On line Tuning Premise and Consequence FIS: Design Fuzzy Adaptive Fuzzy Sliding Mode Controller Based on Lyaponuv Theory

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Abstract

Classical sliding mode controller is robust to model uncertainties and external disturbances. A sliding mode control method with a switching control low guarantees asymptotic stability of the system, but the addition of the switching control law introduces chattering in to the system. One way of attenuating chattering is to insert a saturation function inside of a boundary layer around the sliding surface. Unfortunately, this addition disrupts Lyapunov stability of the closed-loop system. Classical sliding mode control method has difficulty in handling unstructured model uncertainties. One can overcome this problem by combining a sliding mode controller and fuzzy system together. Fuzzy rules allow fuzzy systems to approximate arbitrary continuous functions. To approximate a time-varying nonlinear system, a fuzzy system requires a large amount of fuzzy rules. This large number of fuzzy rules will cause a high computation load. The addition of an adaptive law to a fuzzy sliding mode controller to online tune the parameters of the fuzzy rules in use will ensure a moderate computational load. Refer to this research; tuning methodology can online adjust both the premise and the consequence parts of the fuzzy rules. Since this algorithm for is specifically applied to a robot manipulator.

Keywords: Classical Sliding Mode Controller, Robust, Uncertainties, Chattering Phenomenon, Lyapunov Theory, Fuzzy Sliding Mode Controller, Tuning Fuzzy Sliding Mode Controller, Robotic System.

1. INTRODUCTION AND MOTIVATION

Sliding mode controller (SMC) is one of the influential nonlinear controllers in certain and uncertain systems which are used to present a methodical solution for two main important controllers' challenges, which named: stability and robustness. Conversely, this controller is used in different applications; sliding mode controller has consequent disadvantages ; chattering phenomenon and nonlinear equivalent dynamic formulation; which chattering is caused to some difficulties such as saturation and heat for mechanical parts of robot manipulators or drivers and nonlinear equivalent dynamic formulation in uncertain systems is most important challenge in highly nonlinear uncertain system[1, 5-29]. In order to solve the chattering in the systems output, boundary layer method should be applied so beginning able to recommended model in the main motivation which in this method the basic idea is replace the discontinuous method by saturation (linear) method with small neighbourhood of the switching surface. Slotine and Sastry have introduced boundary layer method instead of discontinuous method to reduce the chattering[21]. Slotine has presented sliding mode with boundary layer to improve the industry application [22]. R. Palm has presented a fuzzy method to nonlinear approximation instead of linear approximation inside the boundary layer to improve the chattering and control the result performance[23]. Moreover, C. C. Weng and W. S. Yu improved the previous method by using a new method in fuzzy nonlinear approximation inside the boundary layer and adaptive method[24]. To remove the above setbacks, control researchers have applied artificial intelligence method (e.g., fuzzy logic, neural network and genetic algorithm) in nonlinear robust controller (e.g., sliding mode controller, backstepping and feedback linearization) besides this technique is very useful in order to implement easily. Estimated uncertainty method is used in term of uncertainty estimator to compensation of the system uncertainties. It has been used to solve the chattering phenomenon and also nonlinear equivalent dynamic. If estimator has an acceptable performance to compensate the uncertainties, the chattering is reduced. Research on estimated uncertainty to reduce the chattering is significantly growing as their applications such as industrial automation and robot manipulator. For instance, the applications of artificial intelligence, neural networks and fuzzy logic on estimated uncertainty method have been reported in [22-30]. Wu et al. [30] have proposed a simple fuzzy estimator controller beside the discontinuous and equivalent control terms to reduce the chattering. Elmali et al. [27]and Li and Xu [29]have addressed sliding mode control with perturbation estimation method (SMCPE) to reduce the classical sliding mode chattering.

In recent years, artificial intelligence theory has been used in sliding mode control systems [31-40, 68]. Neural network, fuzzy logic and neuro-fuzzy are synergically combined with nonlinear classical controller and used in nonlinear, time variant and uncertainty plant (e.g., robot manipulator). Fuzzy logic controller (FLC) is one of the most important applications of fuzzy logic theory. This controller can be used to control nonlinear, uncertain and noisy systems. This method is free of some model-based techniques as in classical controllers [33-38]. The main reasons to use fuzzy logic technology are able to give approximate recommended solution for uncertain and complex systems to easy understanding and flexible. Fuzzy logic provides a method which is able to model a controller for nonlinear plant with a set of IF-THEN rules, or it can identify the control actions and describe them by using fuzzy rules. The applications of artificial intelligence, neural networks and fuzzy logic, on robot arm control have reported in [29-31]. Wai et al. [37-38]have proposed a fuzzy neural network (FNN) optimal control system to learn a nonlinear function in the optimal control law. This controller is divided into three main groups: arterial intelligence controller (fuzzy neural network) which it is used to compensate the system's nonlinearity and improves by adaptive method, robust controller to reduce the error and optimal controller which is the main part of this controller. Mohan and Bhanot [40] have addressed comparative study between some adaptive fuzzy, and a new hybrid fuzzy control algorithm for manipulator control. The applications of fuzzy logic on sliding mode controller have reported in [11-16, 23-30]. Research on applied fuzzy logic methodology in sliding mode controller (FSMC) to reduce or eliminate the high frequency oscillation (chattering), to compensate the unknown system dynamics and also to adjust the linear sliding surface slope in pure sliding mode controller considerably improves the robot manipulator control process [42-47]. H.Temeltas [46] has proposed fuzzy adaption techniques for SMC to achieve robust tracking of nonlinear systems and solves the chattering problem. Conversely system's performance is better than sliding mode controller; it is depended on nonlinear dynamic equation. C. L. Hwang *et al.* [47]have proposed a Tagaki-Sugeno (TS) fuzzy model based sliding mode control based on *N* fuzzy based linear state-space to estimate the uncertainties. A multi-input multi-output (MIMO) FSMC reduces the chattering phenomenon and reconstructs the approximate the unknown system has been presented for a robot manipulator [48]. Investigation on applied sliding mode methodology in fuzzy logic controller (SMFC) to reduce the fuzzy rules and refine the stability of close loop system in fuzzy logic controller has grown specially in recent years as the robot manipulator control [10-15]; [49-55]. Lhee et al. [49]have presented a fuzzy logic controller based on sliding mode controller to more formalize and boundary layer thickness. Emami *et al.* [51]have proposed a fuzzy logic approximate inside the boundary layer. H.K.Lee *et al.* [52] have presented self tuning SMFC to reduce the fuzzy rules, increase the stability and to adjust control parameters control automatically. However the application of FSMC and SMFC are growing but the main SMFC drawback compared to FSMC is calculation the value of sliding surface λ pri-defined very carefully. Moreover, the advantages of SMFC compared to FLC reduce the number of fuzzy rule base and increase the robustness and stability. At last FSMC compare to the SMFC is more suitable for implementation action.

In various dynamic parameters systems that need to be training on-line tuneable gain control methodology is used. On-line tuneable control methodology can be classified into two main groups, namely, traditional adaptive method and fuzzy adaptive method. Fuzzy adaptive method is used in systems which want to training parameters by expert knowledge. Traditional adaptive method is used in systems which some dynamic parameters are known. In this research in order to solve disturbance rejection and uncertainty dynamic parameter, on-line tuneable method is applied to artificial sliding mode controller. F Y Hsu et al. [54]have presented adaptive fuzzy sliding mode control which can update fuzzy rules to compensate nonlinear parameters and guarantee the stability robot manipulator controller. Y.C. Hsueh et al. [43] have presented self tuning sliding mode controller which can resolve the chattering problem without to using saturation function. For nonlinear dynamic systems (e.g., robot manipulators) with various parameters, on-line control technique can train the dynamic parameter to have satisfactory performance. Calculate sliding surface slope is common challenge in classical sliding mode controller and fuzzy sliding mode controller. Research on adaptive (on-line tuneable) fuzzy control is significantly growing, for instance, different adaptive fuzzy controllers have been reported in [40, 56-68]. The adaptive sliding mode controller is used to estimate the unknown dynamic parameters and external disturbances. The applications of adaptive fuzzy sliding mode controller to control the robot manipulators have been reported in [11-20, 67]. Yoo and Ham [58]have proposed a MIMO fuzzy system to help the compensation and estimation the torque coupling. In $n-DOF$ robot manipulator with k membership function for each input variable, the number of fuzzy rules for each joint is equal to $3k^{2n}$ that causes to high computation load and also this controller has chattering. This method can only tune the consequence part of the fuzzy rules. Medhafer et al. [59] have proposed an indirect adaptive fuzzy sliding mode controller to control robot manipulator. This MIMO algorithm, applies to estimate the nonlinear dynamic parameters. If each input variable have K_2 membership functions, the number of fuzzy rules for each joint is $(m+1)K_2^m+K_2$. Compared with the previous algorithm the number of fuzzy rules have reduced by introducing the sliding surface as inputs of fuzzy systems. Y. Guo and P. Y. Woo [51]have proposed a SISO fuzzy system compensate and reduce the chattering. First suppose each input variable with K_2 membership function the number of fuzzy rules for each joint is K_2 which decreases the fuzzy rules and the chattering is reduce. C. M. Lin and C. F. Hsu [61] can tune both systems by fuzzy rules. In this method the number of fuzzy rules equal to K_2 with low computational load but it has chattering. Piltan et al., have proposed a adaptive fuzzy inference sliding mode controller to reduce or eliminate chattering for robot manipulator [67].
This paper is organized as follows:

In section 2, detail of dynamic equation of robot arm, problem statements, objectives and introduced the sliding mode controller are presented. Detail of tuning methodology which online adjusted both the premise and the consequence parts of the fuzzy rules is presented in section 3. In section 4, the simulation result is presented and finally in section 5, the conclusion is presented.

2. ROBOT MANIPULATOR DYNAMICS, OBJECTIVES, PROBLEM STATEMENTS AND SLIDING MODE METHODOLOGY

Robot manipulator dynamic: Robot manipulator is collection of links that connect to each other by joints, these joints can be revolute and prismatic that revolute joint has rotary motion around an axis and prismatic joint has linear motion around an axis. Each joint provides one or more degrees of freedom (DOF). From the mechanical point of view, robot manipulator is divided into two main groups, which called; serial robot links and parallel robot links. In serial robot manipulator, links and joints is serially connected between base and final frame (end-effector). Parallel robot manipulators have many legs with some links and joints, where in these robot manipulators base frame has connected to the final frame. Most of industrial robots are serial links, which in serial robot manipulator the axis of the first three joints has a known as major axis, these axes show the position of end-effector, the axis number four to six are the minor axes that use to calculate the orientation of end-effector, at last the axis number seven to n use to avoid the bad situation. Kinematics is an important subject to find the relationship between rigid bodies (e.g., position and orientation) and end-effector in robot manipulator. The mentioned topic is very important to describe the three areas in robot manipulator: practical application, dynamic part, and control purposed therefore kinematics play important role to design accurate controller for robot manipulators. Robot manipulator kinematics is divided into two main groups: forward kinematics and inverse kinematics where forward kinematics is used to calculate the position and orientation of end-effector with given joint parameters (e.g., joint angles and joint displacement) and the activated position and orientation of end-effector calculate the joint variables in Inverse Kinematics[1]. Dynamic modeling of robot manipulators is used to describe the behavior of robot manipulator, design of model based controller, and for simulation. The dynamic modeling describes the relationship between joint motion, velocity, and accelerations to force/torque or current/voltage and also it can be used to describe the particular dynamic effects (e.g., inertia, coriolios, centrifugal, and the other parameters) to behavior of system[1]. The Unimation PUMA 560 serially links robot manipulator was used as a basis, because this robot manipulator widely used in industry and academic. It has a nonlinear and uncertain dynamic parameters serial link 6 degrees of freedom (DOF) robot manipulator. A nonlinear robust controller design is major subject in this work. The equation of a multi degrees of freedom (DOF) robot manipulator is calculated by the following equation[6]:

$$M(q)\ddot{q} + N(q,\dot{q}) = \tau \tag{1}$$

Where τ is $n \times 1$ vector of actuation torque, M (q) is $n \times n$ symmetric and positive define inertia matrix, $N(q,\dot{q})$ is the vector of nonlinearity term, and q is $n \times 1$ position vector. In equation 1 if vector of nonlinearity term derive as Centrifugal, Coriolis and Gravity terms, as a result robot manipulator dynamic equation can also be written as [10-16]:

$$N(q,\dot{q}) = V(q,\dot{q}) + G(q) \tag{2}$$

$$V(q,\dot{q}) = B(q)[\dot{q}\,\dot{q}] + C(q)[\dot{q}]^2 \tag{3}$$

$$\tau = M(q)\ddot{q} + B(q)[\dot{q}\,\dot{q}] + C(q)[\dot{q}]^2 + G(q) \tag{4}$$

Where,

$B(q)$ is matrix of coriolis torques, $C(q)$ is matrix of centrifugal torque, $[\dot{q}\,\dot{q}]$ is vector of joint velocity that it can give by: $[\dot{q}_1.\dot{q}_2,\dot{q}_1.\dot{q}_3,\dots,\dot{q}_1.\dot{q}_n,\dot{q}_2.\dot{q}_3,\dots]^T$, and $[\dot{q}]^2$ is vector, that it can given by: $[\dot{q}_1^{\ 2},\dot{q}_2^{\ 2},\dot{q}_3^{\ 2},\dots]^T$. In robot manipulator dynamic part the inputs are torques and the outputs are actual displacements, as a result in (4) it can be written as [1, 6, 10-16];

$$\ddot{q} = M^{-1}(q).\{\tau - N(q,\dot{q})\} \tag{5}$$

To implementation (5) the first step is implement the kinetic energy matrix (M) parameters by used of Lagrange's formulation. The second step is implementing the Coriolis and Centrifugal matrix which they can calculate by partial derivatives of kinetic energy. The last step to implement the dynamic equation of robot manipulator is to find the gravity vector by performing the summation of Lagrange's formulation.

The kinetic energy equation (M) is a $n \times n$ symmetric matrix that can be calculated by the following equation;

$$M(\theta) = m_1 J_{v1}^T J_{v1} + J_{\omega 1}^{TC1} I_1 J_{\omega 1} + m_2 J_{v2}^T J_{v2} + J_{\omega 2}^{TC2} I_2 J_{\omega 2} + m_3 J_{v3}^T J_{v3} + J_{\omega 3}^{TC3} I_3 J_{\omega 3} + m_4 J_{v4}^T J_{v4} \cdot m_5 J_{v5}^T J_{v5} + J_{\omega 5}^{TC5} I_5 J_{\omega 5} + m_6 J_{v6}^T J_{v6} + J_{\omega 6}^{TC6} I_6 J_{\omega 6} \tag{6}$$

As mentioned above the kinetic energy matrix in n DOF is a $n \times n$ matrix that can be calculated by the following matrix [1, 6]

$$M(q) = \begin{bmatrix} M_{11} & M_{12} & \cdots & \cdots & \cdots & M_{1n} \\ M_{21} & \cdots & \cdots & \cdots & \cdots & M_{2n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ M_{n.1} & \cdots & \cdots & \cdots & \cdots & M_{n.n} \end{bmatrix} \tag{7}$$

The Coriolis matrix (B) is a $n \times \frac{n(n-1)}{2}$ matrix which calculated as follows;

$$B(q) = \begin{bmatrix} b_{112} & b_{113} & \cdots & b_{11n} & b_{123} & \cdots & b_{12n} & \cdots & \cdots & b_{1.n-1.n} \\ b_{212} & \cdots & \cdots & b_{21n} & b_{223} & \cdots & \cdots & \cdots & \cdots & b_{2.n-1.n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{n.12} & \cdots & \cdots & b_{n.1.n} & \cdots & \cdots & \cdots & \cdots & \cdots & b_{n.n-1.n} \end{bmatrix} \tag{8}$$

and the Centrifugal matrix (C) is a $n \times n$ matrix;

$$C(q) = \begin{bmatrix} C_{11} & \cdots & C_{1n} \\ \vdots & \ddots & \vdots \\ C_{n1} & \cdots & C_{nn} \end{bmatrix} \tag{9}$$

And last the Gravity vector (G) is a $n \times 1$ vector;

$$G(q) = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_n \end{bmatrix} \tag{10}$$

Sliding mode controller (SMC): SMC is a powerful nonlinear controller which has been analyzed by many researchers especially in recent years. This theory was first proposed in the early 1950 by Emelyanov and several co-workers and has been extensively developed since then with the invention of high speed control devices [2]. The main reason to opt for this controller is its acceptable control performance in wide range and solves two most important challenging topics in control which names, stability and robustness [7, 17-20]. Sliding mode control theory for control of robot manipulator was first proposed in 1978 by Young to solve the set point problem ($\dot{q}_d = 0$) by discontinuous method in the following form [19, 3];

$$\tau_{(q,t)} = \begin{cases} \tau_i^+(q,t) & if \ S_i > 0 \\ \tau_i^-(q,t) & if \ S_i < 0 \end{cases} \tag{11}$$

where S_i is sliding surface (switching surface), $i = 1, 2, \dots, n$ for *n*-DOF robot manipulator, $\tau_i(q,t)$ is the i^{th} torque of joint. Sliding mode controller is divided into two main sub controllers: discontinues controller(τ_{dis}) and equivalent controller(τ_{eq}).

Robot manipulators are one of the highly nonlinear and uncertain systems which caused to needed to robust controller. This section provides introducing the formulation of sliding mode controller to robot manipulator based on [1, 6]Consider a nonlinear single input dynamic system of the form [6]:

$$x^{(n)} = f(\vec{x}) + b(\vec{x})u \tag{12}$$

Where u is the vector of control input, $x^{(n)}$ is the n^{th} derivation of x, $x = [x, \dot{x}, \ddot{x}, \dots, x^{(n-1)}]^T$ is the state vector, $f(x)$ is unknown or uncertainty, and $b(x)$ is of known *sign* function. The control problem is truck to the desired state; $x_d = [x_d, \dot{x}_d, \ddot{x}_d, \dots, x_d^{(n-1)}]^T$, and have an acceptable error which is given by:

$$\tilde{x} = x - x_d = [\tilde{x}, \dots, \tilde{x}^{(n-1)}]^T \tag{13}$$

A time-varying sliding surface $s(x,t)$ is given by the following equation:

$$s(x,t) = (\frac{d}{dt} + \lambda)^{n-1}\, \tilde{x} = 0 \tag{14}$$

where λ is the positive constant. To further penalize tracking error integral part can be used in sliding surface part as follows:

$$s(x,t) = (\frac{d}{dt} + \lambda)^{n-1} \left(\int_0^t \tilde{x}\, dt\right) = 0 \tag{15}$$

The main target in this methodology is kept the sliding surface slope $s(x,t)$ near to the zero. Therefore, one of the common strategies is to find input U outside of $s(x,t)$.

$$\frac{1}{2}\frac{d}{dt} s^2(x,t) \le -\zeta |s(x,t)| \tag{16}$$

where ζ is positive constant and in equation (16) forces tracking trajectories is towards sliding condition.

$$\text{If } \mathbf{S(0)>0} \rightarrow \frac{d}{dt} S(t) \le -\zeta \tag{17}$$

To eliminate the derivative term, it is used an integral term from t=0 to t=t_{reach}

$$\int_{t=0}^{t=t_{reach}} \frac{d}{dt} S(t) \le -\int_{t=0}^{t=t_{reach}} \eta \rightarrow S\,(t_{reach}) - S(0) \le -\zeta(t_{reach} - 0) \tag{18}$$

Where t_{reach} is the time that trajectories reach to the sliding surface so, suppose S($t_{reach} = 0$) defined as

$$0 - S(0) \le -\eta(t_{reach}) \rightarrow t_{reach} \le \frac{S(0)}{\zeta} \tag{19}$$

and

$$if\ S(0) < 0 \rightarrow 0 - S(0) \le -\eta(t_{reach}) \rightarrow S(0) \le -\zeta(t_{reach}) \rightarrow t_{reach} \le \frac{|S(0)|}{\eta} \tag{20}$$

Equation (20) guarantees time to reach the sliding surface is smaller than $\frac{|S(0)|}{\zeta}$ since the trajectories are outside of $S(t)$.

$$if\ S_{t_{reach}} = S(0) \rightarrow error(x - x_d) = 0 \tag{21}$$

suppose S is defined as

$$s(x,t) = (\frac{d}{dt} + \lambda)\quad \tilde{x} = (\dot{x} - \dot{x}_d) + \lambda(x - x_d) \tag{22}$$

The derivation of S, namely, $\dot{S}$ can be calculated as the following;

$$\dot{S} = (\ddot{x} - \ddot{x}_d) + \lambda(\dot{x} - \dot{x}_d) \tag{23}$$

suppose the second order system is defined as;

$$\ddot{x} = f + u \rightarrow \dot{S} = f + U - \ddot{x}_d + \lambda(\dot{x} - \dot{x}_d) \tag{24}$$

Where f is the dynamic uncertain, and also since $S = 0$ *and* $\dot{S} = 0$, to have the best approximation ,$\hat{U}$ is defined as

$$\hat{U} = -\hat{f} + \ddot{x}_d - \lambda(\dot{x} - \dot{x}_d) \tag{25}$$

A simple solution to get the sliding condition when the dynamic parameters have uncertainty is the switching control law:

$$U_{dis} = \hat{U} - K(\vec{x},t) \cdot \mathrm{sgn}(s) \quad \textbf{(26)}$$

where the switching function $\mathbf{sgn}(S)$ is defined as

$$sgn(s) = \begin{cases} 1 & s > 0 \\ -1 & s < 0 \\ 0 & s = 0 \end{cases} \quad \textbf{(27)}$$

and the $K(\vec{x},t)$ is the positive constant. Suppose by (26) the following equation can be written as,

$$\frac{1}{2}\frac{d}{dt}s^2(x,t) = \dot{S} \cdot S = [f - \hat{f} - K\mathrm{sgn}(s)] \cdot S = (f - \hat{f}) \cdot S - K|S| \quad \textbf{(28)}$$

and if the equation (27) instead of (28) the sliding surface can be calculated as

$$s(x,t) = (\frac{d}{dt} + \lambda)^2 \left(\int_0^t \tilde{x}\, dt \right) = (\dot{\mathrm{x}} - \dot{\mathrm{x}}_{\mathrm{d}}) + 2\lambda(\dot{\mathrm{x}} - \dot{\mathrm{x}}_{\mathrm{d}}) - \lambda^2(\mathrm{x} - \mathrm{x}_{\mathrm{d}}) \quad \textbf{(29)}$$

in this method the approximation of U is computed as

$$\hat{U} = -\hat{f} + \ddot{x}_d - 2\lambda(\dot{\mathrm{x}} - \dot{\mathrm{x}}_{\mathrm{d}}) + \lambda^2(\mathrm{x} - \mathrm{x}_{\mathrm{d}}) \quad \textbf{(30)}$$

Therefore the switching function $\mathbf{sgn}(S)$ is added to the control law as

$$U = K(\vec{x},t) \cdot \mathrm{Sgn}(S) \quad \textbf{(31)}$$

Based on above discussion, the control law for a multi degrees of freedom robot manipulator is written as:

$$\tau = \tau_{eq} + \tau_{dis} \quad \textbf{(32)}$$

Where, the model-based component τ_{eq} is the nominal dynamics of systems and τ_{eq} can be calculate as follows:

$$\tau_{eq} = [M^{-1}(B + C + G) + \dot{S}]M \quad \textbf{(33)}$$

and τ_{dis} is computed as;

$$\tau_{dis} = K \cdot \mathrm{Sgn}(S) \quad \textbf{(34)}$$

the control output can be written as;

$$\tau = \tau_{eq} + K.\mathrm{Sgn}(S) \quad \textbf{(35)}$$

Figure 1 shows the position classical sliding mode control for robot manipulator. By (34) and (35) the sliding mode control of robot manipulator is calculated as;

$$\tau = [M^{-1}(B + C + G) + \dot{S}]M + K \cdot \mathrm{Sgn}(S) \quad \textbf{(36)}$$

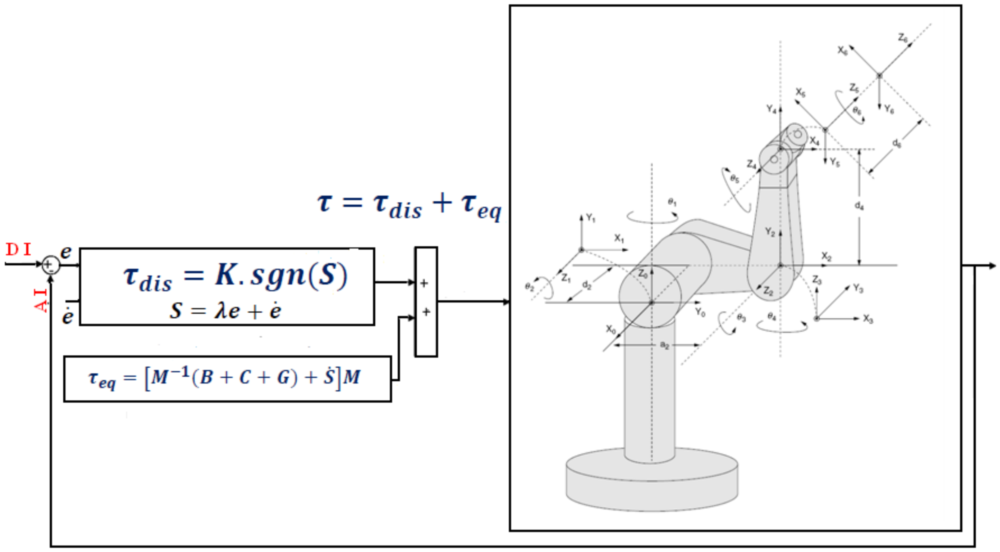

FIGURE 1: Diagram of classical sliding mode controller [3, 9-16, 64-67]

The Lyapunov formulation can be written as follows,

$$V = \frac{1}{2} S^T.M.S \tag{37}$$

the derivation of V can be determined as,

$$\dot{V} = \frac{1}{2} S^T.\dot{M}.S + S^T\ M\dot{S} \tag{38}$$

the dynamic equation of robot manipulator can be written based on the sliding surface as

$$M\dot{S} = -VS + M\dot{S} + VS + G - \tau \tag{39}$$

it is assumed that

$$S^T(\dot{M} - 2V)S = 0 \tag{40}$$

by substituting (39) in (38)

$$\dot{V} = \frac{1}{2} S^T\dot{M}S - S^T VS + S^T(M\dot{S} + VS + G - \tau) = S^T(M\dot{S} + VS + G - \tau) \tag{41}$$

suppose the control input is written as follows

$$\hat{\tau} = \widehat{\tau_{eq}} + \widehat{\tau_{dis}} = [\hat{M}^{-1}(\hat{V} + \hat{G}) + \dot{S}]\hat{M} + K.sgn(S) + K_v S \tag{42}$$

by replacing the equation (42) in (41)

$$\dot{V} = S^T(M\dot{S} + VS + G - \hat{M}\dot{S} - \hat{V}S - \hat{G} - K_v S - Ksgn(S) = S^T\left(\tilde{M}\dot{S} + \tilde{V}S + \tilde{G} - K_v S - Ksgn\right. \tag{43}$$

it is obvious that

$$|\tilde{M}\dot{S} + \tilde{V}S + \tilde{G} - K_v S| \leq |\tilde{M}\dot{S}| + |\tilde{V}S| + |\tilde{G}| + |K_v S| \tag{44}$$

the Lemma equation in robot manipulator system can be written as follows

$$K_u = [|\tilde{M}\dot{S}| + |VS| + |G| + |K_v S| + \eta]_i\ , i = 1,2,3,4,\ldots \tag{45}$$

the equation (40) can be written as

$$K_u \geq \left|[\tilde{M}\dot{S} + VS + G - K_v S]_i\right| + \eta_i \tag{46}$$

therefore, it can be shown that

$$\dot{V} \le -\sum_{i=1}^{n} \eta_i |S_i| \tag{47}$$

Consequently the equation (47) guaranties the stability of the Lyapunov equation

Problem statements: Even though, sliding mode controller is used in wide range areas but, pure it has chattering problem and nonlinear dynamic part challenges. On the other hand, fuzzy logic controller has been used for nonlinear and uncertain systems controlling. Conversely pure fuzzy logic controller (FLC) works in many areas, it cannot guarantee the basic requirement of stability and acceptable performance[8]. Although both SMC and FLC have been applied successfully in many applications but they have some limitations. The boundary layer method is used to reduce or eliminate the chattering and proposed fuzzy Lyapunov estimator method focuses on substitution fuzzy logic system instead of dynamic nonlinear equation to implement easily and avoid mathematical model base controller. To reduce the effect of uncertainty in proposed method, novel antecedent and consequent adaptive method is applied to fuzzy sliding mode controller in robot manipulator.

Objectives: The main goal is to design a novel fuzzy adaptive fuzzy estimation sliding mode methodology which applied to robot manipulator with easy to design and implement. Robot manipulator has nonlinear dynamic and uncertain parameters consequently; following objectives have been pursuit in the mentioned research: To develop a chattering in a position pure variable structure controller against uncertainties, to design and implement a Lyapunov fuzzy structure variable controller in order to solve the equivalent problems with minimum rule base and finally to develop a position fuzzy (antecedent and consequent) adaptive fuzzy estimation sliding mode controller in order to solve the disturbance rejection and reduce the computation load.

3. METHODOLOGY: DESIGN A NOVEL FUZZY (ANTECEDENT AND CONSEQUENT) ADAPTIVE FUZZY ESTIMATION SLIDING MODE CONTROLLER

First part is focused on eliminate the oscillation (chattering) in pure SMC based on linear boundary layer method. To reduce or eliminate the chattering it is used the boundary layer method; in boundary layer method the basic idea is replace the discontinuous method by saturation (linear) method with small neighborhood of the switching surface. This replace is caused to increase the error performance [20-24].

$$B(t) = \{x, |S(t)| \le \emptyset\};\ \emptyset > 0 \tag{48}$$

Where $\emptyset$ is the boundary layer thickness. Therefore, to have a smote control law, the saturation function $Sat(S/_{\emptyset})$ added to the control law:

$$U = K(\vec{x},t).Sat\left(S/_{\emptyset}\right) \tag{49}$$

Where $Sat\left(S/_{\emptyset}\right)$ can be defined as

$$sat\left(S/_{\emptyset}\right) = \begin{cases} 1 & (S/_{\emptyset} > 1) \\ -1 & \left(S/_{\emptyset} < 1\right) \\ S/_{\emptyset} & (-1 < S/_{\emptyset} < 1) \end{cases} \tag{50}$$

Based on above discussion, the control law for a robot manipulator is written as [10-24]:

$$U = U_{eq} + U_r \tag{51}$$

Where, the model-based component U_{eq} is the nominal dynamics of systems and U_{eq} can be calculate as follows:

$$U_{eq} = \left[M^{-1}(B + C + G) + \dot{S}\right]M \tag{52}$$

and U_{sat} is computed as;

$$U_{sat} = K \cdot \mathrm{sat}\left(S/_{\emptyset}\right) \tag{53}$$

the control output can be written as;

$$U = U_{eq} + K.\mathrm{sat}\left({}^{S}/_{\emptyset}\right) = \begin{cases} U_{eq} + K.\mathrm{sgn}(S) & , |S| \geq \emptyset \\ U_{eq} + K.{}^{S}/_{\emptyset} & , |S| < \emptyset \end{cases} \quad \textbf{(54)}$$

Figure 2 is shown classical variable structure which eliminates the chattering using linear boundary layer method.

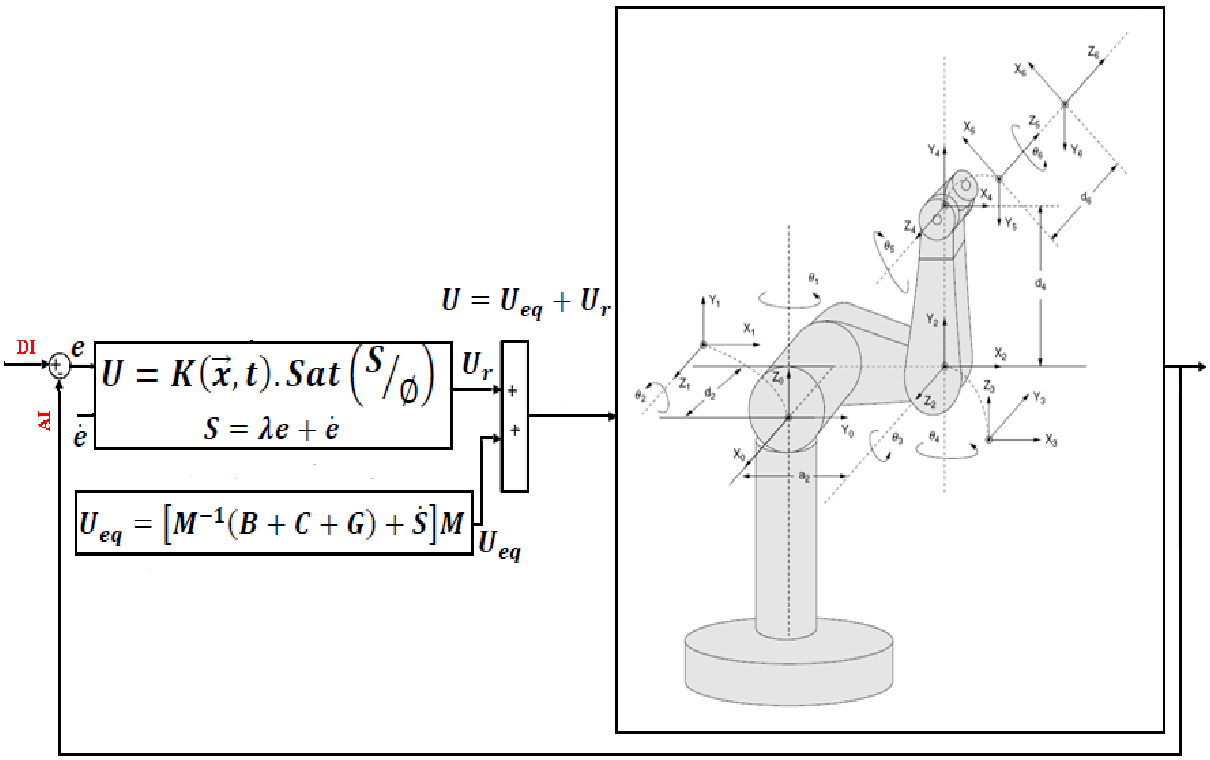

FIGURE 2: Chattering free Block diagram of a SMC: applied to robot manipulator

Second step is focused on design fuzzy estimation variable structure based on Lyapunov formulation. The firs type of fuzzy systems is given by

$$f(x) = \sum_{l=1}^{M} \theta^{l} \mathcal{E}^{l}(x) = \theta^{T} \mathcal{E}(x) \quad \textbf{(55)}$$

Where $\theta = (\theta^{1}, \dots, \theta^{M})^{T}, \mathcal{E}(x) = (\mathcal{E}^{1}(x), \dots, \mathcal{E}^{M}(x))^{T}, and\ \mathcal{E}^{l}(x) =: \prod_{i=1}^{n} \frac{\mu_{A_i^l}(x_i)}{} \sum_{l=1}^{M}(\prod_{i=1}^{n} \mu_{A_i^l}(x_i))$. $\theta^{1}, \dots, \theta^{M}$ are adjustable parameters in (55) . $\mu_{A_1^1}(x_1), \dots, \mu_{A_n^m}(x_n)$ are given membership functions whose parameters will not change over time.

The second type of fuzzy systems is given by

$$f(x) = \frac{\sum_{l=1}^{M} \theta^{l} \left[\prod_{i=1}^{n} \exp\left(-\left(\frac{x_i - \alpha_i^l}{\delta_i^l}\right)^2\right)\right]}{\sum_{l=1}^{M} \left[\prod_{i=1}^{n} \exp\left(-\left(\frac{x_i - \alpha_i^l}{\delta_i^l}\right)^2\right)\right]} \quad \textbf{(56)}$$

Where θ^l , α_i^l *and* δ_i^l are all adjustable parameters. From the universal approximation theorem, we know that we can find a fuzzy system to estimate any continuous function. For the first type of fuzzy systems, we can only adjust θ^l in (55). We define $f^{\wedge}(x|\theta)$ as the approximator of the real function $f(x)$.

$$f^{\wedge}(x|\theta) = \theta^T \varepsilon(x) \quad \textbf{(57)}$$

We define θ^* as the values for the minimum error:

$$\theta^* = \arg\min_{\theta \in \Omega} \left[\sup_{x \in U} |f^{\wedge}(x|\theta) - g(x)| \right] \quad \textbf{(58)}$$

Where Ω is a constraint set for θ. For specific x , $sup_{x \in U} |f^{\wedge}(x|\theta^*) - f(x)|$ is the minimum approximation error we can get.

We used the first type of fuzzy systems (56) to estimate the nonlinear system (20) the fuzzy formulation can be write as below;

$$f(x|\theta) = \theta^T \varepsilon(x) = \frac{\sum_{l=1}^{n} \theta^l [\mu_{A^l}(x)]}{\sum_{l=1}^{n} [\mu_{A^l}(x)]} \quad \textbf{(59)}$$

Where $\theta^1, \dots, \theta^n$ are adjusted by an adaptation law. The adaptation law is designed to minimize the parameter errors of $\theta - \theta^*$. A fuzzy system is designed to compensate the uncertainties of the nonlinear system. The control input is given by

$$\tau = M^{\wedge}\ddot{q}_r + C_1^{\wedge}\dot{q}_r + G^{\wedge} - F^{\wedge}(s) - F_{cp}(s) \quad \textbf{(60)}$$

Where $F^{\wedge}(s) = [f_1^{\wedge}(s_1), \dots, f_m^{\wedge}(s_m)]^T$ and $F_{cp}(S) = [F_{cp1}(s_1), \dots, F_{cp_m}(s_m)]^T$. We define $F = \Delta M\ddot{q}_r + \Delta C_1\dot{q}_r + \Delta G$ where $F = [f_1, \dots, f_m]^T$, $\Delta M = M^{\wedge} - M$, $\Delta C_1 = C_1^{\wedge} - C_1$ and $\Delta G_1 = G^{\wedge} - G$. From Universal Approximation Theorem in (58), there exists an optimal fuzzy controller $f^{\wedge *}_j(s_j)$ such that:

$$f_j = f^{\wedge *}_j(e, \dot{e}) + \Delta_J \quad \textbf{(61)}$$

Where Δ_J is the minimum approximation error.

The fuzzy if-then rules are given by fuzzy rule base. In (59), we assume $\sum_{L=1}^{M} \mu_{A_j^L}(e, \dot{e}) = 1$ and $\varepsilon_j^L(e, \dot{e})$ becomes

$$\varepsilon_j^L(s_j) = \frac{\mu_{A_j^L}(e, \dot{e})}{\sum_{L=1}^{M} [\mu_{A_j^L}(e, \dot{e})]} = \mu_{A_j^L}(e, \dot{e}) \quad \textbf{(62)}$$

Where we define $\varphi_j^L = \mu_{A_j^L}(e, \dot{e})$. The membership function $\mu_{A_j^L}(e, \dot{e})$ is a Gaussian membership function represented by

$$\mu_{A_j^L}(s_j) = \exp\left[-(\sigma_j^l((e, e) - \alpha_j^l))^2\right] \quad \textbf{(63)}$$

Then the fuzzy estimator $f^{\wedge}_j(e, \dot{e})$ is given as

$$f^{\wedge}_j(e, \dot{e}) = \theta_j^T \varphi_j \quad \textbf{(64)}$$

where $\theta_j = [\theta_j^1, \dots, \theta_j^M]^T$, $\varphi_j = [\varphi_j^1, \varphi_j^2, \dots, \varphi_j^M]^T$. We define $\tilde{f}_j$ such that

$$\tilde{f}_j = f_j - f^{\wedge}_j(e, \dot{e})$$

$$
\begin{aligned}
&= f^{\wedge *}_{j}(e,\dot{e}) - f^{\wedge}_{j}(e,\dot{e}) + \Delta_j \\
&= \theta_j^{*T}\varphi_j^* - \theta_j^T\varphi_j + \Delta_j
\end{aligned}
\quad \textbf{(65)}
$$

where θ_j^* and φ_j^* are the optimal values based on Universal Approximation Theorem in (58). We define $\tilde{\theta}_j = \theta_j^* - \theta_j$, $\tilde{\varphi}_j = \varphi_j^* - \varphi_j$ and (6) is rewritten as

$$
\begin{aligned}
\tilde{f}_j &= \left(\theta_j + \tilde{\theta}_j\right)^T\left(\varphi_j + \tilde{\varphi}_j\right) - \theta_j^T\varphi_j + \Delta_j \\
&= \theta_j^T\tilde{\varphi}_j + \tilde{\theta}_j^T\varphi_j + \tilde{\theta}_j^T\tilde{\varphi}_j + \Delta_j
\end{aligned}
\quad \textbf{(66)}
$$

We take Taylor series expansion of φ_j around two vectors α_j and σ_j where $\alpha_j = [\alpha_j^1, \dots, \alpha_j^M]^T$ and $\alpha_j = [\sigma_j^1, \dots, \sigma_j^M]^T$ (α_j^l and σ_j^l are defined in (63)):

$$
\varphi_j^* = \varphi_j + \frac{\partial\varphi_j}{\partial\alpha_j}\tilde{\alpha}_j + \frac{\partial\varphi_j}{\partial\alpha_j}\tilde{\sigma}_j + h.o.t. \quad \textbf{(67)}
$$

where $\tilde{\alpha}_j = \alpha_j^* - \alpha_j$, $\tilde{\varphi}_j = \varphi_j^* - \varphi_j$ and $h.o.t.$ denotes the higher order terms. We rewrite (67) as

$$
\begin{aligned}
\tilde{\varphi}_j &= \frac{\partial\varphi_j}{\partial\alpha_j}\tilde{\alpha}_j + \frac{\partial\varphi_j}{\partial\alpha_j}\tilde{\sigma}_j + h.o.t. \\
&= B_j\tilde{\alpha}_j + C_j\tilde{\sigma}_j + h.o.t.
\end{aligned}
\quad \textbf{(68)}
$$

where

$$
B_j = \begin{bmatrix}
\frac{\partial\varphi_j^1}{\partial\alpha_j^1} & \frac{\partial\varphi_j^2}{\partial\alpha_j^1} & \cdots & \frac{\partial\varphi_j^M}{\partial\alpha_j^1} \\
\frac{\partial\varphi_j^1}{\partial\alpha_j^2} & \frac{\partial\varphi_j^2}{\partial\alpha_j^2} & \cdots & \frac{\partial\varphi_j^M}{\partial\alpha_j^2} \\
\vdots & & & \\
\frac{\partial\varphi_j^1}{\partial\alpha_j^M} & \frac{\partial\varphi_j^2}{\partial\alpha_j^M} & \cdots & \frac{\partial\varphi_j^M}{\partial\alpha_j^M}
\end{bmatrix}, \qquad
C_j = \begin{bmatrix}
\frac{\partial\varphi_j^1}{\partial\alpha_j^1} & \frac{\partial\varphi_j^2}{\partial\alpha_j^1} & \cdots & \frac{\partial\varphi_j^M}{\partial\alpha_j^1} \\
\frac{\partial\varphi_j^1}{\partial\alpha_j^2} & \frac{\partial\varphi_j^2}{\partial\alpha_j^2} & \cdots & \frac{\partial\varphi_j^M}{\partial\alpha_j^2} \\
\vdots & & & \\
\frac{\partial\varphi_j^1}{\partial\alpha_j^M} & \frac{\partial\varphi_j^2}{\partial\alpha_j^M} & \cdots & \frac{\partial\varphi_j^M}{\partial\alpha_j^M}
\end{bmatrix}
\quad \textbf{(69)}
$$

We substitute (68) into (66):

$$
\begin{aligned}
\tilde{f}_j &= \theta_j^T\left(B_j\tilde{\alpha}_j + C_j\tilde{\sigma}_j + h.o.t.\right) + \tilde{\theta}_j^T\varphi_j + \tilde{\theta}_j^T\tilde{\varphi}_j + \Delta_j \\
&= \theta_j^T B_j\tilde{\alpha}_j + \theta_j^T C_j\tilde{\sigma}_j + \tilde{\theta}_j^T\varphi_j + \theta_j^T h.o.t. + \tilde{\theta}_j^T\tilde{\varphi}_j + \Delta_j \\
&= \theta_j^T B_j\tilde{\alpha}_j + \theta_j^T C_j\tilde{\sigma}_j + \tilde{\theta}_j^T\varphi_j + \varepsilon_j
\end{aligned}
\quad \textbf{(70)}
$$

where $\varepsilon_j = \theta_j^T h.o.t. + \tilde{\theta}_j^T\tilde{\varphi}_j + \Delta_j$ is assumed to be bounded by $|\varepsilon_j| \le E_j$. E_j is a constant and the value of E_j uncertain to the designer. We define E^* as the real value and the estimation error is given by

$$
\tilde{E}_j = E_j^* - E_j \quad \textbf{(71)}
$$

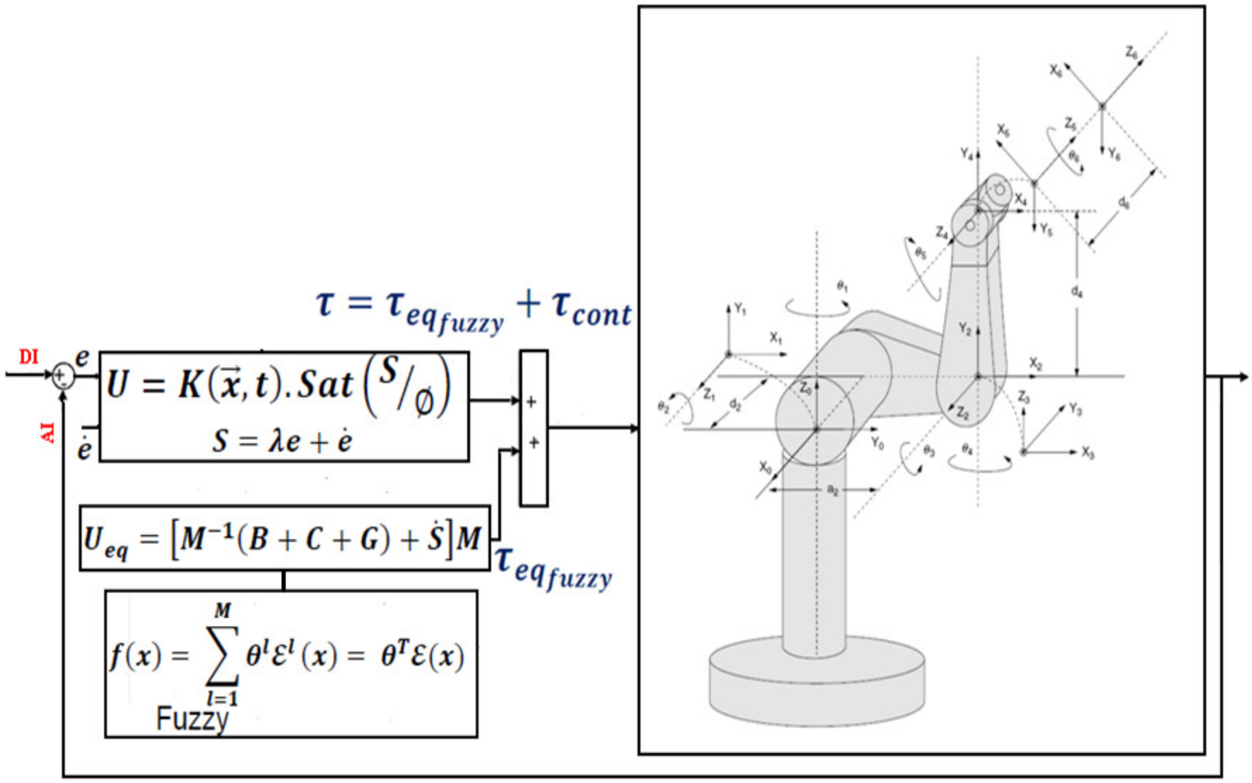

FIGURE 3: Chattering free Block diagram of a fuzzy estimator SMC: applied to robot manipulator

Third step is focused on design fuzzy (antecedent and consequent) adaptive fuzzy estimation sliding mode based on Lyapunov formulation. We produce an adaptation law to online tune the following parameters: θ_j in (64), σ_j^l , α_j^l in (63) and the bound E_j in (71). The adaptation laws are expressed as

$$\dot{\theta}_j = \eta_{j2} s_j \varphi_j \tag{72}$$

$$\dot{\alpha}_j = \eta_{j3} s_j B_j^T \theta_j \tag{73}$$

$$\dot{\sigma}_j = \eta_{j4} s_j C_j^T \theta_j \tag{74}$$

$$f_{cpj}(s_j) = E_j sgn(s_j) \tag{75}$$

$$\dot{E}_j = \eta_{j1} |s_j| \tag{76}$$

where $\eta_{j1}, \eta_{j2}, \eta_{j3}$ and η_{j4} are positive constants; $\theta_j = [\theta_j^1, \theta_j^2, \dots, \theta_j^M]^T$, $\alpha_j = [\alpha_j^1, \alpha_j^2, \dots, \alpha_j^M]^T$, $\sigma_j = [\sigma_j^1, \sigma_j^2, \dots, \sigma_j^M]^T$; B_j , C_j are given in (69); $f_{cpj}(s_j)$ is the compensation term defined in (60).

If the following Lyapunov function candidate defined by:

$$V = \frac{1}{2} s^T M s + \frac{1}{2} \sum_{j=1}^{m} \left(\frac{\tilde{E}_j^2}{\eta_{j1}} + \frac{\tilde{\theta}_j^T \tilde{\theta}}{\eta_{j2}} + \frac{\tilde{\alpha}_j^T \tilde{\alpha}}{\eta_{j3}} + \frac{\tilde{\sigma}_j^T \tilde{\sigma}}{\eta_{j4}} \right) \tag{77}$$

The derivative of V is defined by:

$$\dot{V} = s^T M\dot{s} + \frac{1}{2} s^T \dot{M} s + \sum_{j=1}^{m}\left(\frac{\tilde{E}_j\dot{\tilde{E}}_j}{\eta_{j1}} + \frac{\tilde{\theta}_j^T\dot{\tilde{\theta}}}{\eta_{j2}} + \frac{\tilde{\alpha}_j^T\dot{\tilde{\alpha}}}{\eta_{j3}} + \frac{\tilde{\sigma}_j^T\dot{\tilde{\sigma}}}{\eta_{j4}}\right) \quad \textbf{(78)}$$

where $\tilde{E}_j = E_j^* - E_j$, $\tilde{\theta}_j = \theta_j^* - \theta_j$, $\tilde{\alpha}_j = \alpha_j^* - \alpha_j$, $\tilde{\sigma}_j = \sigma_j^* - \sigma_j$. from robot manipulator formulation and (1):

$$M(q)\ddot{q} + C_1(q,\dot{q})\dot{q} + G(q) = M^{\wedge}\ddot{q}_r + C_1^{\wedge}\dot{q}_r + G^{\wedge} - F^{\wedge}(s) - F_{cp}(s) \quad \textbf{(79)}$$

Since $\dot{M} - 2C_1$ is a skew-symmetric matrix, we can get $s^T M\dot{s} + \frac{1}{2}s^T\dot{M}s = s^T(M\dot{s} + C_1 s)$. From $\dot{q}_r = \dot{q} - s = \dot{q}_d - \lambda e$ and $\ddot{q}_r = \ddot{q} - \dot{s} = \ddot{q}_d - \lambda\dot{e}$

$$M\dot{s} + C_1 s = F - F^{\wedge}(s) - F_{cp}(s) \quad \textbf{(80)}$$

where $= \Delta M\ddot{q}_r + \Delta C_1\dot{q}_r + \Delta G$, $\Delta M = M^{\wedge} - M$, $\Delta C_1 = C_1^{\wedge} - C_1$ and $\Delta G_1 = G^{\wedge} - G$. Then $\dot{V}$ becomes

$$\dot{V} = s^T(M\dot{s} + C_1 s) + \sum_{j=1}^{m}\left(\frac{\tilde{E}_j\dot{\tilde{E}}_j}{\eta_{j1}} + \frac{\tilde{\theta}_j^T\dot{\tilde{\theta}}}{\eta_{j2}} + \frac{\tilde{\alpha}_j^T\dot{\tilde{\alpha}}}{\eta_{j3}} + \frac{\tilde{\sigma}_j^T\dot{\tilde{\sigma}}}{\eta_{j4}}\right)$$

$$= s^T(F - F^{\wedge}(s) - F_{cp}(s)) + \sum_{j=1}^{m}\left(\frac{\tilde{E}_j\dot{\tilde{E}}_j}{\eta_{j1}} + \frac{\tilde{\theta}_j^T\dot{\tilde{\theta}}}{\eta_{j2}} + \frac{\tilde{\alpha}_j^T\dot{\tilde{\alpha}}}{\eta_{j3}} + \frac{\tilde{\sigma}_j^T\dot{\tilde{\sigma}}}{\eta_{j4}}\right)$$

$$= \sum_{j=1}^{m} s_j(f_j - f^{\wedge}_j(s_j) - f_{cpj}) + \sum_{j=1}^{m}\left(\frac{\tilde{E}_j\dot{\tilde{E}}_j}{\eta_{j1}} + \frac{\tilde{\theta}_j^T\dot{\tilde{\theta}}}{\eta_{j2}} + \frac{\tilde{\alpha}_j^T\dot{\tilde{\alpha}}}{\eta_{j3}} + \frac{\tilde{\sigma}_j^T\dot{\tilde{\sigma}}}{\eta_{j4}}\right)$$

$$= \sum_{j=1}^{m} s_j(\theta_j^T B_j\tilde{\alpha}_j + \theta_j^T C_j\tilde{\sigma}_j + \tilde{\theta}_j^T\varphi_j + \varepsilon_j - f_{cpj}) - \sum_{j=1}^{m}\left(\frac{\tilde{E}_j\dot{\tilde{E}}_j}{\eta_{j1}} + \frac{\tilde{\theta}_j^T\dot{\tilde{\theta}}}{\eta_{j2}} + \frac{\tilde{\alpha}_j^T\dot{\tilde{\alpha}}}{\eta_{j3}} + \frac{\tilde{\sigma}_j^T\dot{\tilde{\sigma}}}{\eta_{j4}}\right)$$

$$= \sum_{j=1}^{m}\left[\tilde{\theta}_j^T\left(s_j\varphi_j - \frac{\dot{\theta}_j}{\eta_{j2}}\right) + \tilde{\alpha}_j^T\left(s_j B_j^T\theta_j - \frac{\dot{\alpha}_j}{\eta_{j3}}\right) + \tilde{\sigma}_j^T\left(s_j C_j^T\theta_j - \frac{\dot{\sigma}_j}{\eta_{j4}}\right)\right] + \sum_{j=1}^{m}\left(s_j\varepsilon_j - s_j f_{cpj}\frac{\tilde{E}_j\dot{E}_j}{\eta_{j1}}\right) \quad \textbf{(81)}$$

We substitute the adaption law (72)-(76) in to (81):

$$\dot{V} = \sum_{j=1}^{m}[s_j\varepsilon_j - s_j E_j sgn(s_j) - \tilde{E}_j|s_j|]$$

$$= \sum_{j=1}^{m}[s_j\varepsilon_j - s_j E_j sgn(s_j) - (E_j^* - E_j)s_j sgn(s_j)]$$

$$= \sum_{j=1}^{m}[s_j\varepsilon_j - E_j^* s_j sgn(s_j)]$$

$$= \sum_{j=1}^{m}[|s_j||\varepsilon_j| - E_j^*|s_j|]$$

$$= \sum_{j=1}^{m}[|s_j|(|\varepsilon_j| - E_j^*)] \le 0 \quad \textbf{(82)}$$

where $\dot{V}$ is negative semidefinite . We define $\dot{V}_j = |s_j(t)|(|\varepsilon_j| - E_j^*)$. From $\dot{V}_j \le 0$, we can get $s_j(t)$ is bounded. We assume $|s_j(t)| \le \eta_s$ and rewrite $|s_j(t)|(E_j^* - |\varepsilon_j|) \le -\dot{V}_j$ as

$$s_j(t) \le \frac{1}{E_j^*}|s_j(t)||\varepsilon_j| - \frac{1}{E_j^*}\dot{V}_j \le \frac{\eta_s}{E_j^*}|\varepsilon_j| - \frac{1}{E_j^*}\dot{V}_j \quad \textbf{(83)}$$

Then we take the integral on both sides of (83):

$$\int_0^t |s_j(v)|dv \le \frac{\eta_s}{E_j^*}\int_0^t |\varepsilon_j|dv + \frac{1}{E_j^*}\left(V_j(0) - V_j(t)\right) \le \frac{\eta_s}{E_j^*}\int_0^t |\varepsilon_j|dv + \frac{1}{E_j^*}\left(|V_j(0)| - |V_j(t)|\right) \quad \textbf{(84)}$$

If $\varepsilon_j \in L_1$, we can get $s_j \in L_1$ from (84). Since we can prove $\dot{s}_j$ is bounded (see proof in [15]), we have $\dot{s}_j \in L_\infty$. We introduce the following Barbalat's Lemma[16].

Lemma: let $\emptyset: R \to R$ be a uniformly continues function on $[0,\infty)$. Suppose that $lim_{t\to\infty}\int_0^t \emptyset(\tau)d\tau$ exists and is finite. Then,

$$lim_{t\to\infty}\emptyset(t) = 0$$

Therefore, by using Barbalat's Lemma, we can get $lim_{t\to\infty}s_j(t) = 0$ and $lim_{t\to\infty}e_j(t) = 0$. Figure 4 is shown fuzzy (antecedent and consequent) adaptive fuzzy estimator sliding mode controller.

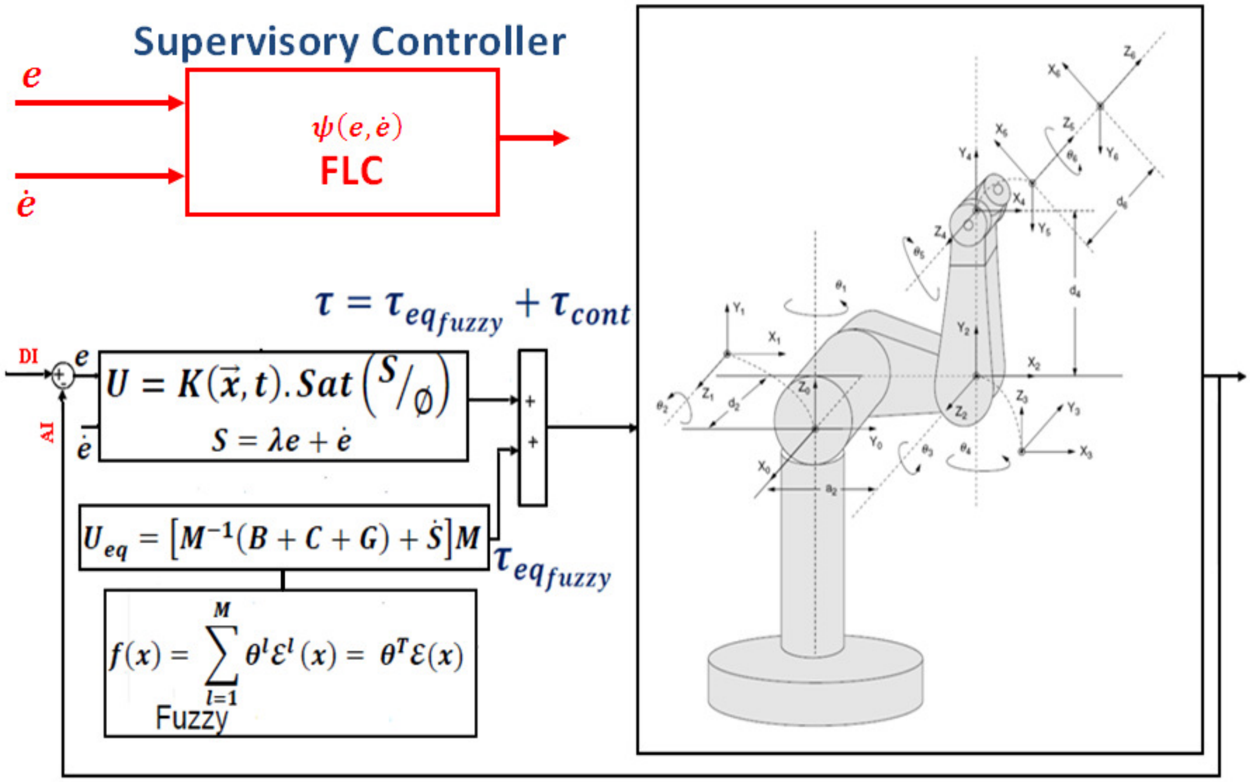

FIGURE 4: Chattering free Block diagram of a fuzzy adaptive (consequent and antecedent) fuzzy estimator SMC: applied to robot manipulator

4. RESULTS

This methodology can online adjust both the premise and the consequent parts of the fuzzy rules. In this method we choose $\hat{M}, \hat{B}$ and $\hat{C}$ for compensation. We define five membership functions for each input variable based on

$$\mu_{A_j^l}(S_j) = \exp[-(\sigma_j^l(S_j-\alpha_j^l))^2] \quad \textbf{(85)}$$

Sliding mode controller (SMC) and proposed fuzzy (premise and the consequent) adaptive fuzzy estimator sliding mode controller were tested to sinus response trajectory. The simulation was implemented in Matlab/Simulink environment. Link/joint trajectory and disturbance rejection are compared in these controllers. These systems are tested by band limited white noise with a predefined 40% of relative to the input signal amplitude.

Link/joint Trajectory: Figure 5 shows the link/joint trajectory in SMC and proposed methodology without disturbance for sinus trajectory. (This techniques is applied to first two link of PUMA robot manipulator)

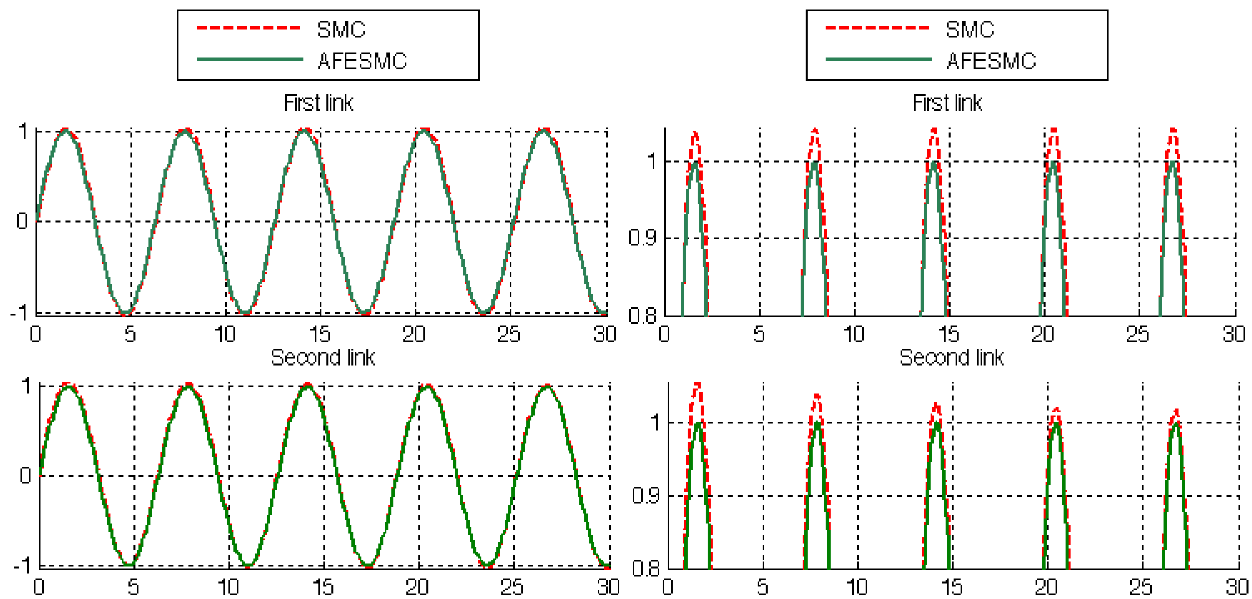

FIGURE 5: Proposed method **(**AFESMC) and SMC trajectory: applied to robot manipulator

By comparing sinus response, Figure 5, in SMC and proposed controller, the proposed controller's overshoot **(0%)** is lower than SMC's **(3%).**

Disturbance rejection: Figure 6 is indicated the power disturbance removal in SMC and proposed controller. Besides a band limited white noise with predefined of 40% the power of input signal is applied to the sinus SMC and proposed controller; it found slight oscillations in SMC trajectory responses.

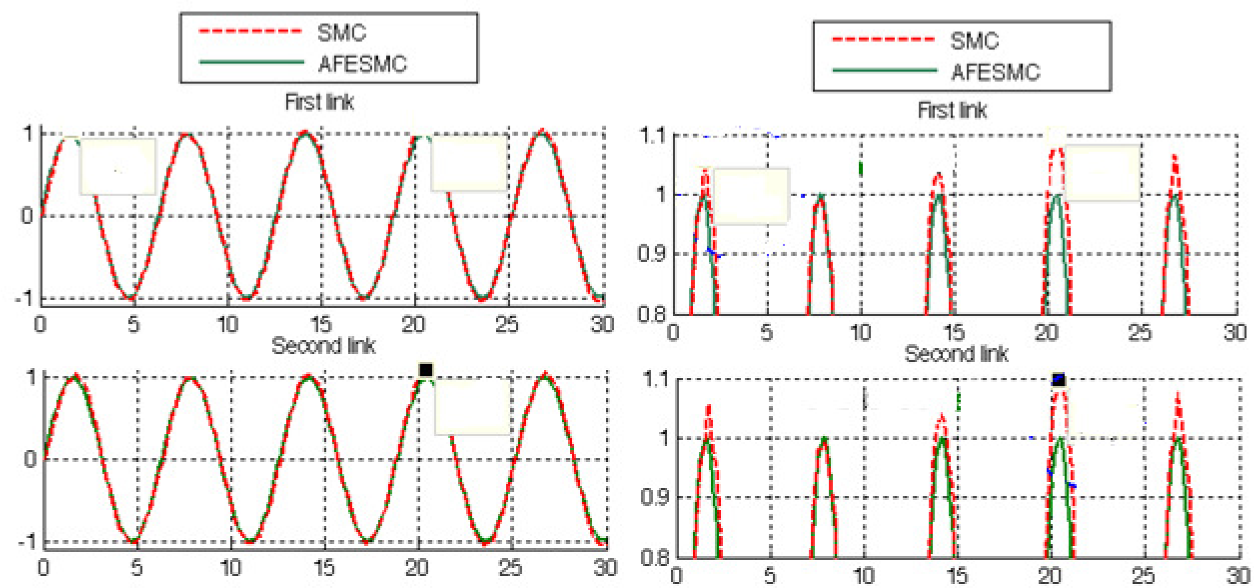

FIGURE 6: Proposed controller (AFESMC) and SMC trajectory with external disturbance: applied to robot manipulator

5 CONCLUSIONS

In this paper, a fuzzy (premise and consequent) adaptive robust control fuzzy estimator sliding mode method is proposed in order to design a high performance robust controller in the presence of structured uncertainties and unstructured uncertainties. The approach improves performance by using the advantages of sliding mode control, fuzzy logic estimation method and adaptive control while the disadvantages attributed to these methods are remedied by each other. This is achieved without increasing the complexities of the overall design and analysis of the controller. The proposed controller attenuates the effort of model uncertainties from both structured uncertainties and unstructured uncertainties. Thus, transient performance and final tracking accuracy is guaranteed by proper design of the controller. The results revealed that adaption of fuzzy rules weights reduce the model uncertainties significantly, and hence farther improvements of the tracking performance can be achieved. This algorithm created a methodology of learning both the premise and the consequence part of fuzzy rules. In this method chattering is eliminated.

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