Compensating Joint Configuration through Null Space Control in Composite Weighted Least Norm Solution

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Abstract

We have presented a methodology for compensating joint configuration by composite weighting in different sub spaces. It augments the weighted least norm solution by weighted residual of the current joint rate and preferred pose rate in null space, so that we can arrive at a solution which is able to handle both joint limits and preferred joint configuration simultaneously satisfying the primary task. The null space controller is formulated in conjunction with the work space controller to achieve the objective. The contribution of null space has been discussed in the formulation in two different situations including joint limits, workspace and near configuration singularities.

Keywords: Null Space Controller, Weighted Least Norm, Joint Limit, Singularity, Joint Configuration.

1. INTRODUCTION

A robotic manipulator in general sense or an articulated serial structure in particular is kinematically redundant when the number of operational space variables necessary to specify a given task, is less than the number of joints. Redundancy leads to infinite solutions for the joint space but offers greater flexibility and dexterity in motion as different constraint based or goal based criteria can be formulated as sub tasks in the solution. Two kinds of approaches have been reported in the literature to deal with this situation. One is set to exploit the null space of the Jacobian matrix in the homogeneous solution that infuses self motion of joints without affecting the task space. Typical method of this kind is gradient projection method (GPM) [1][2]. In GPM the anti-gradient of a quadratic cost function, is projected in the null space of the task Jacobian, which is reminiscent of the projected gradient method for constrained minimization. The other approach is weighted least norm (WLN) approach [3][4], which minimizes the weighted norm of joint rate. In both the cases the primary task is to follow the prescribed trajectory and there may be multiple secondary tasks or nested subtasks with priority fixation [5] [6].

GPM has been used in Joint Limit Avoidance (JLA), obstacle avoidance [7], visual servoing [4]. WLN which was introduced in JLA in [3], has been successfully exploited by others with single or multiple criteria and Close Loop Inverse Kinematics (CLIK)[8]. Null space based motion control [9] has been studied with configuration optimization [10], influence of un-weighted and inertia weighted pseudoinverse [11], proportional-integral-derivative (PID) controller considering passivity [12], task priority implementation based on behavioral scheme [13]. An elaborate discussion with illustrations on various pros and cons of different approaches for operational space control with null space contributions has been reported in [14].

In practice, many subtasks are often needed for the control of manipulator. For example both the joint limits and the joint configuration became the basic requirements where human motion analysis is concerned. In many cases, local optimality of GPM may not provide good performance to all prioritized subtasks. WLN method is effective only for the joints limits but direct optimization of the weighted norm sum of all tasks may lead to the poor performance for all tasks. The ability of WLN to effectively handle joint limits and the self motion from null space, motivate us to presents a methodology of composite weighted least norm (CWLN) solution in conjunction with GPM. It is so called because the formulation tries to minimize the primary task objective of weighted norm of joint rate in range space and the weighted residual of the current joint rate $(\dot{q})$ and preferred pose rate $(\dot{q}_r)$ in null space (hence composite weighting in different sub spaces) so that we can arrive at a solution which is able to handle both joint limits and preferred joint configuration simultaneously satisfying the primary task.

This paper is organized as follows: Section II formulates the CWLN method from classical redundancy control methods. Section III discusses stability of the CWLN method and its regularized version. The case studies are illustrated in Section IV. Section V concludes the paper.

2. COMPOSITE WEIGHTED LEAST NORM

We focus on first and second order kinematics for the time variant task space defined as $x(t) \in \Re^{m\times 1}$ and joint space $q(t) \in \Re^{n\times 1}$ related by the direct kinematic non linear and transcendental vector function $k_t(q)$, whose time differentiation will define the non square analytic Jacobian matrix $J(q) \triangleq J_t^{ij}(q) \triangleq \partial k_t^j / \partial q_i \in \Re^{m\times n}$; $\forall n > m$, with its assumption of bounded higher order terms and linearization. We denote the desired task space positions, velocities, and accelerations as $x_d, \dot{x}_d$ *and* $\ddot{x}_d$ respectively and reference or preferred joint configuration as q_r Dropping the subscript *t* for brevity, the classical forward kinematics differential relationships can be expressed as

$$\dot{x} = J(q)\dot{q};\ and\ \ddot{x} = J(q)\ddot{q} + \dot{J}(q,\dot{q})\dot{q} \tag{1}$$

and inverse kinematics least norm (LN) general solution as

$$\dot{q} = \dot{q}_p + \dot{q}_h = J^{\dagger}\dot{x} + (I - J^{\dagger}J)\xi_1;\ \ddot{q} = J^{\dagger}(\ddot{x}_d - \dot{J}\dot{q}) + (I - J^{\dagger}J)\xi_2 \tag{2}$$

where $\dot{q}_p \in \Re(J)$ is particular solution, $\dot{q}_h \in \aleph(J)$ is homogeneous solution, $J^{\dagger} \triangleq J^T(JJ^T)^{-1}$ is the right pseudoinverse of the Jacobian, ξ_1 *and* $\xi_2 \in \Re^{n\times 1}$ are arbitrary vectors and $(I - J^{\dagger}J)$ is the null space projector. The Weighted Least Norm (WLN) solution formulates the problem as $min(\dot{q})[\mathrm{H}_1(\dot{q})] = min(\dot{q}) \lVert \dot{q} \rVert^2 = min(\dot{q})[\dot{q}^T W_1 \dot{q}]$, *st* $(\dot{x} - J\dot{q}) = 0$, $\forall W_1 \in \mathbb{R}^{n\times n}$ is the symmetric positive definite weighing matrix. To stabilize the ill posed condition of LN or WLN solution near singularities, Tikhonov like regularization has been used, which makes a trade off between tracking accuracy and the feasibility of the joint velocities, known as classical Damped Least Square (DLS) solution. The trade off parameter is the damping factor α. If the objective is specified through a configuration rate dependent performance criteria $\mathrm{H}_2(\dot{q})$, set to be the closest to some particular pose, hence forth called the reference configuration (q_r) the problem can be reformulated as $min(\dot{q})[\mathrm{H}_2(\dot{q})] = min(\dot{q})[(1/2)(\dot{q}-\dot{q}_r)^T W_2(\dot{q}-\dot{q}_r)];\ \ s.t\ \ J\dot{q}=\dot{x};\ \ \forall W_2 \in \mathbb{R}^{n\times n}$ .In our approach an augmented objective function has been formulated by combining configuration rate dependent performance criteria $\mathrm{H}_2(\dot{q})$ for pose optimization and $\mathrm{H}_1(\dot{q})$ for joint limit avoidance, subjected to the requirement of primary task space $(\dot{x}-J\dot{q})=0$, as $\forall \mathrm{H}_3(\dot{q}) = \mathrm{H}_1(\dot{q}) + \mathrm{H}_2(\dot{q})\ and\ \forall (W_1, W_2) \in \mathbb{R}^{n\times n}$, henceforth called as Composite Weighted Least Norm Solution *(CWLNS)* as,

$$min(\dot{q})\mathrm{H}_3(\dot{q}) = min(\dot{q})[(1/2)\dot{q}^T W_1 \dot{q} + (1/2)(\dot{q}-\dot{q}_r)^T W_2(\dot{q}-\dot{q}_r)];\ s.t\ \ J\dot{q}=\dot{x} \tag{3}$$

To solve this optimization problem with equality constraint, it should satisfy both the necessary condition $\nabla_{\dot{q}}L=0$ and sufficient condition $\nabla^2_{\dot{q}}L>0$, where the Lagrangian is $L(\dot{q},\lambda) = \mathrm{H}_3(\dot{q}) + \lambda(J\dot{q}-\dot{x})$ and we can directly evaluate $\nabla^2_{\dot{q}}L = (W_1+W_2) > 0$, which is true for minimization. Putting the value of $\dot{q}$ from $\nabla_{\dot{q}}L=0$ in the expression $\nabla_{\lambda}L=0$, we get λ. Substituting λ back in $\dot{q}$ from $\nabla_{\dot{q}}L=0$, and $\forall J^{\hbar} \triangleq W^{-1}J^T(JW^{-1}J^T)^{-1},\ \ \forall W \triangleq (W_1+W_2)$, $\forall \xi_1 \triangleq \dot{q}_r$, the general solution of CWLS reduces to [Appendix-I.A]

$$\dot{q} = J^{\hbar}\dot{x} + (I - J^{\hbar}J)W^{-1}W_2\xi_1 \tag{4}$$

It is trivial to show $(I - J^{\hbar}J)W^{-1}W_2$ is the null space projector of reference joint rate vector $\dot{q}_r$ and hence no impact on task space as $JJ^{\hbar}=I$. The optimization in the direction of the anti-gradient of scalar configuration dependent performance criteria $\mathrm{H}_3(q)$ can also be set up by minimizing $\mathrm{H}_3(q)$ for weighted reference configuration (q_r) as

$$\mathrm{H}_3(q) = (1/2)(q-q_r)^T W_2(q-q_r);\Rightarrow \nabla_q \mathrm{H}_3(q) = W_2(q-q_r) \tag{5}$$

and for a positive scalar k_H and $\forall \xi_1' \triangleq -k_H\ (W_1+W_2)^{-1}W_2\nabla_q\mathrm{H}_3(q)$ the GPM flavor of CWLS formulation is

$$\dot{q} = J^{\hbar}\dot{x} + (I - J^{\hbar}J)W^{-1}W_2\xi_1';\ \ \ddot{q} = J^{\hbar}(\ddot{x}_d - \dot{J}\dot{q}) + (I - J^{\hbar}J)\xi_2 \tag{6}$$

Using Eq.(2), the relation $\dot{J}J^{\hbar} = -J\dot{J}^{\hbar}$ and after simplification we can establish the relation between ξ_2 and ξ_1 as.

$$\xi_2 = \dot{J}^{\hbar}J(\dot{q}-\xi_1) + \dot{\xi}_1' \tag{7}$$

The diagonal elements (w_1^i) of W_1 has been utilized to implement JLA [2][3] with a modified sigmoid function to vary smoothly from -1 to 1. If τ is the threshold parameter for each joint, the activation limits are defined as $q^{th}_{i,max} = (q_{i,max}-\tau)$ and $q^{th}_{i,min} = (q_{i,min}+\tau)$. If $\Delta q^{th}_i = (q^{th}_{i,max} - q^{th}_{i,min})$ is the activation range of i^th joint, then $w_i = 1 + \mu\,|\,h(q_i)\,|$, where $\mu =$ *a large positive gain* ,

$$h(q_i) \triangleq \begin{cases} -\varphi_i & \forall q_i < q^{th}_{i,min} + \overline{\varepsilon} \\ 0 & \forall -\varepsilon_0 < \varphi_i < \varepsilon_0 \\ \varphi_i & otherwise \end{cases};\ \forall \varphi_i = \frac{1}{1+e^{a(q^{th}_{i,max}-q_i)(q_i-q^{th}_{i,min})/\Delta q^{th}_i}}, \forall a > 0 \tag{8}$$

In general μ should be large enough to make the $1/w_i$ near to zero when JLA is activated, so that $\dot{q}_i \to 0$ as in this case $h(q_i)$ is bounded between ± 1. In this case the role of $\overline{\varepsilon}$ is to smoothen $h(q_i)$ when changes from φ_i to $-\varphi_i$. Away from the joint limits when $\varphi_i \approx 0$, w_i may still have oscillations due to large gain μ and oscillatory q_i, which is smoothened by implementing $\varepsilon_0 \approx 1e-4$.

The role of the term $(W_1+W_2)^{-1}W_2$ in the null space of Jacobian needs to be discussed. Starting with $[W_2, W_1] \in I_{n\times n}$, if we increase W_1 (which will occur during JLA activation), keeping W_2 constant then since $\| W_1 \| \to \infty$, $\| (W_1+W_2)^{-1}W_2 \| \to 0$ resulting diminishing contribution from null space. On the other hand, if we increase W_2, keeping W_1 constant, which will occur most of the time when the joint is away from its limits, $\| (W_1+W_2)^{-1}W_2 \| \to 1$, since $(W_1+W_2) \approx W_2$.

3. CONTROL SCHEMES AND STABILITY

Introducing Proportional (K_P) and Derivative (K_D) error control in Eq.(6) by positive definite diagonal gain matrices and task space error $e \triangleq x_d - x = x_d - \kappa(q)$, we can arrive at the second order close loop kinematic scheme (Figure-1[a]) with error system [9][11][13][14]

$$\ddot{q} = J^h(\ddot{x}_d - \dot{J}\dot{q} + K_D\dot{e} + K_P e) + (I - J^h J)\xi_2; \quad \ddot{e} + K_D\dot{e} + K_P e = 0 \tag{9}$$

[a]

[b]

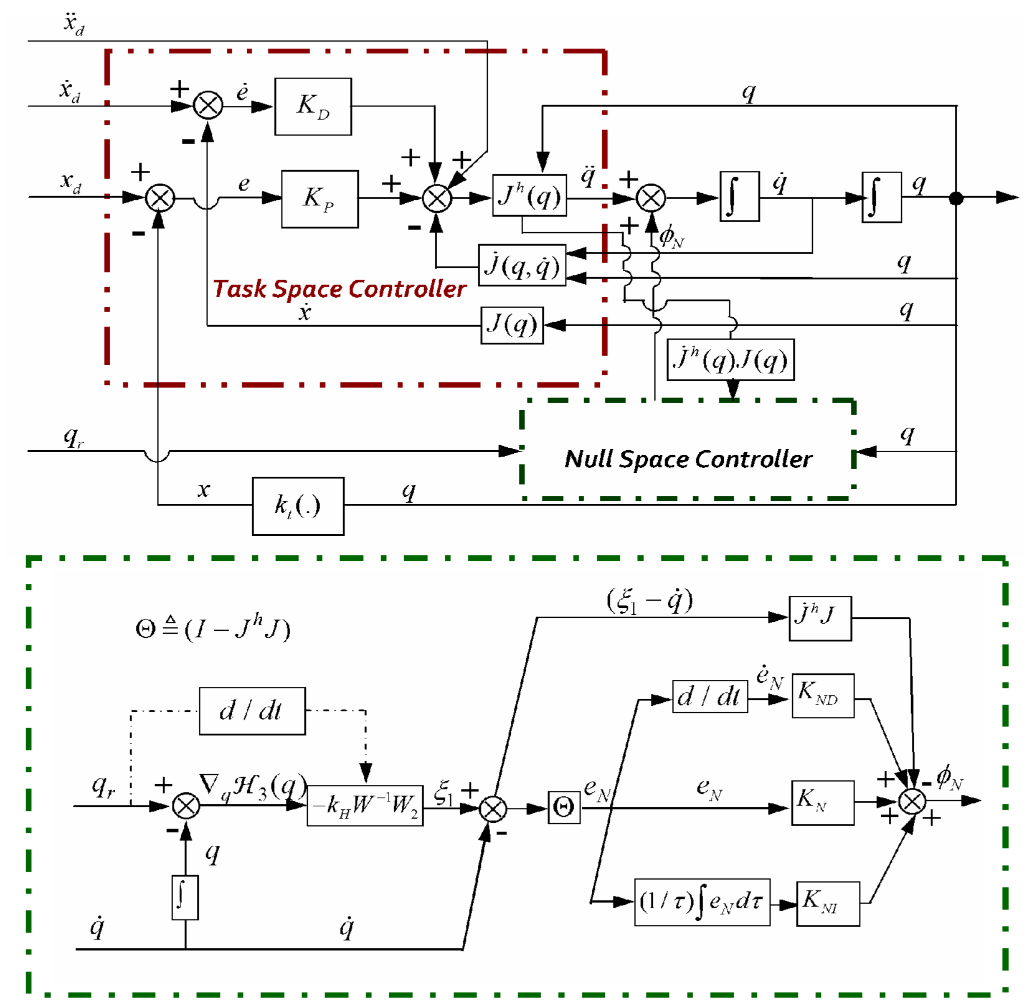

FIGURE 1: [a] Schematic implementation for 2nd order resolution in CWLS solution. [b] Null space controller schematic. ϕ_N is the null space contribution.

Continuous time stability can be analyzed by Lyapunov second or direct method for Eq.(9) by selecting Lyapunov candidate function $V(e)=(1/2)e^T Ke+V_2$, $\forall V(e)>0$ and $V_2=(1/2)\beta^2\dot{q}^T K_{NS}\dot{q}$ resulting $\dot{V}(e)=e^T K\dot{e}+\dot{V}_2$. V_2 is included to ensure that the system does not go unstable in the Null Space Motion. K , K_{NS} are symmetric positive definite diagonal matrices for task space and null space respectively. Substituting the value of $e \; and \; \dot{e}$ in expression of $\dot{V}(e)$ and after simplification and substitution of $JJ^h = I; \; and \; J(I_{n\times n}-J^h J)\xi_1 = O$ we can establish $\dot{V}(e)=-e^T K^T K_p e+\dot{V}_2$.Considering the case of a constant reference $(\dot{x}_d = 0)$, the function $\dot{V}(e)$ is negative definite, under the assumption of full rank for J and β is so chosen such that $\dot{V}_2$ is negative , indicates solution is stable in Lyapunov sense. If we consider the regularized version [8] of CWLN solution, $\because JJ^{h^*} \neq I; \; J(I - J^h J)=O$, and $(I - JJ^{h^*})\neq O$ the error system reduces to

$$\ddot{e}+K_D\dot{e}+K_P e = N[\ddot{x}_d - \dot{J}\dot{q}+K_D\dot{e}+K_P e]; \; \forall \mathbf{N} \in (I-JJ^{h^*})\neq O \tag{10}$$

In defining the null space controller (Figure-1[b]), the first question that has to be answered is how many sub tasks the null space can simultaneously handle? If we choose k sub tasks each of rank r_k, the limit is $\sum_{i=1}^{k} r_i = n$. Once all the dof's are exhausted, it is useless to put additional low priority tasks, as their contribution will be always projected in to null space or they can even corrupt the primary task. Dropping the regularizing term for the time being and defining the null space error e_N, the null space contribution as ϕ_N is

$$\phi_N = (I-J^h J)[\dot{\xi}_1 + K_N e_N - \dot{J}^h J(\xi_1-\dot{q})]; \; \forall e_N \in (I-J^h J)(\xi_1-\dot{q}) \tag{11}$$

Defining a Lyapunov positive definite candidate function $V(e_N)=(1/2)e_N^T e_N$, or $\dot{V}(e_N)=e_N^T\dot{e}_N$, substituting the values of $\dot{e}_N$ in $\dot{V}(e_N)$ and after simplification we can establish $\dot{V}=-K_N e_N^T e_N$, [Appendix-I.B] which is negative definite for positive definite symmetric null space proportional gain matrix K_N, which implies that the proposed controller in Eq.(11) stabilizes null space motion as long as the Jacobian is full rank.

4. RESULTS AND DISCUSSION

To illustrate the performance, we discuss the results of null space optimized $q_{cwls\_opt}(t)$ form in Eq.(6) and its canonical $q_{cwls\_ref}(t)$ form in Eq.(4), for a planar serial 3RRR manipulator following two distinct types of trajectories, namely, the trajectory resembling the motion of finger tip (Γ_1) and lamniscate trajectory (Γ_s). The particular solution $q_{cwls\_p}(t)$ and CWLN solution with joint limit activation $q_{cwls\_opt\_jla}(t)$ are also plotted to understand the contribution of null space and self motion.

In both the cases the link parameters in Denavit Hardenberg standard convention is $l_i=[1.5,0.9,0.7]cm$, $\alpha_i=[0,0,0], d_i=[0,0,0] \; and \; \theta_i=[q_1,q_2,q_3]$. Γ_1 is analytically generated by joint space vector $q_t(t)=[0.3t^2+0.2;0.5t^2+0.5;0.7t^2+0.3]$ and the reference joint space vector is $q_r(t)=[0.4t+0.2;1.0t+0.5;1.0t+0.5]$ with values far away from $q_t(t)$. JLA parameters in Eq.(8) are $\tau=0.1,\; a=100,\; \bar{\varepsilon}=0.3,\; \varepsilon_0=1e-4,\quad q_{max}=[0.8\;\; 1.8\; 2.6]^{\mathrm{T}},\; q_{min}=[-0.5\;\; -0.5\;\; -0.5]^{\mathrm{T}},\quad \mu = 1e+7.$ Initial values of $W_1 = I_{3\times3}$ and $W_2 = diag[45.0\; 45.0\; 45.0]$, resulting $(W_1+W_2)^{-1}W_2 = 0.978$. The task space controller parameters are $K_P = diag[1\;\; 1]*0.07/dt;$, $K_D = diag[1\;\; 1]*0.9$ and

$K_I = diag[1\ 1]*0.1$. The null space controller parameters are $k_H = 0.95,\ K_{NP} = diag[45\ 45\ 45],\ K_{ND} = diag[4\ 4\ 4]\ \ and\ K_{NI} = diag[10\ 10\ 10]$.

$q_{cwls\_opt}(t)$ solution for Γ_1 recovers the joint configuration better than $q_{cwls\_ref}(t)$ and it is in good agreement with $q_t(t)$ Figure-2[a]-[c]. The particular solution $q_{cwls\_p}(t)$ (range space) fails to follow $q_t(t)$ after $t \approx 0.5s$. The null space error e_N for q_1, rapidly converges from -0.7 at $t = 0s$ to -0.01 at $t = 0.02s$ and remains steady with a peak response at $t = 1.7s$ after which it again converges to zero (Figure-2[e]). The peak in e_N time history corresponds near configuration singularity in joint space between $1.3s \le t \le 1.7s$, in which σ_{min} (min svd(J)) drops from 1.2 to 0.56. The effect of $K_{ND}\dot{e}_N$ term is more prominent in contributing to ξ_2 and finally in null space acceleration ϕ_a .

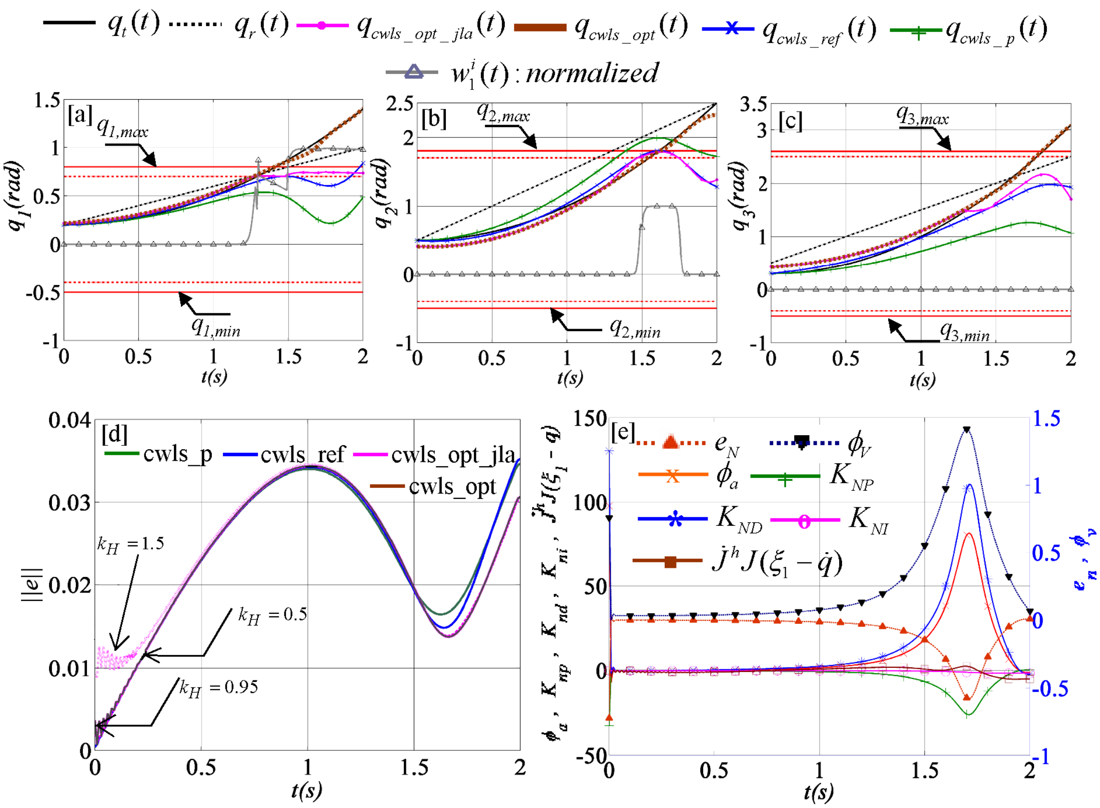

FIGURE 2: [a]-[c]: Time history of joint configurations with null space contribution for finger tip trajectory. Horizontal dotted lines represents joint activation threshold values q_{max}^{th} and q_{min}^{th} .[d] Time history of task space error norm $\| e(t) \|$.[e] Null space response for q_1 with out JLA, left Y-axis for variables e_N *and null space velocity* ϕ_V .

The contribution of $K_{NI}\frac{1}{\tau}\int e_N d\tau$ is insignificant here and contribution from $\dot{J}^h J(\xi_1 - \dot{q})$ is difficult to interpret in this case as its value is seen rising only during the configuration singularity period. The net effect of these terms is reflected in ϕ_a . Here ϕ_V is used to evaluate $\dot{q}(t)$ as a 1st order resolution and from which we can evaluate e_N and subsequently ϕ_a in the 2nd order resolution. Thus the null space interaction between $1.3s \le t \le 1.7s$, which raises ϕ_V and ϕ_a shifts the recovered joint space trajectory towards q_r and q_t in $q_{cwls\_opt}(t)$. This response can be utilized for an event where some preferred poses are desired in joint space, keeping the task space error minimum. Increase in the value of the scalar k_H , results in initial oscillations in the solution as reflected in the Figure-2[d].

To observe the response of W_1 near joint limits, its normalized value is additionally plotted in Figure-2[a]-[c]. When q_1 , reaches its joint threshold limit $q_{1,max}^{th} = 0.7 rad$, at $t = 1.26s$, the normalized value of w_1^1 in Eq.(8) increases from zero at $t = 1.17s$ to 0.6 at $t = 1.26s$. The first diagonal element of $(W_1 + W_2)^{-1} \to 0$, arresting further motion. The null space controller contribution is drastically scaled down as $(W_1 + W_2)^{-1} \to 0$, and the solution finally dominated by the particular solution. Arresting of motion near $q_{1,max}^{th}$ results oscillations in joint accelerations in second order formulation which amplifies oscillations in φ^1 , by the term μ . This is because we have formulated the JLA algorithm based on the joint configuration as $\varphi_t = f(q_t, q_{max}^{th}, q_{max}, q_{min}^{th}, q_{min})$. This will only occur when q_1 over shoots $q_{1,max}^{th}$ in k^{th} time step gets damped and returned back to lower value in $(k+1)^{th}$ time step, until it is gradually damped out. This behavior has been reduced by implementing the term ε_0 in Eq.(8). For joint 2, the $q_{cwls\_p}(t)$ solution overshoots the limit and $q_{cwls\_ref}(t)$ touches the maximum limit. For joint 3, JLA is not actuated for $q_{cwls\_opt}(t)$ as it is well under actuation threshold limit.

For the Regularized Composite Least Square (RCWLS) solution, the lamniscate trajectory (Γ_s) simulates the condition of reaching workspace singularity condition, crossing it and then moving away form it as the trajectory is closed and has two distinct lobes which results in multimodal joint space trajectories. Moreover this particular case is extreme as $q_t(t)$ and $q_r(t)$ differs both in amplitude and phase. The iteration started with $W_1 = I_{3\times3}$, $W_2 = diag[75.0\ 75.0\ 75.0]$, $q_{i,min} = [-1.5\ \ -0.5\ -0.5]$, $q_{i,max} = [2\ \ 2.3\ \ 2.3]$, $\tau = 0.25\, rad$, $a = 75$; $\overline{\varepsilon} = 0.4$; $\varepsilon_0 = 1e-4; \mu = 1e+7$, $K_P = diag[45\ 45]$, $K_D = diag[0.45\ 0.45]$, and $K_I = diag[0.1\ 0.1]$. The null space controller parameters are $k_H = 0.95$, $K_{NP} = diag[45\ 45\ 45]$, $K_{ND} = diag[2.5\ 2.5\ 2.5]$, and $K_{NI} = diag[1.0\ 1.0\ 1.0]$.

The first workspace singularity crossing occurs between $0.08s \le t \le 0.3s$ when the tip crosses from A to B in Γ_s (Figure-3[d]) and second workspace singularity occurs between $1.1s \le t \le 1.5s$ when the tip crosses from C to D. In between these two, the solution faces near configuration singularity when it crosses from P to Q between $0.6s \le t \le 0.8s$ and from R to S between $1.6s \le t \le 1.8s$. It is to be mentioned here that initial high oscillating acceleration between $0.0s \le t \le 0.05s$ in $\| e \|$ is due to the task space gains. In the near configuration singularity cases (pq and rs) in Figure-3[e] which lowers $\sigma_m(t)$ between $0.6s \le t \le 0.8s$ and $1.6s \le t \le 1.8s$, the damping parameter $\alpha(t)$ does not interfere $\forall \varepsilon = 0.5$, the threshold value to initiate damping and $\alpha(t) = f(\sigma_m, \varepsilon)$.

$q_{cwls\_opt\_jla}(t)$ solution increased $\| e \|$ between $1.3s \le t \le 1.5s$ due to the simultaneous occurrences of JLA for q_3 and singularity crossings from C to D. It should be noted that in the expression of $J^{h*} = (W_1 + W_2)^{-1} J^T (J (W_1 + W_2)^{-1} J^T + \alpha^2 I_{m\times m})^{-1}$, increase of $\alpha(t)$ to α_{max} during singularity keeping $W_1\ and\ W_2$ to its initial values , will reduce the over all value of J^{h*} . On the contrary, during JLA, increase of the diagonal element $w_{1,3}$ of the weighing matrix W_1 to a very high value (Oe+7), will only make the third row of J^{h*} approaching to zero in order to make that particular joint immobile but the other two rows of J^{h*} may increase or decrease as per the action of the task space controller.

So the combined effect is overall damping of J^{h*} due to $\alpha(t)$ and the third row is approaching zero. This increases the task space error between $1.3s \le t \le 1.5s$ in comparison to $q_{cwls\_opt}(t)$, where only singularity avoidance is active. The null space contribution from $(\phi_v\ and\ \phi_a)$ has been considerably diminished as high gain of W_1 during JLA makes $(W_1+W_2)^{-1} \to 0$ and W_2 remains constant in the null space. Further increase of value of $W_2\ and\ k_H$ and null space gain parameters results in increased oscillation in initial joint velocity and acceleration and also increases $\|e\|$.The task space and null space gains are kept on the higher side in the simulation which causes initial oscillations in joint space in some cases. It has been verified that reducing these gains eliminates these initial oscillations except during near singular or singularity crossings. The role of the weighing matrices $W_1\ and\ W_2$ has been defined with a bias to higher gain of W_2 which will amplify the null space contribution and in doing this the $(W_1+W_2)^{-1}W_2$ term is advantageously used.

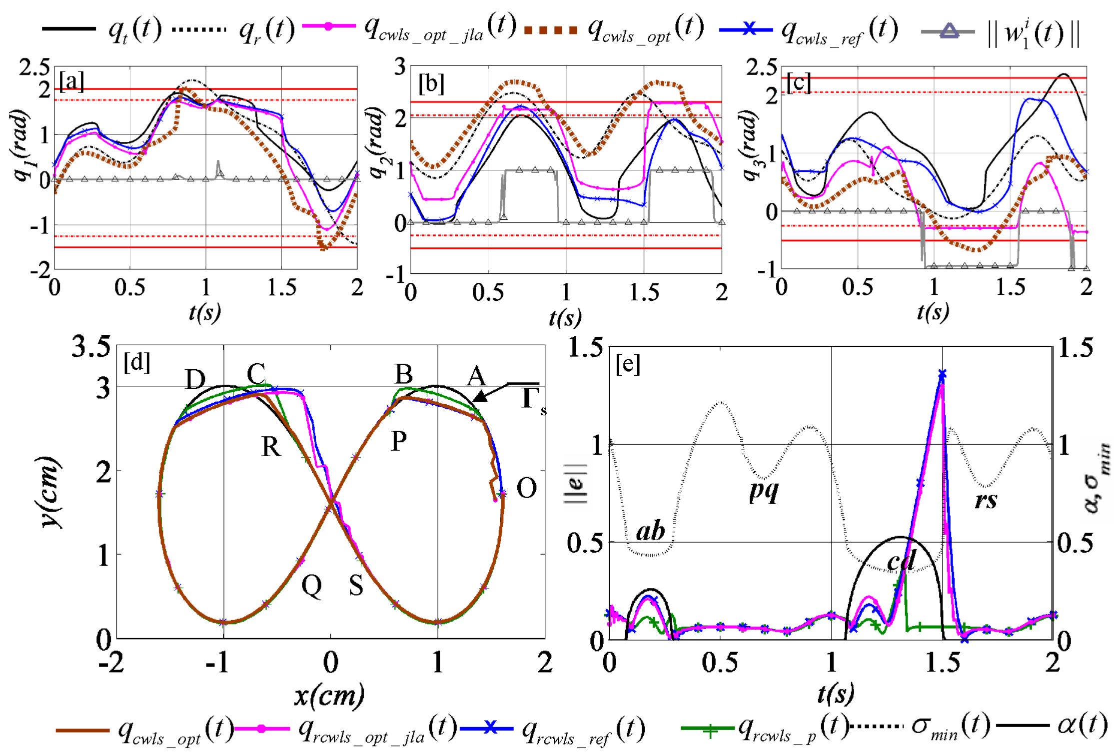

FIGURE 3: [a]-[c]: Time history of joint configurations with null space contribution for lamniscate trajectory Γ_s. Horizontal dotted lines represents joint activation threshold values q_{max}^{th} and q_{min}^{th} .[d] Trajectory trace for the solutions. Analytical trajectory generating workspace singularity Γ_s is OABPQDCRSO. [e] Time histories for $\|e\|$, α and $\sigma_m \triangleq min\ svd(J)$ values.

In a hypothetical situation, we want to see the response when the reference signal (q_r) in joint space approximates the analytical joint trajectory $(q_r = 0.9q_t)$, as in the earlier cases q_r is generated with considerably deviation from q_t. For trajectory Γ_1, (Figure-4: Top row) both

$q_{cwls\_opt}(t)$ and $q_{cwls\_ref}(t)$ solutions remain in between q_t and q_r, and the difference between them can be neglected where as the particular solution deviates significantly q_t as before.

Similar responses obtained from trajectory Γ_s for lamniscate path (Figure-4: Bottom row) for $q_r = 0.9q_t$. In this situation, the null space error e_N, for trajectory Γ_1, remains stable at ≈ 0 until it briefly oscillates in near configuration singularity period between $1.3s \le t \le 1.7s$ (Figure-5: Left) and between $0.6s \le t \le 0.7s$ and $1.5s \le t \le 1.7s$ for lamniscate trajectory Γ_s (Figure-5: Right). During these time periods there is a surge in ϕ_V and ϕ_a injecting the null space contribution in the solution. For the remaining time in all cases, the null space contribution is ≈ 0, which is desired as the recovered joint space trajectory is in between q_t and q_r (Figure-4: Top Row).

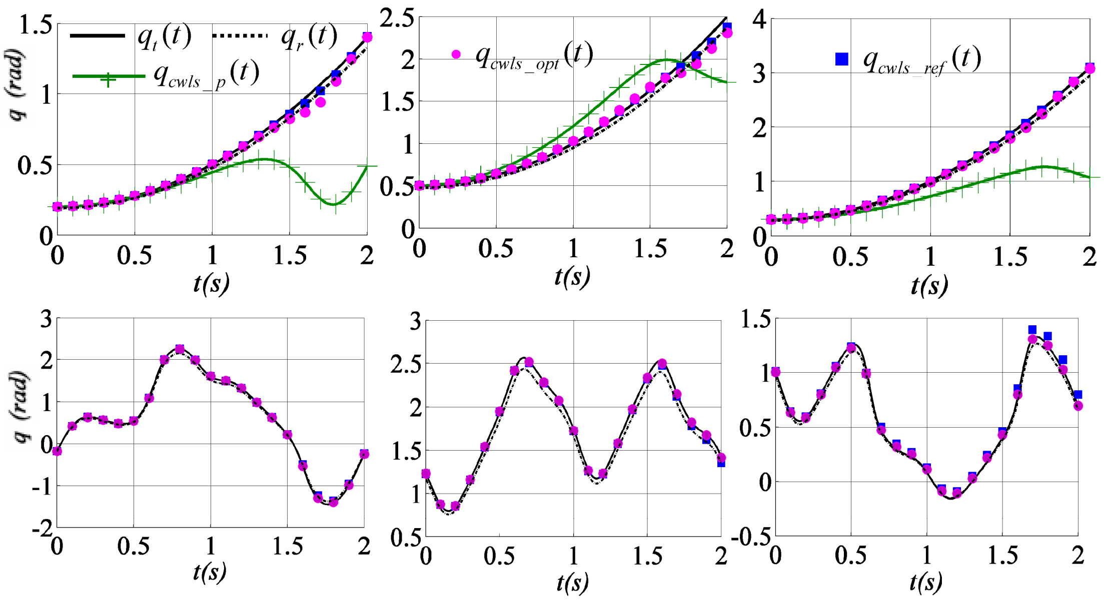

FIGURE 4: Top: Time history of joint configurations for trajectory Γ_1 with the special case of $q_r = 0.9q_t$. Bottom: Time history of joint configurations for trajectory Γ_s with the special case of $q_r = 0.9q_t$. All results are for $q_{cwls\_opt}(t)$ solution. Columns from left represent joints q_1, q_2 *and* q_3 respectively.

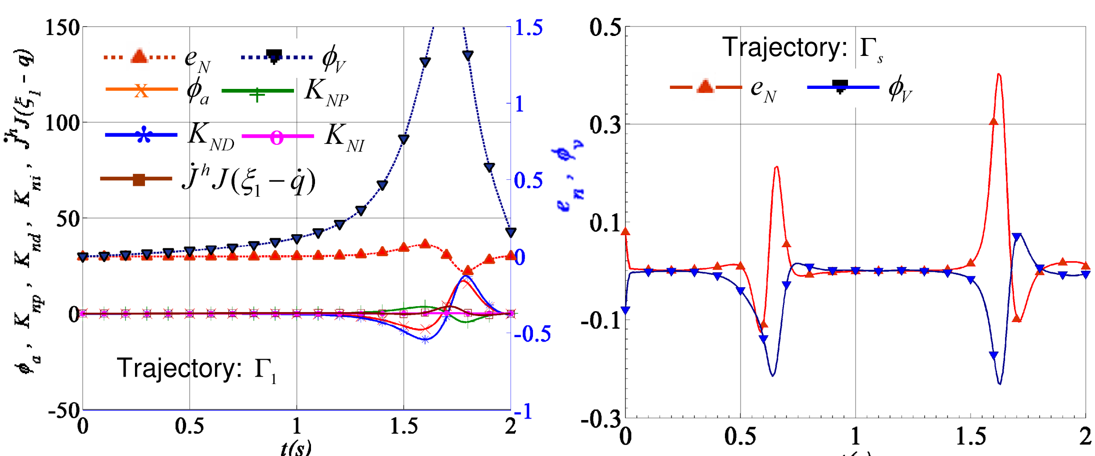

FIGURE 5: Left : Null space response for Γ_1 when $(q_r = 0.9q_t)$ Left Y-axis for variables e_N *and* ϕ_V . Right: Null space response for variables e_N *and* ϕ_V for lamniscate trajectory Γ_s . All results are for $q_{cwls\_opt}(t)$ solution.

5. CONCLUSION

By composite weighting the range and null space we can arrive at a solution which is able to handle both joint limits and preferred joint configuration, simultaneously satisfying the primary task. The solution lies between $q_t \ and \ q_r$ also shifts the recovered joint space to wards the reference configuration q_r without JLA. In this formulation the role of W_2 and q_r is of paramount importance as it controls the contribution form null space along with scalar k_H. It has been observed that null space velocities ϕ_V and acceleration ϕ_a are shooting up antagonistically to e_N which signifies that the null space controller is working and there is self motion contribution form null space when e_N is facing a drift from asymptotic stability. This enables the CWLS framework to retrieve the desired joint configuration given the desired task space and preferred joint rate $(\dot{q}_r)$ without considering any joint dependency.

The response can be utilized for an event where some preferred poses are desired in joint space, keeping the task space error minimum, which can be exploited for recovering various human postures where the motion workspace is limited and there is practical difficulty in mounting optical markers or inertial motion sensors due to limited space availability or hindrance in natural articulation. A typical application in this regard is recovering human palmer grasps (full closure of fist) postures which are currently under study. The task is challenging, as in human palmer grasp motion, apart from it's high dimensionality, the problem is much more aggravated by limited workspace space availability, cross finger occlusion, constraints in finger joint motion and full traversal of joint motion ranges. The state of the art motion tracking technologies using optical or inertial sensing for retrieving position and orientation data from each joint sometimes becomes infeasible for this particular grasp mode, due to space limitations and slip, which results in restricting natural articulation.

The limitation with $(W_1 + W_2)^{-1}$ term is with the activation of JLA, it reduces the null space contribution. Sensitivity of k_H parameter is another issue and hence its bound has been kept in between 0.75-0.95 for most of the cases as it is additionally coupled with the term $(W_1 + W_2)^{-1} W_2$. The other important limitation observed in CWLS scheme is its dependency on initial configuration. Hence it will require an initial configuration close to the analytical solution.

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APPENDIX- I.A CWLS derivation

Objective **:** $min(\dot{q})\mathrm{H}_3(\dot{q}) = min(\dot{q})[(1/2)\dot{q}^T W_1 \dot{q} + (1/2)(\dot{q}-\dot{q}_r)^T W_2(\dot{q}-\dot{q}_r)];\ s.t\ \ J\dot{q} = \dot{x}$

$\forall (W_1, W_2) \in \mathbb{R}^{n\times n}$ and positive diagonal positive definite ,

$$Lagrangian: L(\dot{q},\lambda) = \mathrm{H}(\dot{q}) + \lambda(J\dot{q}-\dot{x}) = [(1/2)\dot{q}^T W_1 \dot{q} + (1/2)(\dot{q}-\dot{q}_r)^T W_2(\dot{q}-\dot{q}_r)] + \lambda(J\dot{q}-\dot{x})$$

$$\nabla_{\dot{q}} L = \frac{\partial L^T}{\partial \dot{q}} \Rightarrow W_1\dot{q} + W_2(\dot{q}-\dot{q}_r) + J^T\lambda = 0 \ and\ \ \nabla_\lambda L = J\dot{q}-\dot{x} = 0 \ with\ \nabla^2_{\dot{q}} L = (W_1+W_2) > 0$$

$$\Rightarrow W_1\dot{q} + W_2(\dot{q}-\dot{q}_r) + J^T\lambda = 0 \ \Rightarrow (W_1+W_2)\dot{q} = W_2\dot{q}_r - J^T\lambda;\ \Rightarrow \dot{q} = (W_1+W_2)^{-1}(W_2\dot{q}_r - J^T\lambda)$$

$$Putting\ the\ value\ of\ \dot{q}\ in\ \nabla_\lambda L = 0;\ J(W_1+W_2)^{-1}(W_2\dot{q}_r - J^T\lambda) = \dot{x};$$

$$\Rightarrow \lambda = (J(W_1+W_2)^{-1}J^T)^{-1}[J(W_1+W_2)^{-1}W_2\dot{q}_r - \dot{x}] \ and\ \forall W \triangleq (W_1+W_2), \lambda = (JW^{-1}J^T)^{-1}[JW^{-1}W_2\dot{q}_r - \dot{x}]$$

$$\Rightarrow \dot{q} = W^{-1}J^T(JW^{-1}J^T)^{-1}\dot{x} + (I - W^{-1}J^T(JW^{-1}J^T)^{-1}J)W^{-1}W_2\dot{q}_r$$

$$\forall J^{\hbar} \triangleq W^{-1}J^T(JW^{-1}J^T)^{-1}; \forall \xi_1 \triangleq (W_1+W_2)^{-1}W_2\dot{q}_r;$$

$$\dot{q} = J^{\hbar}\dot{x} + (I - J^{\hbar}J)(W_1 + W_2)^{-1}W_2\dot{q}_r = J^{\hbar}\dot{x} + (I - J^{\hbar}J)\xi_1$$

APPENDIX- I.B Null space Lyapunov stability.

Differentiating the null space error term e_N in Eq.(11),

$\dot{e}_N = (I - J^h J)(\dot{\xi}_1 - \ddot{q}) - (\dot{J}^h J + J^h \dot{J})(\xi_1 - \dot{q})$;

rewriting $(\xi_1 - \dot{q})$ and substituting in $\dot{e}_N$,

$\dot{e}_N = (I - J^h J)(\dot{\xi}_1 - \ddot{q}) - \dot{J}^h J e_N - J^h \dot{J} e_N - (\dot{J}^h J J^h + J^h \dot{J} J^h) J(\xi_1 - \dot{q});\ \because J e_N = 0\ \ as\ e_N \in \mathcal{N}(J),$ and

$JJ^h = I,\ \Rightarrow \dot{J}J^h + J\dot{J}^h = 0$; which after simplification

$$\dot{e}_N = (I - J^h J)(\dot{\xi}_1 - \ddot{q}) - J^h \dot{J} e_N - (I - J^h J)\dot{J}^h J(\xi_1 - \dot{q})$$

Now $(I - J^h J)\dot{J}^h J(\xi_1 - \dot{q}) = (I - J^h J)(\dot{\xi}_1 + K_N e_N) - \phi_N$ from Eq. (11)

$$or,\ \dot{e}_N = (I - J^h J)(\dot{\xi}_1 - \ddot{q}) - J^h \dot{J} e_N - (I - J^h J)\dot{\xi}_1 - (I - J^h J)K_N e_N + \phi_N$$

Substituting the value of $\ddot{q}$ from Eq.(6)

$$\dot{e}_N = (I - J^h J)[(\dot{\xi}_1 - J^h((\ddot{x}_d - \dot{J}\dot{q}) + \ddot{q}_N)] - J^h \dot{J} e_N - (I - J^h J)\dot{J}^h J(\xi_1 - \dot{q})]$$

$$= (I - J^h J)\dot{\xi}_1 - (I - J^h J)J^h(\ddot{x}_d - \dot{J}\dot{q}) - (I - J^h J)\ddot{q}_N - J^h \dot{J} e_N - (I - J^h J)\dot{J}^h J(\xi_1 - \dot{q})$$

$$= (I - J^h J)\dot{\xi}_1 - (I - J^h J)\ddot{q}_N - J^h \dot{J} e_N - (I - J^h J)\dot{\xi}_1 - (I - J^h J)K_N e_N + \phi_N$$

$$= -J^h \dot{J} e_N - (I - J^h J)K_N e_N;\ \ \because (I\text{ - }J^h J)\ddot{q}_N\ projects\ \ddot{q}_N in\ N(J)\ and\ = \phi_N$$

$$\therefore \dot{e}_N = -(I - J^h J)K_N e_N - J^h \dot{J} e_N\ \ .$$

Defining a Lyapunov positive definite candidate function :

$V = (1/2)e_N^T e_N \Rightarrow \dot{V} = e_N^T \dot{e}_N$ and substituting the values of $\dot{e}_N$

$$\dot{V} = e_N^T(-(I - J^h J)K_N e_N - J^h \dot{J} e_N) = -e_N^T(I - J^h J)K_N e_N - e_N^T J^h \dot{J} e_N)$$

$$= -e_N^T K_N e_N + e_N^T J^h J K_N e_N - e_N^T J^h \dot{J} e_N = -K_N e_N^T e_N + e_N^T J^h (K_N J - \dot{J})e_N$$

$$\because e_N^T J^h = [(I - J^h J)(\xi_1 - \dot{q})]^T J^h = (\xi_1 - \dot{q})^T (I - J^h J)^T J^h = (\xi_1 - \dot{q})^T (I - J^h J)J^h = 0$$

$$\Rightarrow \dot{V} = -K_N e_N^T e_N$$

which is negative definite for positive definite symmetric null space proportional gain matrix K_N, which implies that the proposed controller stabilizes null space motion as long as the Jacobian is full rank.