Design Auto Adjust Sliding Surface Slope: Applied to Robot Manipulator

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Abstract

The main target in this paper is to present the nonlinear methods in order to control the robot manipulators and also the related results. Also the important role of sliding surface slope in sliding mode fuzzy control of robot manipulator should be considered. Sliding mode controller (SMC) is a significant nonlinear controller in certain and uncertain dynamic parameters systems. To solve the chattering phenomenon, this paper complicated two methods to each other; boundary layer method and applied fuzzy logic in sliding mode methodology. To remove the chattering sliding surface slope also played important role so this paper focused on the auto tuning this important coefficient to have the best results by applied mathematical model free methodology. Auto tuning methodology has acceptable performance in presence of uncertainty (e.g., overshoot=0%, rise time=0.8 s, steady state error = 1e-9 and RMS error=0.0001632).

Keywords: Sliding Mode Controller, Fuzzy Logic Methodology, Sliding Mode Fuzzy Methodology, Robotic Manipulator, Auto Tuning Sliding Mode Fuzzy Controller.

1. INTRODUCTION, BACKGROUND and MOTIVATION

A robot system without any controllers does not to have any benefits, because controller is the main part in this sophisticated system. The main objectives to control of robot manipulators are stability, and robustness. Lots of researchers work on design the controller for robotic manipulators to have the best performance. Control of any systems divided in two main groups: linear and nonlinear controller [1-2, 4, 6-7, 9].

However, one of the important challenging in control algorithms is design linear behavior controller to easier implementation for nonlinear systems but these algorithms have some limitation such as controller working area must to be near the system operating point and this adjustment is very difficult specially when the dynamic system parameters have large variations, and when the system has hard nonlinearities [1-4]. Most of robot manipulators which work in industry are usually controlled by linear PID controllers. But the robot manipulator dynamic functions are, nonlinear with strong coupling between joints (low gear ratio), structure and unstructured uncertainty, and multi- inputs multi-outputs (MIMO) which, design linear controller is very difficult especially if the velocity and acceleration of robot manipulator be high and also when the ratio between joints gear be small [1-2]. To eliminate above problems in physical systems most of control researcher go toward to select nonlinear robust controller [1-4].

One of the most important powerful nonlinear robust controllers is sliding mode controller (SMC) [1-4, 7-9]. However Sliding mode control methodology was first proposed in the 1950 but this controller has been analyzed by many researchers in recent years [16, 18-19]. The main reason to select this controller in wide range area is have an acceptable control performance and solve some main challenging topics in control such as resistivity to the external disturbance and stability. Sliding mode controller is divided into two main sub controllers: discontinues controller(τ_{dis}) and equivalent controller(τ_{eq}) [24, 28, 30, 38 – 44]. Discontinues controller causes an acceptable tracking performance at the expense of very fast switching. In the theory of infinity fast switching can provide a good tracking performance but it also can provide some problems (e.g., system instability and chattering phenomenon). After going toward the sliding surface by discontinues term, equivalent term help to the system dynamics match to the sliding surface[1, 6]. When all dynamic and physical parameters are known or limitation unknown the controller works superbly and output responses are good quality; practically a large amount of systems have unlimited or highly uncertainties and sliding mode controller with estimator methodology reduce this kind of limitation. However, this controller has above advantages but, pure sliding mode controller has following disadvantages i.e.chattering problem, sensitivity, and equivalent dynamic formulation [3]. Chattering phenomenon can causes some problems such as saturation and heat the mechanical parts of robot manipulators or drivers. To reduce or eliminate the chattering, various papers have been reported by many researchers which classified into two most important methods: boundary layer saturation method and estimated uncertainties method [1]. In boundary layer saturation method, the basic idea is the discontinuous method replacement by saturation (linear) method with small neighborhood of the switching surface. This replacement caused to increase the error performance against with the considerable chattering reduction. Slotine and Sastry have introduced boundary layer method instead of discontinuous method to reduce the chattering[2]. Slotine has presented sliding mode with boundary layer to improve the industry application [2]. Estimated uncertainty method used in term of uncertainty estimator to compensation of the system uncertainties. It has been used to solve the chattering phenomenon and also nonlinear equivalent dynamic. If estimator has an acceptable performance to compensate the uncertainties, the chattering is reduced. Research on estimated uncertainty to reduce the chattering is significantly growing as their applications such as industrial automation and robot manipulator. For instance, the applications of artificial intelligence, neural networks and fuzzy logic on estimated uncertainty method have been reported in [10-15, 20, 36]. Wang et al. [5] have proposed a simple fuzzy estimator controller beside the discontinuous and equivalent control terms to reduce the chattering. Their design had three main parts i.e. equivalent, discontinuous and fuzzy estimator tuning part which has reduced the chattering very well. After the invention of fuzzy logic theory in 1965 by Zadeh (Zadeh, 1997), this theory was used in wide range area because Fuzzy logic controller (FLC) is one of the most important applications of fuzzy logic theory. Application of fuzzy logic to automatic control was first reported in [10], where, based on Zadeh's proposition, Mamdani buit a controller for a steam engine and boiler combination by synthesizing a set of linguistic expressions in the form of IF-THEN rules as follows: IF (system state) THEN (control action), which will be referred to as "Mamdani controller' hereafter. In Mamdani's controller the knowledge of the system state (the IF part) and the set of actions (the THEN part) are obtained from the experienced human operators [11]. This controller

can be used to control of nonlinear, uncertain, and noisy systems. However pure FLC works in many engineering applications but, it cannot guarantee two most important challenges in control, namely, stability and acceptable performance [4]. In the past three decades, more diversified application domains for fuzzy logic controllers have been created, which range from water cleaning process, home appliances such as air conditioning systems and online recognition of handwritten symbols [10-15, 20, 36]. Some researchers applied fuzzy logic methodology in sliding mode controllers (FSMC) to reduce the chattering and solve the nonlinear dynamic equivalent problems in pure sliding mode controller and the other researchers applied sliding mode methodology in fuzzy logic controller (SMFC) to improve the stability of systems, therefore FSMC is a controller based on SMC but SMFC works based on FLC [17, 21-44].

Adaptive control used in systems whose dynamic parameters are varying and/or have unstructured disturbance and need to be training on line. Adaptive fuzzy inference system provide a good knowledge tools for adjust a complex uncertain nonlinear system with changing dynamics to have an acceptable performance. Combined adaptive method to artificial sliding mode controllers can help to controllers to have a better performance by online tuning the nonlinear and time variant parameters [21-44]. Many dynamic systems to be controlled have unknown or varying uncertain parameters. For instance, robot manipulators may carry large objects with unknown inertial parameters. Generally, the basic objective of adaptive control is to maintain performance of the closed-loop system in the presence of uncertainty (e.g., variation in parameters of a robot manipulator). The above objective can be achieved by estimating the uncertain parameters (or equivalently, the corresponding controller parameters) on-line, and based on the measured system signals. The estimated parameters are used in the computation of the control input. An adaptive system can thus be regarded as a control system with on-line parameter estimation [17, 21-30]. In conventional nonlinear adaptive controllers, the controller attempts to learn the uncertain parameters of particular structured dynamics, and can achieve fine control and compensate for the structure uncertainties and bounded disturbances. On the other hand, adaptive control techniques are restricted to the parameterization of known functional dependency but of unknown Constance. Consequently, these factors affect the existing nonlinear adaptive controllers in cases with a poorly known dynamic model or when the fast real-time control is required. Adaptive control methodologies and their applications to the robot manipulators have widely been studied and discussed in the following references [19-44]. The combined adaptive sliding mode controllers (robust adaptive controllers) have been studied by Slotine and Coetsee [3], Piltan et al. [4, 9], and Wang [5], as a method to overcome the unmodelled dynamics and external disturbances. However, the combined adaptive controllers need a linearly parameterized model of the system under investigation and a priori knowledge of the bounds of uncertainties. Furthermore, the large number of parameters and adaptation gain (i.e., design parameter) corresponding to each unknown parameter introduces more complexity. The problem of application time and computation burden can lead to severe stability and robustness problem (e.g., when the fast real-time control is required). In addition to the above mentioned issued that need to be addressed, there is another issue which makes it necessary to develop a new uncertainty estimation method. That is because, currently, the control theory uses powerful tools which no longer rely on the parameterized dynamic model of the system, such as fuzzy modeling, neural networks and neuro-fuzzy modeling [16-44].

In this research we will highlight a new auto adjust sliding surface slope derived in the Lyapunov sense. This algorithm will be analyzed and evaluated on robotic manipulators. Section 2, is served as a problem statements, robot manipulator dynamics and introduction to the pure sliding mode controller with proof of stability and its application to robot manipulator. Part 3, introduces and describes the methodology algorithms and proves Lyapunov stability. Section 4 presents the simulation results of this algorithm applied to a 3 degree-of-freedom robot manipulator and the final section is describe the conclusion.

2. ROBOT MANIPULATOR DYNAMICS, PROBLEM STATEMENTS AND SLIDING MODE CONTROLLER FORMULATION

Robot Manipulator Dynamic Formulation: The equation of an *n-DOF* robot manipulator governed by the following equation [1, 3, 16-28, 30, 38-40]:

$$M(q)\ddot{q} + N(q,\dot{q}) = \tau \quad (1)$$

Where τ is actuation torque, M (q) is a symmetric and positive define inertia matrix, $N(q,\dot{q})$ is the vector of nonlinearity term. This robot manipulator dynamic equation can also be written in a following form:

$$\tau = M(q)\ddot{q} + B(q)[\dot{q}\,\dot{q}] + C(q)[\dot{q}]^2 + G(q) \quad (2)$$

Where B(q) is the matrix of coriolios torques, C(q) is the matrix of centrifugal torques, and G(q) is the vector of gravity force. The dynamic terms in equation (2) are only manipulator position. This is a decoupled system with simple second order linear differential dynamics. In other words, the component $\ddot{q}$ influences, with a double integrator relationship, only the joint variableq_i, independently of the motion of the other joints. Therefore, the angular acceleration is found as to be [3, 16-28]:

$$\ddot{q} = M^{-1}(q).\{\tau - N(q,\dot{q})\} \quad (3)$$

Sliding Mode Control: This technique is very attractive from a control point of view. The central idea of sliding mode control (SMC) is based on nonlinear dynamic equivalent. It has assumed that the desired motion trajectory for the manipulator $q_d(t)$, as determined, by a path planner. Defines the tracking error as [4-9, 18, 21, 31-44]:

$$e(t) = q_d(t) - q_a(t) \quad (4)$$

Where e(t) is error of the plant, $q_d(t)$ is desired input variable, that in our system is desired displacement, $q_a(t)$ is actual displacement. Consider a nonlinear single input dynamic system of the form [6]:

$$x^{(n)} = f(\vec{x}) + b(\vec{x})u \quad (5)$$

Where u is the vector of control input, $x^{(n)}$ is the nth derivation of$x$, $x = [x, \dot{x}, \ddot{x}, \ldots, x^{(n-1)}]^T$ is the state vector, $f(x)$ is unknown or uncertainty, and $b(x)$ is of known *sign* function. The control problem is truck to the desired state; $x_d = [x_d, \dot{x}_d, \ddot{x}_d, \ldots, x_d{}^{(n-1)}]^T$, and have an acceptable error which is given by:

$$\tilde{x} = x - x_d = [\tilde{x}, \ldots, \tilde{x}^{(n-1)}]^T \quad (6)$$

A time-varying sliding surface $s(x,t)$ is given by the following equation:

$$s(x,t) = (\frac{d}{dt} + \lambda)^{n-1}\,\tilde{x} = 0 \quad (7)$$

where λ is the positive constant. To further penalize tracking error integral part can be used in sliding surface part as follows:

$$s(x,t) = (\frac{d}{dt} + \lambda)^{n-1}\left(\int_0^t \tilde{x}\,dt\right) = 0 \quad (8)$$

The main target in this methodology is kept the sliding surface slope $s(x,t)$ near to the zero. Therefore, one of the common strategies is to find input U outside of $s(x,t)$.

$$\frac{1}{2}\frac{d}{dt}s^2(x,t) \le -\zeta|s(x,t)| \quad (9)$$

where ζ is positive constant.

$$\text{If } S(0)>0 \rightarrow \frac{d}{dt}S(t) \le -\zeta \quad (10)$$

To eliminate the derivative term, it is used an integral term from t=0 to t=t_{reach}

$$\int_{t=0}^{t=t_{reach}} \frac{d}{dt}S(t) \le -\int_{t=0}^{t=t_{reach}} \eta \rightarrow S(t_{reach}) - S(0) \le -\zeta(t_{reach} - 0) \quad (11)$$

Where t_{reach} is the time that trajectories reach to the sliding surface so, suppose S($t_{reach} = 0$) defined as

$$0 - S(0) \leq -\eta(t_{reach}) \rightarrow t_{reach} \leq \frac{S(0)}{\zeta} \tag{12}$$

and

$$if\ S(0) < 0 \rightarrow 0 - S(0) \leq -\eta(t_{reach}) \rightarrow S(0) \leq -\zeta(t_{reach}) \rightarrow t_{reach} \leq \frac{|S(0)|}{\eta} \tag{13}$$

Equation (13) guarantees time to reach the sliding surface is smaller than $\frac{|S(0)|}{\zeta}$ since the trajectories are outside of $S(t)$.

$$if\ S_{t_{reach}} = S(0) \rightarrow error(x - x_d) = 0 \tag{14}$$

suppose S is defined as

$$s(x,t) = (\frac{d}{dt} + \lambda) \quad \tilde{x} = (\dot{x} - \dot{x}_d) + \lambda(x - x_d) \tag{15}$$

The derivation of S, namely, $\dot{S}$ can be calculated as the following;

$$\dot{S} = (\ddot{x} - \ddot{x}_d) + \lambda(\dot{x} - \dot{x}_d) \tag{16}$$

suppose the second order system is defined as;

$$\ddot{x} = f + u \rightarrow \dot{S} = f + U - \ddot{x}_d + \lambda(\dot{x} - \dot{x}_d) \tag{17}$$

Where f is the dynamic uncertain, and also since $S = 0\ and\ \dot{S} = 0$, to have the best approximation ,$\hat{U}$ is defined as

$$\hat{U} = -\hat{f} + \ddot{x}_d - \lambda(\dot{x} - \dot{x}_d) \tag{18}$$

A simple solution to get the sliding condition when the dynamic parameters have uncertainty is the switching control law:

$$U_{dis} = \hat{U} - K(\vec{x}, t) \cdot sgn(s) \tag{19}$$

where the switching function $sgn(S)$ is defined as

$$sgn(s) = \begin{cases} 1 & s > 0 \\ -1 & s < 0 \\ 0 & s = 0 \end{cases} \tag{20}$$

and the $K(\vec{x}, t)$ is the positive constant. Suppose by (20) the following equation can be written as,

$$\frac{1}{2}\frac{d}{dt}s^2(x,t) = \dot{S} \cdot S = [f - \hat{f} - Ksgn(s)] \cdot S = (f - \hat{f}) \cdot S - K|S| \tag{21}$$

and if the equation (13) instead of (12) the sliding surface can be calculated as

$$s(x,t) = (\frac{d}{dt} + \lambda)^2 \left(\int_0^t \tilde{x}\, dt\right) = (\dot{x} - \dot{x}_d) + 2\lambda(\dot{x} - \dot{x}_d) - \lambda^2(x - x_d) \tag{22}$$

in this method the approximation of U is computed as

$$\hat{U} = -\hat{f} + \ddot{x}_d - 2\lambda(\dot{x} - \dot{x}_d) + \lambda^2(x - x_d) \tag{23}$$

Based on above discussion, the control law for a multi degrees of freedom robot manipulator is written as:

$$\tau = \tau_{eq} + \tau_{dis} \tag{24}$$

Where, the model-based component τ_{eq} is the nominal dynamics of systems and τ_{eq} can be calculate as follows:

$$\tau_{eq} = [M^{-1}(B + C + G) + \dot{S}]M \tag{25}$$

Where [15-44]

$$\tau_{eq} = \begin{bmatrix} \tau_{eq1} \\ \tau_{eq2} \\ \tau_{eq3} \\ \tau_{eq4} \\ \tau_{eq5} \\ \tau_{eq6} \end{bmatrix}, M^{-1} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & 0 & 0 & 0 \\ M_{21} & M_{22} & M_{23} & 0 & 0 & 0 \\ M_{31} & M_{32} & M_{33} & 0 & M_{35} & 0 \\ 0 & 0 & 0 & M_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{66} \end{bmatrix}^{-1}$$

$$B+C+G=\begin{bmatrix} b_{112}\dot{q}_1\dot{q}_2+b_{113}\dot{q}_1\dot{q}_3+0+b_{123}\dot{q}_2\dot{q}_3 \\ 0+b_{223}\dot{q}_2\dot{q}_3+0+0 \\ 0 \\ b_{412}\dot{q}_1\dot{q}_2+b_{413}\dot{q}_1\dot{q}_3+0+0 \\ 0 \\ 0 \end{bmatrix}+\begin{bmatrix} C_{12}\dot{q}_2^{\,2}+C_{13}\dot{q}_3^{\,2} \\ C_{21}\dot{q}_1^{\,2}+C_{23}\dot{q}_3^{\,2} \\ C_{31}\dot{q}_1^{\,2}+C_{32}\dot{q}_2^{\,2} \\ 0 \\ C_{51}\dot{q}_1^{\,2}+C_{52}\dot{q}_2^{\,2} \\ 0 \end{bmatrix}+\begin{bmatrix} 0 \\ g_2 \\ g_3 \\ 0 \\ g_5 \\ 0 \end{bmatrix}$$

$$\dot{S}=\begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \\ \dot{S}_4 \\ \dot{S}_5 \\ \dot{S}_6 \end{bmatrix} \text{ and } M=\begin{bmatrix} M_{11} & M_{12} & M_{13} & 0 & 0 & 0 \\ M_{21} & M_{22} & M_{23} & 0 & 0 & 0 \\ M_{31} & M_{32} & M_{33} & 0 & M_{35} & 0 \\ 0 & 0 & 0 & M_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{66} \end{bmatrix}$$

and τ_{dis} is computed as;

$$\tau_{dis}=K\cdot \mathrm{sgn}(S) \tag{26}$$

where

$$\tau_{dis}=\begin{bmatrix} \tau_{dis1} \\ \tau_{dis2} \\ \tau_{dis3} \\ \tau_{dis4} \\ \tau_{dis5} \\ \tau_{dis6} \end{bmatrix}, K=\begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \\ K_5 \\ K_6 \end{bmatrix}, (S)=\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} \; and \; S=\lambda e+\dot{e}$$

The result scheme is shown in Figure 1.

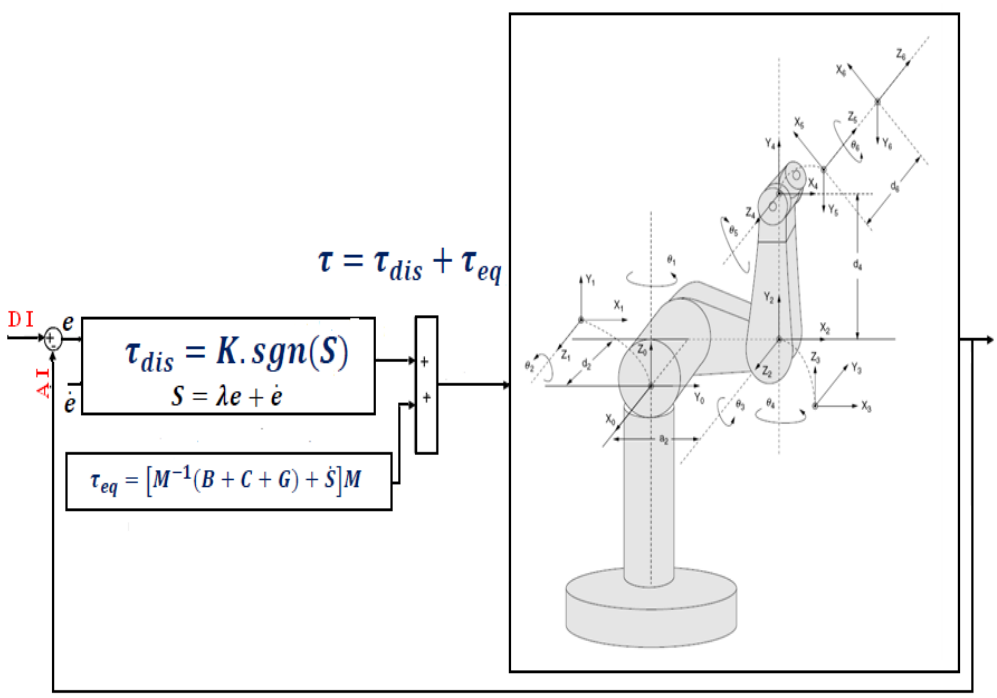

FIGURE :1 Block diagram of Sliding Mode Controller (SMC)

Problem Statement: Even though, SMC is used in wide range areas but, pure SMC has the chattering phenomenon disadvantages to reduce or eliminate the chattering this paper focouses on applied fuzzy logic methodology in sliding mode controller with minimum rule base after that sliding surface slope which has play important role in remove the chattering is auto adjusted.

Proof of Stability: The proof of Lyapunov function can be determined by the following equations. The dynamic formulation of robot manipulate can be written by the following equation

$$\tau = M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) \tag{27}$$

the lyapunov formulation can be written as follows,

$$V = \frac{1}{2}S^T.M.S \tag{28}$$

the derivation of V can be determined as,

$$\dot{V} = \frac{1}{2}S^T.\dot{M}.S + S^T\,M\dot{S} \tag{29}$$

the dynamic equation of robot manipulator can be written based on the sliding surface as

$$M\dot{S} = -VS + M\dot{S} + VS + G - \tau \tag{30}$$

it is assumed that

$$S^T(\dot{M} - 2V)S = 0 \tag{31}$$

by substituting (30) in (29)

$$\dot{V} = \frac{1}{2}S^T\dot{M}S - S^TVS + S^T(M\dot{S} + VS + G - \tau) = S^T(M\dot{S} + VS + G - \tau) \tag{32}$$

suppose the control input is written as follows

$$\hat{\tau} = \widehat{\tau_{eq}} + \widehat{\tau_{ds}} = [\widetilde{M^{-1}}(\hat{V} + \hat{G}) + \dot{S}]\hat{M} + K.sgn(S) + K_vS \tag{33}$$

by replacing the equation (33) in (32)

$$\dot{V} = S^T(M\dot{S} + VS + G - \hat{M}\dot{S} - \hat{V}S - \hat{G} - K_vS - Ksgn(S) = S^T\left(\tilde{M}\dot{S} + \tilde{V}S + \tilde{G} - K\right. \tag{34}$$

it is obvious that

$$|\tilde{M}\dot{S} + \tilde{V}S + \tilde{G} - K_vS| \leq |\tilde{M}\dot{S}| + |\tilde{V}S| + |\tilde{G}| + |K_vS| \tag{35}$$

the Lemma equation in robot manipulator system can be written as follows

$$K_u = [|\tilde{M}\dot{S}| + |VS| + |G| + |K_vS| + \eta]_i, i = 1, 2, 3, 4, \ldots \tag{36}$$

the equation (31) can be written as

$$K_u \geq \left|[\tilde{M}\dot{S} + VS + G - K_vS]_i\right| + \eta_i \tag{37}$$

therefore, it can be shown that

$$\dot{V} \leq -\sum_{i=1}^{n}\eta_i|S_i| \tag{38}$$

Consequently the equation (38) guaranties the stability of the Lyapunov equation.

3. METHODOLOGY: DESIGN AUTO ADJUST SLIDING SURFACE SLOPE SLIDING MODE FUZZY CONTROLLER

First Step: Improve Chattering Free Sliding Mode Controller: To reduce or eliminate the chattering in this research is used the boundary layer method; in boundary layer method the basic idea is replace the discontinuous method by saturation (linear) method with small neighborhood of the switching surface.

$$B(t) = \{x, |S(t)| \leq \emptyset\}; \emptyset > 0 \tag{39}$$

Where $\emptyset$ is the boundary layer thickness. Therefore the saturation function $\mathrm{Sat}(S/\emptyset)$ is added to the control law as

$$U = K(\vec{x}, t) \cdot \mathrm{Sat}\left(S/\emptyset\right) \tag{40}$$

Where $\mathrm{Sat}\left(S/\emptyset\right)$ can be defined as

$$\mathrm{sat}\left(S/\emptyset\right) = \begin{cases} 1 & (S/\emptyset > 1) \\ -1 & (S/\emptyset < 1) \\ S/\emptyset & (-1 < S/\emptyset < 1) \end{cases} \tag{41}$$

Based on above discussion, the control law for a multi degrees of freedom robot manipulator is written as:

$$\tau = \tau_{eq} + \tau_{sat} \tag{42}$$

Where, the model-based component τ_{eq} is the nominal dynamics of systems and τ_{eq} can be calculate as follows:

$$\tau_{eq} = \left[M^{-1}(B + C + G) + \dot{S}\right]M \tag{43}$$

Where

$$\tau_{eq} = \begin{bmatrix} \tau_{eq1} \\ \tau_{eq2} \\ \tau_{eq3} \\ \tau_{eq4} \\ \tau_{eq5} \\ \tau_{eq6} \end{bmatrix}, M^{-1} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & 0 & 0 & 0 \\ M_{21} & M_{22} & M_{23} & 0 & 0 & 0 \\ M_{31} & M_{32} & M_{33} & 0 & M_{35} & 0 \\ 0 & 0 & 0 & M_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{66} \end{bmatrix}^{-1}$$

$$B + C + G = \begin{bmatrix} b_{112}\dot{q}_1\dot{q}_2 + b_{113}\dot{q}_1\dot{q}_3 + 0 + b_{123}\dot{q}_2\dot{q}_3 \\ 0 + b_{223}\dot{q}_2\dot{q}_3 + 0 + 0 \\ 0 \\ b_{412}\dot{q}_1\dot{q}_2 + b_{413}\dot{q}_1\dot{q}_3 + 0 + 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} C_{12}\dot{q}_2^{\,2} + C_{13}\dot{q}_3^{\,2} \\ C_{21}\dot{q}_1^{\,2} + C_{23}\dot{q}_3^{\,2} \\ C_{31}\dot{q}_1^{\,2} + C_{32}\dot{q}_2^{\,2} \\ 0 \\ C_{51}\dot{q}_1^{\,2} + C_{52}\dot{q}_2^{\,2} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ g_2 \\ g_3 \\ 0 \\ g_5 \\ 0 \end{bmatrix}$$

$$\dot{S} = \begin{bmatrix} \dot{S}_1 \\ \dot{S}_2 \\ \dot{S}_3 \\ \dot{S}_4 \\ \dot{S}_5 \\ \dot{S}_6 \end{bmatrix} \text{ and } M = \begin{bmatrix} M_{11} & M_{12} & M_{13} & 0 & 0 & 0 \\ M_{21} & M_{22} & M_{23} & 0 & 0 & 0 \\ M_{31} & M_{32} & M_{33} & 0 & M_{35} & 0 \\ 0 & 0 & 0 & M_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & M_{66} \end{bmatrix}$$

and τ_{sat} is computed as;

$$\tau_{sat} = K \cdot \mathrm{sat}\left(S/\emptyset\right) \tag{44}$$

where

$$\tau_{sat}=\begin{bmatrix}\tau_{dis1}\\ \tau_{dis2}\\ \tau_{dis3}\\ \tau_{dis4}\\ \tau_{dis5}\\ \tau_{dis6}\end{bmatrix}.K=\begin{bmatrix}K_1\\ K_2\\ K_3\\ K_4\\ K_5\\ K_6\end{bmatrix}.\left(S/_{\emptyset}\right)=\begin{bmatrix}\frac{S_1}{\emptyset_1}\\ \frac{S_2}{\emptyset_2}\\ \frac{S_3}{\emptyset_3}\\ \frac{S_4}{\emptyset_4}\\ \frac{S_5}{\emptyset_5}\\ \frac{S_6}{\emptyset_6}\end{bmatrix}\ and\ S=\lambda e+\dot{e}$$

by replace the formulation (44) in (42) the control output can be written as;

$$\tau=\tau_{eq}+K.\mathrm{sat}\left(S/_{\emptyset}\right)=\begin{cases}\tau_{eq}+K.\mathrm{sgn}(S) & ,|S|\geq\emptyset\\ \tau_{eq}+K.S/_{\emptyset} & ,|S|<\emptyset\end{cases} \quad (45)$$

Figure 2 shows the chattering free sliding mode control for robot manipulator. By (45) and (43) the sliding mode control of PUMA 560 robot manipulator is calculated as;

$$\tau=\left[M^{-1}(B+C+G)+\dot{S}\right]M+K\cdot\mathrm{sat}\left(S/_{\emptyset}\right) \quad (46)$$

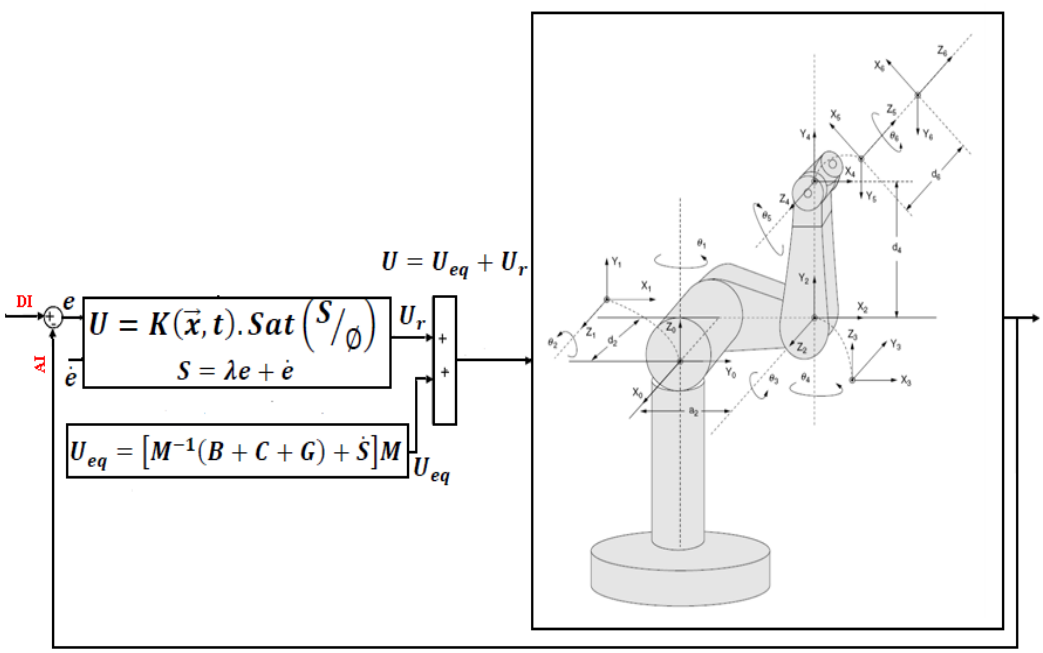

FIGURE 2: Block diagram of chattering free Sliding Mode Controller (SMC)

Second Step, Design Sliding Mode Fuzzy Controller: As shown in Figure 1, sliding mode controller divided into two main parts: equivalent controller, based on dynamics formulation of robot manipulators and sliding surface saturation part based on saturation continuous function to reduce the chattering. Boundary layer method (saturation function) is used to reduce the chattering. Reduce or eliminate the chattering regarding to reduce the error is play important role in this research therefore boundary layer method is used beside the equivalent part to solve the chattering problem besides reduce the error.

Combinations of fuzzy logic systems with sliding mode method have been proposed by several researchers. SMFC is fuzzy controller based on sliding mode method for easy implementation, stability, and robustness. Control rules for SMFC can be described as:

IF S is<ling.var> THEN U is<ling.var> **(47)**

Table 1 is shown the fuzzy rule table for SMFC, respectively:

S	NB	NM	NS	Z	PS	PM	PB
T	NB	NM	NS	Z	PS	PM	PB

Table 1. Rule table (SMFC)

A block diagram for sliding mode fuzzy controller is shown in Figure 3.

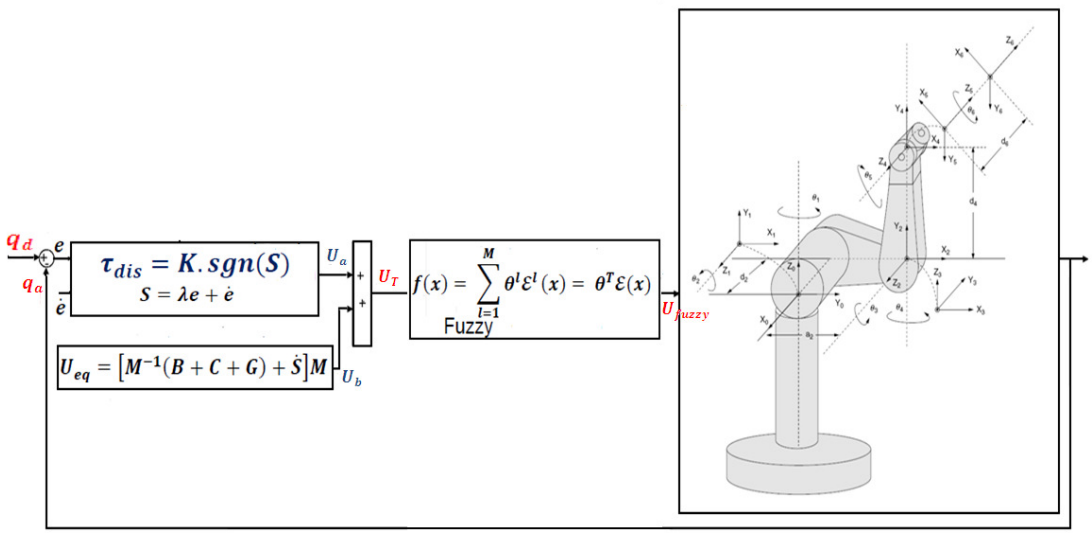

FIGURE3: Block Diagram of sliding mode Fuzzy Controller with Minimum Rule Base

It is basic that the system performance is sensitive to the sliding surface slope λ for sliding mode fuzzy controller. For instance, if large value of λ are chosen the response is very fast but the system is very unstable and conversely, if small value of λ considered the response of system is very slow but the system is very stable. Therefore, calculate the optimum value of λ for a system is one of the most important challenging works. SMFC has two most important advantages i.e. the number of rule base is smaller and Increase the robustness and stability.
In this method the control output can be calculated by

$$U_T = U_{dis} + U_{eq} \quad (48)$$

Where U_{eq} the nominal compensation is term and U_{dis} is the output of sliding function [9].

Third Step; Auto Tuning Sliding Surface Slope: All conventional controller have common difficulty, they need to find and estimate several nonlinear parameters. Tuning sliding surface slope can tune by mathematical automatically the scale parameters using mathematical model free method. To keep the structure of the controller as simple as possible and to avoid heavy computation, in this design model free mathematical supervisor tuner is selected. For nonlinear, uncertain, and time-variant plants (e.g., robot manipulators) adaptive method can be used to self adjusting the surface slope and gain updating factors. Research on adaptive sliding mode fuzzy controller is significantly growing, for instance, the different ASMFC have been reported in [5]; [10-12]. It is a basic fact that the system performance in SMFC is sensitive to sliding surface slope, λ. Thus, determination of an optimum λ value for a system is an important problem. If the

system parameters are unknown or uncertain, the problem becomes more highlighted. This problem may be solved by adjusting the surface slope and boundary layer thickness of the sliding mode controller continuously in real-time. To keep the structure of the controller as simple as possible and to avoid heavy computation, a new supervisor tuner based on updated by a new coefficient factor kn is presented. In this method the supervisor part tunes the output scaling factors using gain online updating factors. The inputs of the supervisor term are error and change of error $(e, \dot{e})$ and the output of this controller is U, which it can be used to tune sliding surface slope, λ.

$$k_n = e^2 - \frac{(r_v - r_{vmin})^5}{1 + |e|} + r_{vmin} \tag{49}$$

$$r_v = \frac{\left(de(k) - de(k-1)\right)}{de(.)} = \frac{\ddot{e}(t)}{\dot{e}(.)} \tag{50}$$

$$de(.) = \begin{cases} de(k); & if\ de(k) \geq de(k-1) \\ de(k-1) & if\ de(k) < de(k-1) \end{cases}$$

$$S_{new} = \lambda_{new} \times e_{new} + \dot{e}_{new}$$

$$e_{new} = e \times \lambda_{new}$$

$$\lambda_{new} = \lambda \times K_n$$

In this way, the performance of the system is improved with respect to the SMFC controller.

In this method the tunable part tunes the sliding surface slope. However pure sliding mode controller has satisfactory performance in a limit uncertainty but tune the performance of this controller in highly nonlinear and uncertain parameters (e.g., robot manipulator) is a difficult work which proposed methodology can solve above challenge by applied adaptive. The lyapunov candidate formulation for our design is defined by:

$$V = \frac{1}{2} S^T M S + \frac{1}{2} \sum_{j=1}^{M} \frac{1}{\gamma_{sj}} \phi^T . \phi_j \tag{51}$$

Where γ_{sj} is positive coefficient, $\phi = \theta^* - \theta;\ \theta^*\ is\ minimum\ error\ \&\ \theta\ is\ adjustable\ parameter$

Since $\dot{M} - 2V$ is skew-symetric matrix, we can get

$$S^T M \dot{S} + \frac{1}{2} S^T \dot{M} S = S^T (M \dot{S} + V S) \tag{52}$$

From following two functions:

$$\tau = M(q)\ddot{q} + V(q, \dot{q})\dot{q} + G(q) \tag{53}$$

And

$$\tau = \hat{M}\ddot{q}_r + \hat{V}\dot{q}_r + \hat{G} - AS - K \tag{54}$$

We can get:

$$M(q)\ddot{q} + V(q, \dot{q})\dot{q} + G(q) = \hat{M}\ddot{q}_r + \hat{V}\dot{q}_r + \hat{G} - AS - K \tag{55}$$

Since; $\dot{q}_r = \dot{q} - S\ \&\ \ddot{q}_r = \ddot{q} - \dot{S}$ then

$$M\dot{S} + (V + A)S = \Delta f - K \tag{56}$$

$$M\dot{S} = \Delta f - K - VS - AS$$

The derivative of V defined by;

$$\dot{V} = S^T M \dot{S} + \frac{1}{2} S^T \dot{M} S + \sum_{j=1}^{M} \frac{1}{\gamma_{sj}} \phi^T . \dot{\phi}_j \tag{57}$$

$$\dot{V} = S^T(M\dot{S} + VS) + \sum_{j=1}^{M} \frac{1}{\gamma_{sj}} \phi^T.\dot{\phi}_j$$

$$\dot{V} = S^T(\Delta f - K - VS - AS + VS) + \sum_{j=1}^{M} \frac{1}{\gamma_{sj}} \phi^T.\dot{\phi}_j$$

$$\dot{V} = \sum_{j=1}^{M} [S_j(\Delta f_1 - K_n)] - S^T AS + \sum_{j=1}^{M} \frac{1}{\gamma_{sj}} \phi^T.\dot{\phi}_j$$

suppose K_n is defined as follows

$$k_n = e^2 - \frac{(r_v - r_{vmin})^2}{1+|e|} + r_{vmin} \quad (58)$$

Based on $\phi = \theta^* - \theta \rightarrow \dot{\theta} = \dot{\theta}^* - \dot{\phi}$

$$\dot{V} = \sum_{j=1}^{M} [S_j(\Delta f_1 - (\theta^*)^T \zeta(S_j) k_n] - S^T AS + \sum_{j=1}^{M} \frac{1}{\gamma_{sj}} \phi_j^T [\gamma_{sj}.S_j.\zeta_j(S_j) + \dot{\phi}_j]) \quad (59)$$

where $\dot{\theta}_j = \gamma_{sj} S_j \zeta_j(S_j)$ is adaption law, $\dot{\phi}_j = -\dot{\theta}_j = -\gamma_{sj} S_j \zeta_j(S_j)$
consequently $\dot{V}$ can be considered by

$$\dot{V} = \sum_{j=1}^{m} [S_j \Delta f_j - ((\theta_j^*)^T \zeta_j(S_j) k_n)] - S^T AS \quad (60)$$

If the minimum error can be defined by

$$e_{mj} = \Delta f_j - \left((\theta_j^*)^T \zeta_j(S_j)\right)$$

□**(61)**□□$\dot{V}$ is intended as follows

(61)□□$\dot{V}$ is intended as follows

$\dot{V}$ is intended as follows

$$\begin{aligned} \dot{V} &= \sum_{j=1}^{m} [S_j e_{mj}] - S^T AS \\ &\leq \sum_{j=1}^{m} |S_j||e_{mj}| - S^T AS \\ &= \sum_{j=1}^{m} |S_j||e_{mj}| - k_n S_j^2 \\ &= \sum_{j=1}^{m} |S_j|(|e_{mj}| - a_j S_j) \end{aligned} \quad (62)$$

For continuous function $g(x)$, and suppose $\varepsilon > 0$ it is defined mathematical model free in form of $k_n|f(x) - g(x)| < \varepsilon$

the minimum approximation error (e_{mj}) is very small.

$$that \quad k_n|S_j| > e_{mj} \ (S_j \neq 0) \qquad then \quad \dot{V} < 0 \ for \ (S_j \neq 0)$$

4 SIMULATION RESULTS

Classical sliding mode control (SMC) and adaptive sliding mode fuzzy control (ASMFC) are implemented in Matlab/Simulink environment. In theses controllers changing updating factor performance, tracking performance, error, and robustness are compared.

Changing Sliding Surface Slope Performance: For various value of sliding surface slope (λ) in SMC and ASMFC the trajectory performances have shown in Figures 4 and 5.

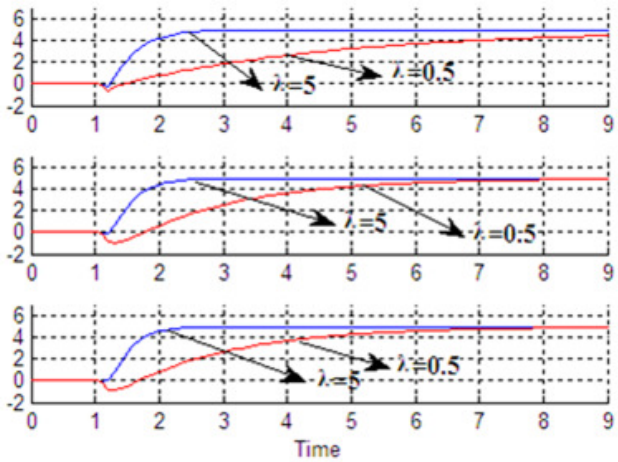

FIGURE 4 : Sliding surface slope in SMC: applied to 3DOF's robot manipulator

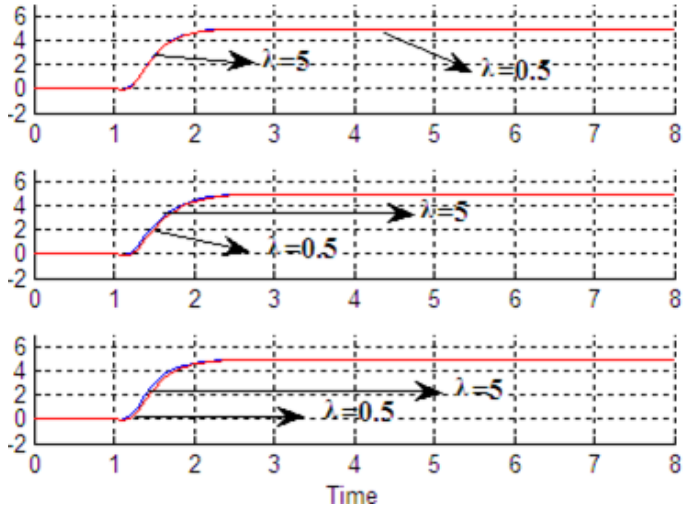

FIGURE 5: Sliding surface slope in ASMFC: applied to 3DOF's robot manipulator

Figures 4 and 5 are shown trajectory performance with different sliding surface slope; it is seen that AFSMC has the better performance in comparison with classical SMC.

Tracking Performances

From the simulation for first, second and third trajectory without any disturbance, it was seen that SMC and ASMFC have the about the same performance because this system is worked on certain environment and in sliding mode controller also is a robust nonlinear controller with acceptable performance. Figure 6 shows tracking performance without any disturbance for SMC and ASMFC.

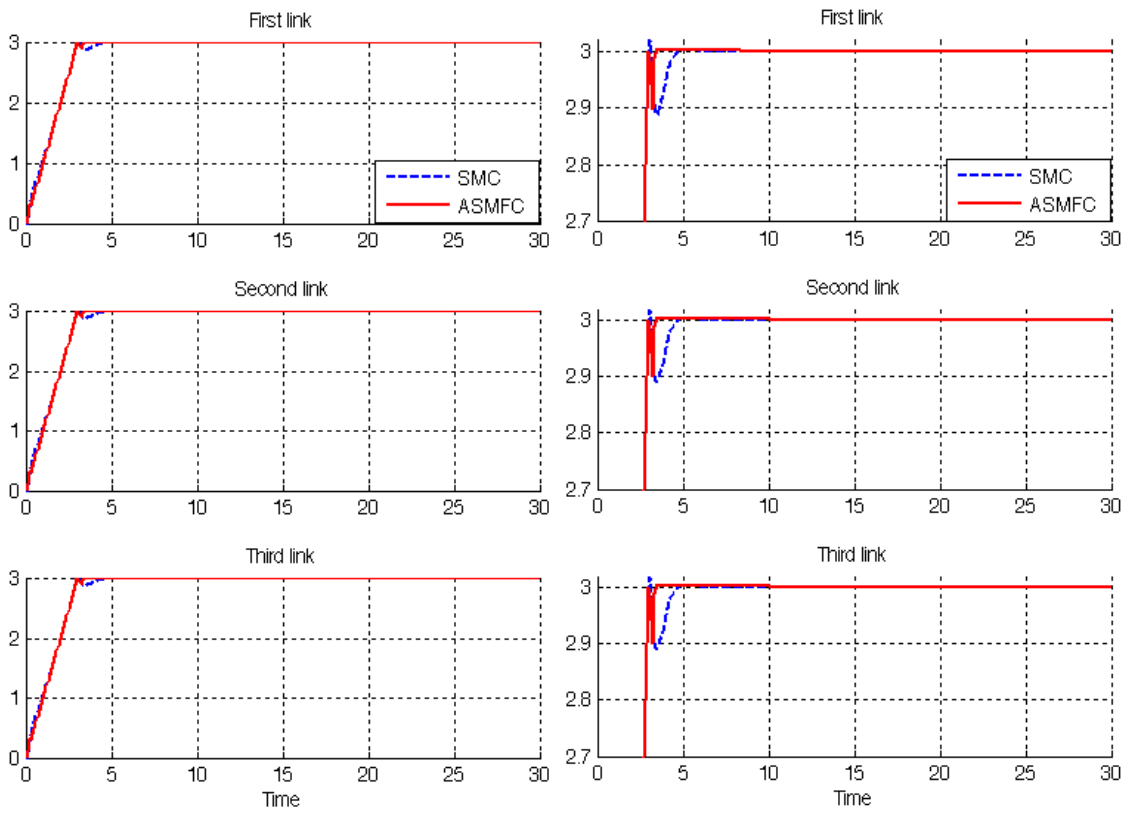

FIGURE 6 : SMC Vs. ASMFC: applied to 3DOF's robot manipulator

By comparing trajectory response trajectory without disturbance in SMC and ASMFC, it is found that the SMFC's overshoot **(0%)** is lower than IDC's **(3.33%)** and the rise time in both of controllers are the same.

Disturbance Rejection

Figure 7 has shown the power disturbance elimination in SMC and ASMFC. The main targets in these controllers are disturbance rejection as well as the remove the chattering phenomenon. A band limited white noise with predefined of 40% the power of input signal is applied to the SMC and SMFC. It found fairly fluctuations in SMC trajectory responses.

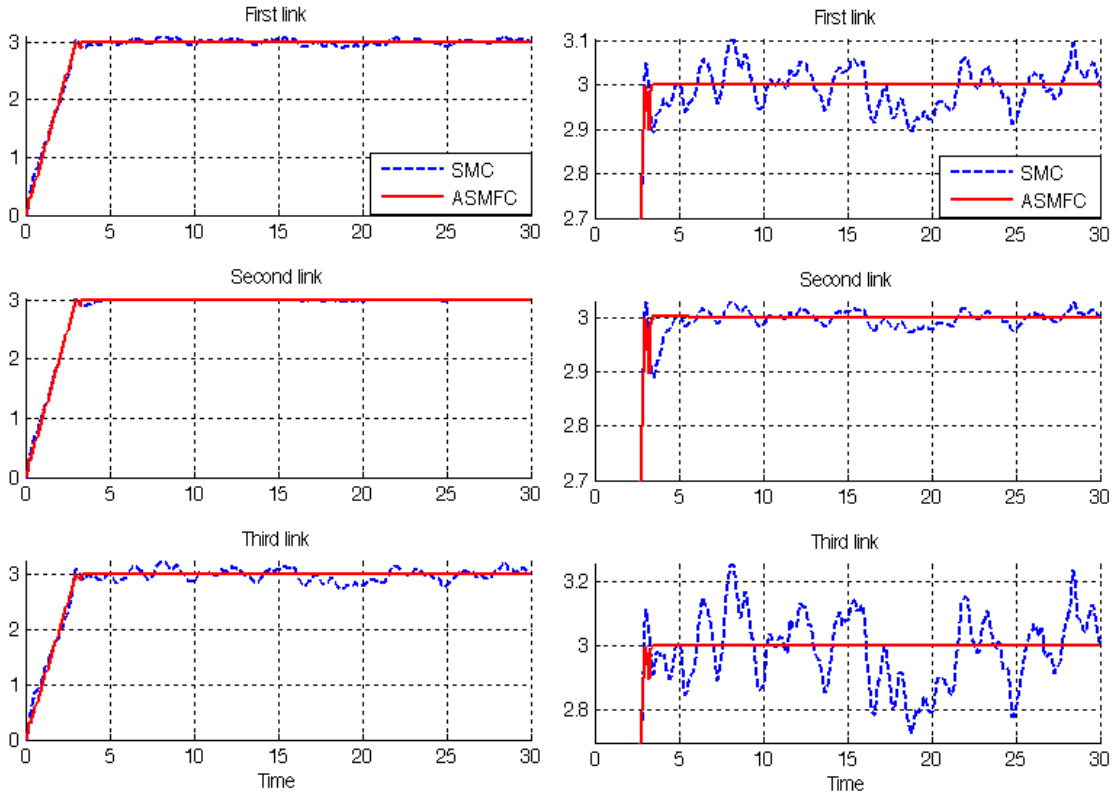

FIGURE 7: SMC Vs. ASMFC in presence of uncertainty: applied to robot manipulator.

Among above graph relating to trajectory following with external disturbance, SMC has fairly fluctuations. By comparing some control parameters such as overshoot and rise time it found that the ASMFC's overshoot **(0%)** is lower than SMC's **(12.4%)**, although both of them have about the same rise time**.**

5 CONCLUSIONS

This research presents a design auto adjust sliding surface slope in sliding mode fuzzy controller (ASMFC) with improved in sliding mode controller which offers a model-free sliding mode controller. The sliding mode fuzzy controller is designed as 7 rules Mamdani's error-based to estimate the uncertainties in nonlinear equivalent part. To eliminate the chattering with regard to the uncertainty and external disturbance applied mathematical self tuning method to sliding mode fuzzy controller for adjusting the sliding surface slope coefficient (λ). In this research new λ is obtained by the previous λ multiple gains updating factor (K_n) which it also is based on error and change of error and also the second derivation of error. The proof of stability in this method is discussed. As a result auto adjust sliding surface slope in sliding mode fuzzy controller has superior performance in presence of structure and unstructured uncertainty (e.g., overshoot=0%, rise time=0.9 s, steady state error = 1e-7 and RMS error=0.00016) and eliminate the chattering.

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