{bmatrix} = n + k$ (20)

Theorem 1: If conditions 1 and 2 are satisfied, there exists a robust fuzzy observer in the form of (16) for the plant (15), which can make the steady estimator error dynamics as small as any desired accuracy. The derivative gain is such that $S = \overline{E} + \overline{L}_d\overline{C}$ is non-singular and the gain $\overline{L}_p$ is solved from the following linear matrix inequalities if there exists a common positive definite matrix $P \in R^{(n+k)\times(n+k)}$ and matrix Y_i such that

$$\begin{bmatrix} \overline{A}_i S^{-T}P + PS^{-1}\overline{A}_i - \overline{C}^T Y_i^T - Y_i\overline{C} + I & -PS^{-1}M \\ -M^T S^{-T}P & -\gamma^2 I\end{bmatrix} < 0 \tag{21}$$

with $\gamma > 0$ then $\overline{L}_p = SP^{-1}Y_i$

Proof: Under the conditions 1 and 2, the trio $\left(\overline{E}, \overline{A}_i, \overline{C}\right)$ is completely observable. Then derivative gain must be chosen such that S is non-singular.

For $rank(E)=m,$ singular value decomposition of descriptor state matrix *E* gives two orthogonal matrices Γ and Ξ such that

$$E=\Gamma\begin{bmatrix}\Delta_m & 0\\ 0 & 0\end{bmatrix}\Xi^T=\Gamma\begin{bmatrix}I_m & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\Delta_m & 0\\ 0 & I_{n-m}\end{bmatrix}\Xi^T$$

where $\Delta_m=diag(\Psi_1,\Psi_2,...,\Psi_m)$ with $\Psi_k>0,k=1,2,....m.$

Let

$$\Theta=\Xi\begin{bmatrix}\Delta_m^{-1} & 0\\ 0 & I_{n-m}\end{bmatrix}$$

Then, we get

$$\Gamma^T E\,\Theta=\begin{bmatrix}I_m & 0\\ 0 & 0\end{bmatrix},$$

$$C\,\Theta=\begin{bmatrix}C_1 & C_2\end{bmatrix},$$

In order that trio (*E, A, C*) is completely observable,

$$rank(C_2)=n-m$$

Thus, one has derivative gain as below [12]:

$$L_d=\Gamma\begin{bmatrix}0\\ \gamma_d\left(C_2^T C_2\right)^{-1}C_2^T\end{bmatrix} \tag{22}$$

where γ_d is any positive number.

We can compute as below [12]:

$$E+L_d C=\Gamma\begin{bmatrix}I_m & 0\\ \gamma_d\left(C_2{}^T C_2\right)^{-1}C_2{}^T C_1 & \gamma_d I_{n-m}\end{bmatrix} \tag{23}$$

which implies that $E+L_d C$ is non-singular. Now by using $\overline{E},\overline{L}_d$, and $\overline{C}$, we can say that $S=\overline{E}+\overline{L}_d\overline{C}$ is non-singular too.

Stability analysis is performed using Lyapunov direct method. The convergence conditions of state estimation error are formulated in the form of LMI (linear matrix inequality).The proportional and integral gains are determined from obtained LMI (21).

When a system is quadratically stable it implies that it is stable. However, the reverse is not necessarily true. So, the conditions obtained using the Lyapunov function are only sufficient. The unforced T-S model is quadratically stable if the Lyapunov function decreases and tends to zero when time approaches to infinity for all trajectories of error in our case.

The goal of robust actuator fault estimation is to determine proportional and integral gains (together with derivative gains) that cause the asymptotic convergence of error towards zero as time tends to infinity in case of disturbances and perturbations.

Consider the following Lyapunov function candidate,

$$V(\overline{e})=\overline{e}^T P\overline{e},$$

$$\dot{V}(\bar{e}) = \bar{e}^T P\dot{\bar{e}} + \dot{\bar{e}}^T P\bar{e}$$

$$= \bar{e}^T P\left[\left(\sum_{i=1}^{p} w_i(z) S^{-1}\left(\bar{A}_i - \bar{L}_p\bar{C}\right)\bar{e} - S^{-1}M\bar{d}\right)\right] + \left[\sum_{i=1}^{p} w_i(z)\left(\bar{e}^T\left(\left(\bar{A}_i - \bar{L}_p\bar{C}\right)^T S^{-T}\right)\right) - \bar{d}^T M^T S^{-T}\right] P\bar{e}$$

$$= \sum_{i=1}^{p} w_i(z)\bar{e}^T PS^{-1}\left(\bar{A}_i - \bar{L}_p\bar{C}\right)\bar{e} - \bar{e}^T PS^{-1}M\bar{d} + \sum_{i=1}^{p} w_i(z)\bar{e}^T\left(\bar{A}_i - \bar{L}_p\bar{C}\right)^T S^{-T}P\bar{e} - \bar{d}^T M^T S^{-T}P\bar{e}$$

$$= \sum_{i=1}^{p} w_i(z)\bar{e}^T\left[PS^{-1}\left(\bar{A}_i - \bar{L}_p\bar{C}\right) + \left(\bar{A}_i - \bar{L}_p\bar{C}\right)^T S^{-T}P\right]\bar{e} - 2\bar{e}^T PS^{-1}M\bar{d}$$

Define $\bar{L}_p = SP^{-1}\bar{Y}_i$ and consider the Lyapunov function V such that

$$\dot{V}(\bar{e}) + \bar{e}^T\bar{e} - \gamma^2\bar{d}^T\bar{d} \le 0$$

Thus integrating this expression leads to

$$V(\bar{e}(\infty)) - V(\bar{e}(0)) \le \int_0^\infty \gamma^2\bar{d}^T\bar{d} - \bar{e}^T\bar{e}\,dt.$$

Since T-S model is asymptotically stable and with zero initial conditions, we obtain

$$0 < \int_0^\infty \gamma^2\bar{d}^T\bar{d} - \bar{e}^T\bar{e}\,dt$$

which is equivalent to

$$\int_0^\infty \bar{e}^T\bar{e} < \gamma^2\int_0^\infty \bar{d}^T\bar{d} \quad \text{Or} \quad \|\bar{e}\|_{T_f} < \gamma\|\bar{d}\|_{T_f} \tag{24}$$

Now, the stability conditions are obtained as in (25) below

$$\begin{aligned}\dot{V}(\bar{e}) + \bar{e}^T\bar{e} - \gamma^2\bar{d}^T\bar{d} &= \sum_{i=1}^{p} w_i(z)\bar{e}^T\left[PS^{-1}\left(\bar{A}_i - \bar{L}_p\bar{C}\right) + \left(\bar{A}_i - \bar{L}_p\bar{C}\right)^T S^{-T}P\right]\bar{e} - 2\bar{e}^T PS^{-1}M\bar{d} + \bar{e}^T\bar{e} - \gamma^2\bar{d}^T\bar{d} \\ &= \sum_{i=1}^{p} w_i(z)\begin{bmatrix}\bar{e}^T & \bar{d}^T\end{bmatrix}\begin{bmatrix}\bar{A}_i S^{-T}P + PS^{-1}\bar{A}_i - \bar{C}^T Y_i^T - Y_i\bar{C} + I & -PS^{-1}M \\ -M^T S^{-T}P & -\gamma^2 I\end{bmatrix}\begin{bmatrix}\bar{e} \\ \bar{d}\end{bmatrix}\end{aligned} \tag{25}$$

4. T-S FUZZY DESCRIPTOR PD OBSERVER

Consider the following fuzzy descriptor system with proportional and derivative weights of the output estimation error

$$\begin{aligned} E\dot{\hat{x}} &= \sum_{i=1}^{p} w_i(z)\left(A_i\hat{x} + Bu + L_p(y - C\hat{x}) + L_d\left(\dot{y} - C\dot{\hat{x}}\right)\right), \\ \hat{y} &= C\hat{x}, \\ \dot{\hat{y}} &= C\dot{\hat{x}} \end{aligned} \tag{26}$$

where L_p and L_d are respectively the proportional and derivative gain matrices. Derivative gain is determined using (22).

In order to obtain the estimation of actuator fault, we introduce fault as an auxiliary state vector in (11) and get the following augmented system,

$$\begin{aligned} \bar{E}\dot{\bar{x}} &= \sum_{i=1}^{p} w_i(z)\bar{A}_i\bar{x} + \bar{B}u + \bar{D}d, \\ y &= \bar{C}\bar{x} \end{aligned} \tag{27}$$

where $x_a = Ff_{a,}\ \bar{x} = \begin{bmatrix} x \\ x_a \end{bmatrix}, \bar{A}_i = \begin{bmatrix} A_i & I_3 \\ 0 & 0 \end{bmatrix}, \bar{E} = \begin{bmatrix} I_3 & 0 \\ 0 & I_3 \end{bmatrix}, F = I_3, \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \bar{D} = \begin{bmatrix} D & 0 \end{bmatrix}$.

If there exists an observer as (28) for the plant (27), then actuator fault and the states of the system can be estimated simultaneously.

$$\begin{aligned}
\dot{\Phi} &= \sum_{i=1}^{p} w_i(z)\left(\bar{A}_i - L_p\bar{C}\right)\hat{\bar{x}} + \bar{B}u + L_p y, \\
\hat{\bar{x}} &= \left(\bar{E} + L_d\bar{C}\right)^{-1}\left(\Phi + L_d y\right), \\
\dot{\hat{\bar{x}}} &= \left(\bar{E} + L_d\bar{C}\right)^{-1}\left(\dot{\Phi} + L_d \dot{y}\right), \\
&= \left(\bar{E} + L_d\bar{C}\right)^{-1} \cdot \left(\sum_{i=1}^{p} w_i(z)\left(\left(\bar{A}_i - L_p\bar{C}\right)\hat{\bar{x}} + \bar{B}u + L_p y + L_d \dot{y}\right)\right)
\end{aligned} \quad (28)$$

Add $L_d\dot{y}$ to the both sides of (27), and then we have

$$\dot{\bar{x}} = \left(\bar{E} + L_d\bar{C}\right)^{-1} \cdot \left(\sum_{i=1}^{p} w_i(z)\left(\left(\bar{A}_i - L_p\bar{C}\right)\bar{x} + \bar{B}u + \bar{D}d + L_p y + L_d \dot{y}\right)\right) \quad (29)$$

The dynamic state error equation is:

$$\begin{aligned}
\dot{\bar{e}} &= \dot{\bar{x}} - \dot{\hat{\bar{x}}} \\
&= \left(\bar{E} + \bar{L}_d\bar{C}\right)^{-1}\left(\sum_{i=1}^{p} w_i(z)\left(\left(\bar{A}_i - \bar{L}_p\bar{C}\right)\bar{e} + \bar{D}d\right)\right)
\end{aligned}$$

Theorem 2: If conditions 1 and 2 are satisfied, there exists a robust fuzzy observer in the form of (28) for the plant (27), which can make the steady estimator error dynamics as small as any desired accuracy. The derivative gain is such that $S = \bar{E} + L_d\bar{C}$ is non-singular and the gain L_p is solved from the following linear matrix inequalities if there exists a common positive definite matrix $P \in R^{(n+k)\times(n+k)}$ and matrix Y_i such that

$$\begin{bmatrix} \bar{A}_i S^{-T} P + PS^{-1}\bar{A}_i - \bar{C}^T Y_i^T - Y_i\bar{C} + I & -PS^{-1}\bar{D} \\ -\bar{D}^T S^{-T} P & -\gamma^2 I \end{bmatrix} < 0 \quad (30)$$

with $\gamma > 0$ then $L_p = SP^{-1}Y_i$

Proof: Proof is similar to theorem 1. Derivative gain and proportional gains are obtained in the same fashion as in theorem 1. The only difference is that we have removed multiple integrals.

Remark: As we have introduced the actuator fault as an auxiliary state vector in the plant (11), the matrix $F = I_3$ and not the inverse of inertia matrix *J.* The actuator fault can be directly isolated from estimated state $\hat{\bar{x}}$.

Since $x_a = Ff_{a,}\ \bar{x} = \begin{bmatrix} x \\ x_a \end{bmatrix}$

Therefore,

$\hat{x}_a = \begin{bmatrix} 0 & I_3 \end{bmatrix}\hat{\bar{x}}$ then $\hat{f}_a = F^{-1}\begin{bmatrix} 0 & I_3 \end{bmatrix}\hat{\bar{x}}$

5. SIMULATION RESULTS

In order to obtain the T-S fuzzy model, suitable operating points are chosen in the vicinity of zero and we employ triangular membership function in this case. The actuator faults [4] along three axes are

$$f_{ax} = \begin{cases} 0 & t \le 0 \\ 0.01(t-60)+0.4 & 60 < t \le 80 \\ 0.8 & t > 80 \end{cases}$$

$$f_{ay} = \begin{cases} 0 & 0 < t < 30 \\ 0.03(t-30) & 30 \le t \le 50 \\ -0.02(t-50)+0.6 & 50 < t \le 80 \\ 0 & t > 80 \end{cases} \tag{31}$$

$$f_{az} = \begin{cases} 0 & t \le 40 \\ 0.2\sin((0.523t)-1.57) & t > 40 \end{cases}$$

The simulation data is borrowed from [4]. The proportional and integral gains are obtained using proper index γ for fuzzy PMID. In case of fuzzy PD, only proportional gains are determined using (30).

The T-S Fuzzy PMID Descriptor Observer shows satisfactory performance when $q \geq 1$. The simulation results shown below are obtained using MATLAB/SIMULINKsoftware.

In order to get the T-S fuzzy model, linearization method is employed using suitably choosen 8 different operating points in the vicinity of equilibrium point (0,0,0). The reasonable index γ is choosen in such a way that feasible solution is obtained for LMIs in (21) & (30).

Derivative gain is obtained from (22) and proportional-integral gains are determined from (21) for fuzzy descriptor PMID observer. The fuzzy descriptor system (15) and fuzzy descriptor observer (16) are simulated in SIMULINK to get the outputs shown in this section. Similar procedure is followed for fuzzy descriptor PD observer using (22), (27), (28), and (30).

Actuator fault estimation using T-S fuzzy descriptor PMID and PD observers is shown in figures 1 – 6. It can be infered from these figures that proposed fuzzy descriptor PMID observer outperforms fuzzy descriptor PD observer. So, the multiple integral actions introduced in observer estimate the fault more better.

In order to have better idea, the figures for estimated error are also obtained that clearly represents that fuzzy descriptor PMID performs better than fuzzy descriptor PD observer.

The derivative gain can give us more design degrees of freedom. It can make us obtain fuzzy PMID and PD observer only with original coefficient matrices together with proportional or proportional-integral gains. Further, the effects of faults and disturbances are reduced with smaller values of $\bar{S}^{-1}$ as we increase the derivative gain. Due to such effect, faults are estimated more accurately.

In the present design, we have to take only the original system matrices into consideration which clearly indicates that the simultaneous observer is state-space observer. So it is more easy in computation and reliable in many applications [12].

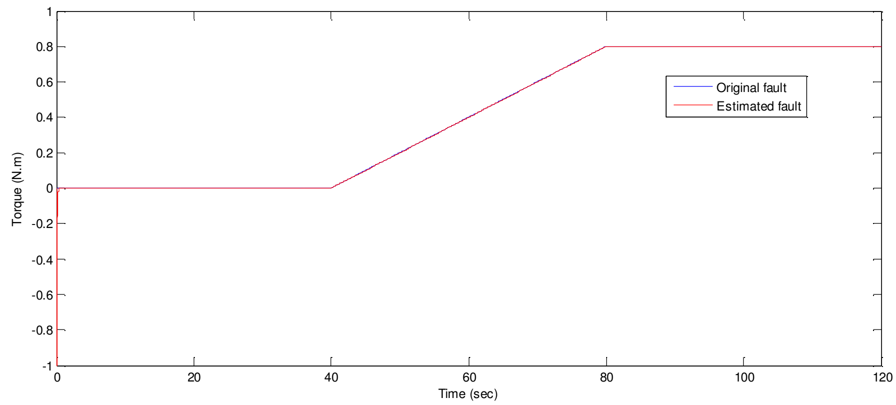

FIGURE 1: Actuator fault estimation along X-axis using T-S fuzzy descriptor PMID observer

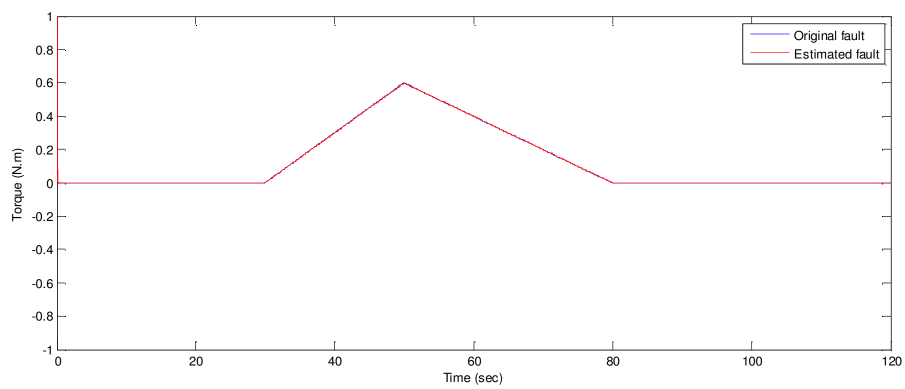

FIGURE 2: Actuator fault estimation along Y-axis using T-S fuzzy descriptor PMID observer

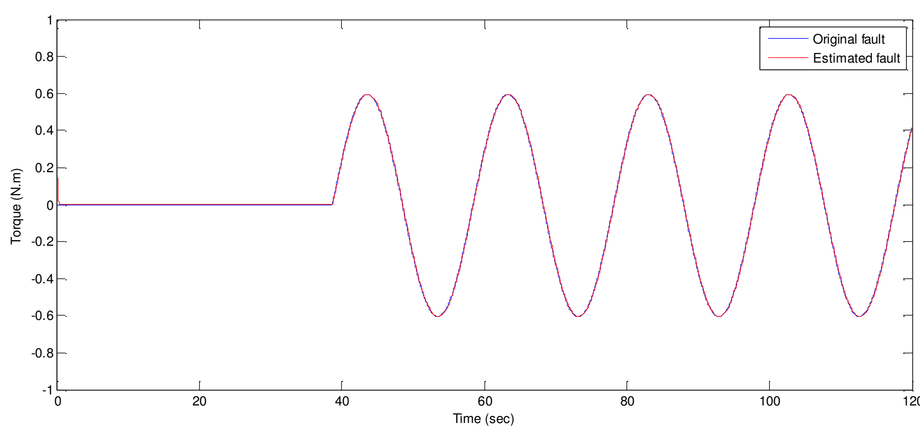

FIGURE 3: Actuator fault estimation along Z-axis using T-S fuzzy descriptor PMID observer

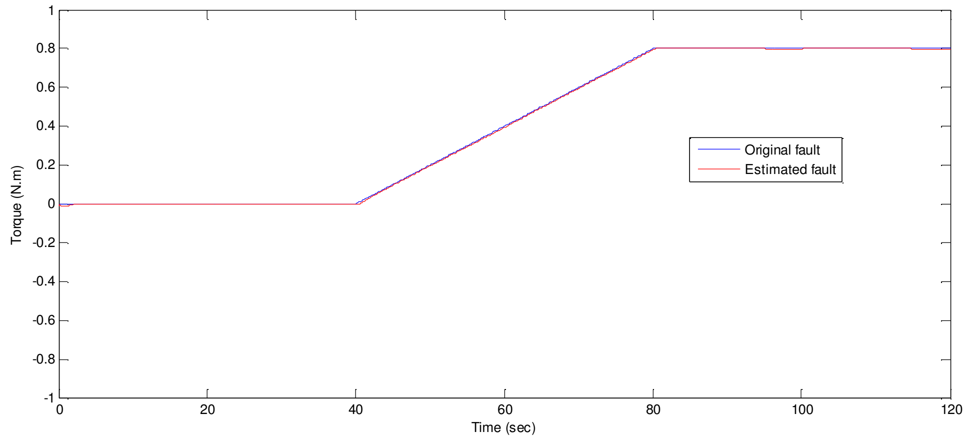

FIGURE 4: Actuator fault estimation along X-axis using T-S fuzzy descriptor PD observer

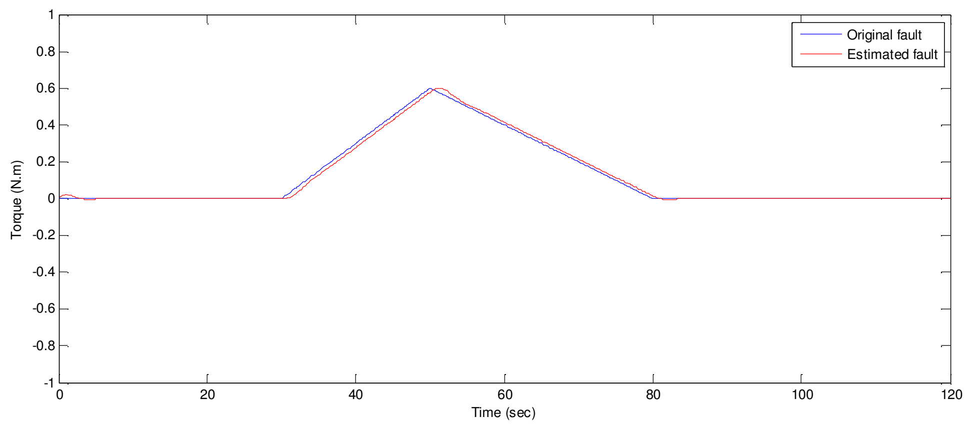

FIGURE 5: Actuator fault estimation along Y-axis using T-S fuzzy descriptor PD observer

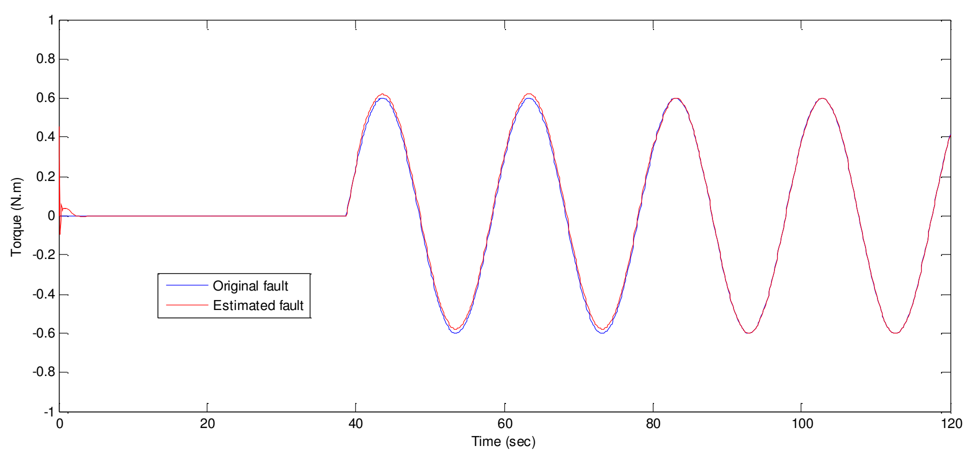

FIGURE 6: Actuator fault estimation along Z-axis using T-S fuzzy descriptor PD observer

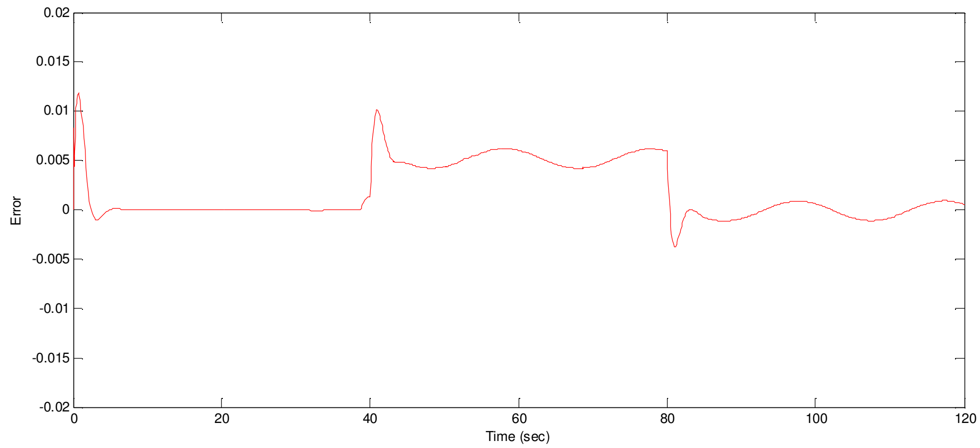

FIGURE 7: Estimated error for fault along X-axis using T-S fuzzy descriptor PD observer

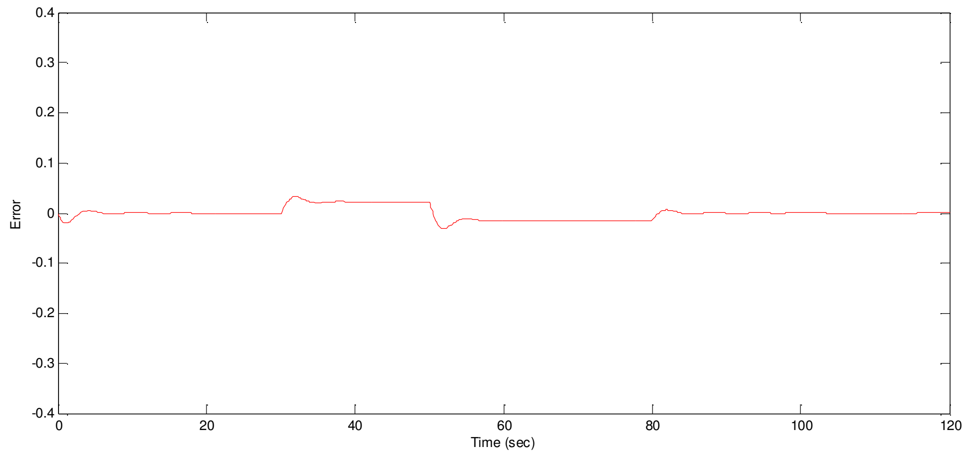

FIGURE 8: Estimated error for fault along Y-axis using T-S fuzzy descriptor PD observer

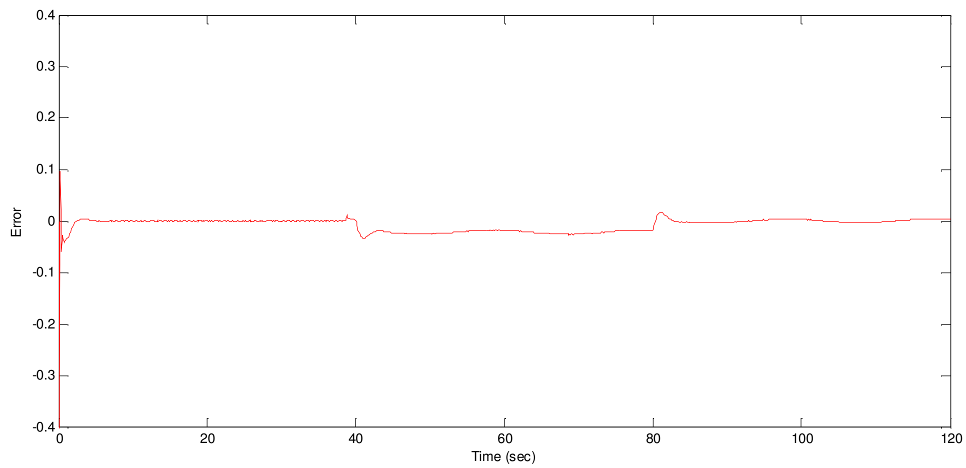

FIGURE 9: Estimated error for fault along Z-axis using T-S fuzzy descriptor PD observer

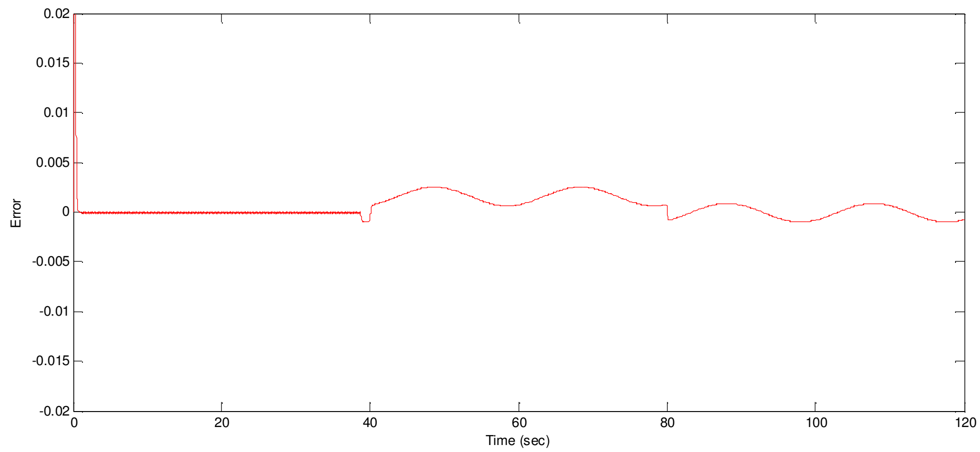

FIGURE 10: Estimated error for fault along X-axis using T-S fuzzy descriptor PMID observer

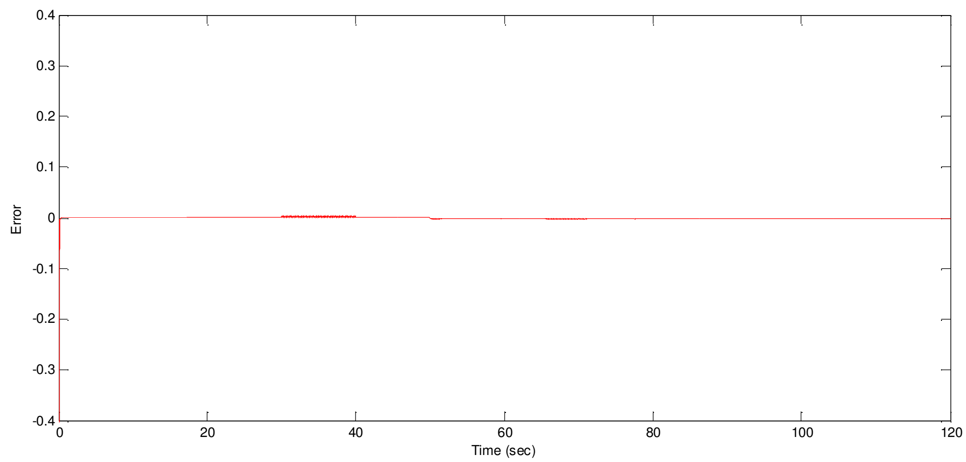

FIGURE 11: Estimated error for fault along Y-axis using T-S fuzzy descriptor PMID observer

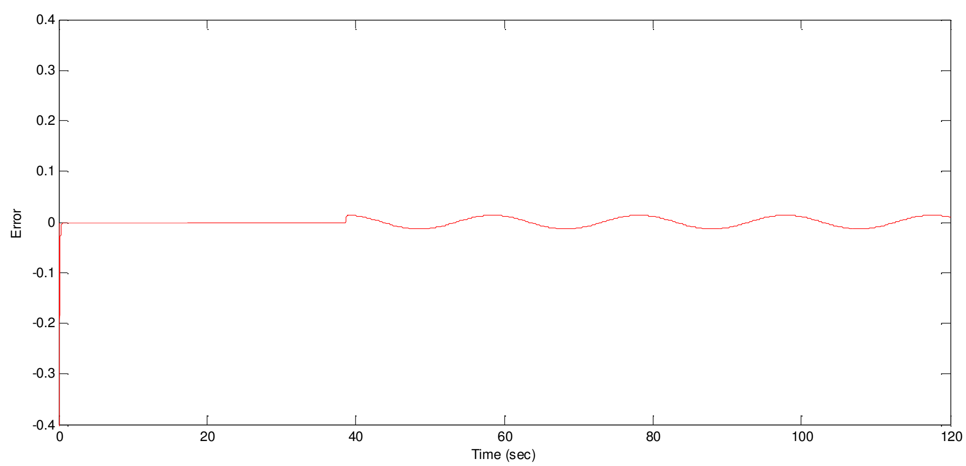

FIGURE 12: Estimated error for fault along Z-axis using T-S fuzzy descriptor PMID observer

In our proposed work, the multiple integral actions assumed for fault estimation reduced as compared to other observers in the literature available. We have assumed q=1 and as it can be noticed that [4] has assumed q=2. The less number of integral actions considered in our work is sufficient and better enough for observer design to reduce computational cost. As we increase the order of the faults, less integral actions would be required as per proposed work. Such difference arises due to derivative gain added in previously available fuzzy PMI observer.

The more design degrees of freedom given by derivative gain reduces the multiple integral actions and make observer design simpler. It should also be noticed that together with γ, we have γ_d index that makes fault estimation better. The main contribution of this paper is design of fuzzy PMID and fuzzy PD observers.

The results shown above in figures 1-6 support the proposed methods. It can be noticed that the estimated error in the fuzzy PMID observer is less than the fuzzy PD observer. The comparison of fig.7 and fig. 10 gives better picture that error in fig.7 is more. Similar conclusions can be made about figures for estimated error.

The figures 1-3 clearly show the good estimation of original fault along three dimensions. In order to get better notion, fig. 2 and fig. 5 should be compared. In fig. 2 original fault is estimated far better than in fig. 5. Thus, the conclusion can be drawn that the fuzzy PMID is better than fuzzy PD observer in terms of estimation of time varying faults.

Artificial neural networks are better approximator of nonlinear systems as compared to fuzzy logic methods. The extension of this research would include the construction of neuro-fuzzy observer. The fuzzy weights determined using linearization method can act as weights of neural networks and by choosing suitable activation function, artificial neural networks can be brought into play.

A better fault tolerant scheme can be designed for such observers. We have considered only the fault estimation. Fault diagnosis is another essential extension.

The continuous time-invariant system is considered for fault estimation here, the discrete time or continuous time variant systems can also be considered giving better application in real world problems.

The time delay systems (A (t + Δt), B (t + Δt), etc) with reduced order observer can also be designed providing less computational costs for observer design. While considering nonlinear models, modeling uncertainty should be taken into consideration which is of importance in the field of fault diagnosis.

6. CONCLUSION

The fuzzy descriptor proportional multiple integral derivative (PMID) and proportional derivative (PD) observers are proposed to estimate the actuator fault of satellite attitude control systems. The convergence condition of state estimation error is formulated in the form of LMI. The proposed observers are robust since they have been synthesized to decouple and attenuate both the effects of disturbances and fault approximated error. The main contribution can be noticed in terms of more design degrees of freedom added by derivative gain which enhances the system response. Simulation study reveals fuzzy descriptor PMID outperforms fuzzy descriptor PD observer in terms of robust actuator fault estimation.

7. REFERENCES

[1] W. Ai-Guo, D. Guang-Ren and F. Yan-Ming, “Generalized PID observer design for descriptor linear systems,” IEEE Transactions on Systems, Man, and Cybernetics, Vol. 37, No. 5, pp. 1390-1395, 2007.

[2] W. Ai-Guo and D. Guang-Ren, “PMID observer design of descriptor linear systems,” In Proceedings of the 26th Chinese Control Conference, 2007, pp. 161-165.

[3] S. Ding. (2008), Model-based fault diagnosis techniques (1st edition), [Online]. Available: http://0www.springerlink.com.oasis.lib.tamuk.edu/content/?k=model+based+fault+diagnosis [Feb., 2011]

[4] Y. Fu, Y. Cheng and B. Jiang, “Robust actuator fault estimation for satellite control system via fuzzy proportional multiple-integral observer method,” in Proceedings of the 29th Chinese Control Conference, 2010, pp. 3942-3946.

[5] J. Gun-Ping, W. Suo-Ping and S. Wen-Zhong, “Design of observer with integrators for linear systems with unknown input disturbances,” IEE Electronic Letters, Vol. 36, No. 13, pp. 1168-1169, 2000.

[6] Y. Jinyong and L. Zhiyuan, “Fault reconstruction based on sliding mode observer for linear descriptor systems,” In Proceedings of the 7th Asian Control Conference, 2009, pp. 1132-1137.

[7] J. Lia-hui, T. Fa, and L. Chen, “Fault reconstruction based on sliding mode observers,” World Congress on Computer Science and Information Engineering, IEEE Computer Society, pp. 479-484, 2009.

[8] Z. Lendek, T.M. Guerra, R. Babuska, and B. De Schutter (2011), Stabilty Analysis and Nonlinear Observer Design using Takagi-Sugeno Fuzzy Models (1st edition), [Online]. Available: http://0-www.springerlink.com [April, 2011]

[9] K. Meinghed, J. Yame, and C. Aubrun, “Modifica-tion of an adaptive observer to improve its performance in actuator fault detection,” In Proceedinds of 18th Mediterranean Conference on Control & Automation, Morocco, 2008,pp. 75-81.

[10] B. Shafai and R L. Carroll, “Design of proportional integral observer for linear time- varying multivariable systems,” In Proceedings of 24th IEEE conference on Decision Control, 1985, pp. 597-599.

[11] T. Takagi and M. Sugeno, “Fuzzy identification of the systems and its application to modeling and control,” IEEE Transactions on Systems, Man, and Cybernetics, Vol. SMC-15, No. 1, pp. 116-132, 1985.

[12] Z. Gao and S.X.Ding, “Fault estimation and fault-tolerant control for descriptor systems via proportional, multiple-integral and derivative observer design,” IET Control Theory Appl., Vol. 1, No. 5, pp. 1208-1218, 2007.

[13] Christopher Edwards, Thomas Lombaerts and Hafid Smaili, “Fault Tolerant Flight Control,” Lecture notes in Control and Information Sciences, 2010.

[14] S. X. Ding, P. Zhang, B. Huang, E. L. Ding, and P. M. Frank, “An approach to norm and statistical methods based residual evaluation,” In Proceedings of the 10th IEEE International Conference on Methods and Models in Automation and Robotics, pp. 777 - 780, 2004.

[15] X. Ding and P. M. Frank, "Frequency domain approach and threshold selector for robust model-based fault detection and isolation," In Proceedings of the 1st IFAC Symp. Safe-process, 1991.

[16] X. Ding and P. M. Frank, "An adaptive observer-based fault detection scheme for nonlinear systems," in Proceedings of the 12th IFAC World Congress, Sydney, pp. 307-312, 1993.

[17] P. M. Frank and X. Ding, "Frequency domain approach to optimally robust residual generation and evaluation for model-based fault diagnosis," Automatica, vol. 30, pp. 789-904, 1994.

[18] E. Frisk and L. Nielsen, "Robust residual generation for diagnosis including a reference model for residual behavior," Automatica, vol. 42, pp. 437-445, 2006

[19] Ashish Teewari, "Atmospheric and Space Flights," Second edition, 2007, pp. 370-372.

[20] Ajit K. Mandal, "Introduction to Control Engineering," Third edition, 2006, pp. 255-275.

[21] Guido Herrmann, Matthew C. Turner and Ian Postlethwaite, "Mathematical Methods for Robust & Nonlinear Control," Second edition, 2007, pp. 170-175.

[22] Kevin M. Passino and Stephen Yurkovich, "Fuzzy Control," First edition, 1998, pp. 121-130.

[23] Nguyen Tan Tien, "Applied Nonlinear Control-Fundamentals of Lyapunov Theory," Second edition, 2002, pp. 68-79.

[24] Stephen Boyd, "Linear Matrix Inequalities in System and Control Theory," 1994, pp. 25-33.

[25] S. Linder, B. Shafai and M. Saif, "Estimating and Accomodating Unknown Actuator Faults with PI Observer," in Proceedings of the 1988 IEEE International Conference on Control Applications, 1998.

[26] G. Duan and G. Liu, "Eigenstructure assignment design for proportional integral observers: continuous time case," in Proceedings of IEE Control Theory Appl. Vol. 148, No. 3, May 2001.

[27] K. Krishna and P. Kabore, "On the design of integral and proportional integral observers," in Proceedings of the American Control Conference, June 2009.

[28] D. Ichalal, B. Marx and J. Ragot, "Observer based actuator fault tolerant control for nonlinear Takagi-Sugeno systems: an LMI approach," in Proceedings of Mediterranean Conference on Control & Automation (MED), pp. 1278-1283, June 2010.

[29] S.R. Beale and B. Shafai, "Robust control system design with the proportional integral observer," in Proceedings of the 27th conference on Decision and Control," pp. 554-557, 1988.

[30] Q. Wu and M. Saif, "Robust fault Diagnosis for a Satellite using Neural Sliding Mode Observer," in Proceedings of the 44th IEEE Conference on Decision and Control, pp. 7668-7673, 2005.

[31] B. Marx, D. Koeing and D. Georges, "Robust fault diagnosis for linear descriptor systems using proportional and integral observers," in Proceedings of the 42nd IEEE Conference on Decision and Control, pp. 457-462, 2003.