Flocking Control In Networks of Multiple VTOL Agents with Nonlinear and Under-actuated Features

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Abstract

In this paper, the problem of flocking control in networks of multiple Vertical Take-Off and Landing (VTOL) agents with nonlinear and under-actuated features is addressed. Compared with the widely used double-integrator model, the VTOL agents are distinguished with nonlinear and under-actuated dynamics and cannot be linearly parameterized. A unified and systematic procedure is employed to design the flocking controllers by using the backstepping technique to guarantee multi-agents to arrive at a fixed formation and converge in a desired geometric pattern whose centroid move along a desired trajectory. Finally, some numerical simulations are provided to illustrate the effectiveness of the new design.

Keywords: Flocking, Multi-agent System, Under-actuated, VTOL, Formation.

1. INTRODUCTION

Over the last decades, tremendous interests have been paid to the problem of flocking in biology, physics, and computer science. Engineering applications of flocking include search, rescue, coverage, surveillance, sensor networks, and cooperative transportation [1]. The analysis of flocking is inspired by scenes of animals in nature, such as birds, fishes, and bacteria. Compared with a single animal, multiple animals provide advantages over their monolithic counterparts for many reasons. Perhaps the greatest benefit is their resilience against entirety failures and their ability to adapt to unknown environments [2]. Thus, a variety of algorithms have been proposed for the coordination of multiple agent systems and the flocking control problem has been employed in many fields.

While, there remain several open problems to be solved before the complete of flocking controller. Work on control of multi-agent systems mainly focuses on linear systems with first-order [3] or second-order [4] dynamics, and the work on Vertical Take-Off and Landing (VTOL) is immediately inspired by the recent results in coordinated control of multi-agent systems. Some nonlinear or under-actuated models are also used for underwater vehicles [5] and wheeled robots [6]. Furthermore, the second-order nonholonomy in hovercraft systems of flocking is also considered [7].

However, only a little work has been done on nonlinear systems with nonlinear couplings and under-actuated systems. Under-actuated systems are those with fewer inputs than their degrees of freedom. Controllability, for instance, which is usually implied in systems with full control, is not easy to determine in an under-actuated system. Control synthesis for an under-actuated system is also more complex than that for a system with full actuated features [8][9]. Formation control of VTOL Unmanned Aerial Vehicles (UAVs) with communication delays are presented in [10] and formation control of VTOL UAVs without linear velocity measurements are required to track a desired reference linear velocity and maintain a desired formation [11]. In [12], a fault tolerant control approach is proposed for the formation control system of UAVs. In addition, several other methods have also been applied to multiple agents. In [13], a distributed smooth time-varying feedback control law is proposed for multiple mobile robots. In [14], formation control of several mobile robots is addressed with the dynamic feedback linearization technique.

In this paper, we propose a backstepping control framework for multiple VTOL agents on an undirected graph. We are particularly settling the following two issues: (1) flocking: VTOL agents cooperatively attain a desired formation shape with all their velocities converging to a common unspecified constant; (2) formation control: similar to the case in flocking, position and velocity converge to a desired timed trajectory. To the best of our knowledge, flocking control for multi-agents with nonlinear and under-actuated features has not been paid extensively attentions in the coordinated control problems. The main difficulty encountered lies in the fact that the under actuation renders many flocking/consensus results instead of deriving from simple agents with full-actuation and finding a suitable (nominal) Lyapunov function for the backstepping design. Compared with the results in [5]-[14], the proposed framework in this paper has the following advantages:

1. Each VTOL agent is a high-order system with nonlinear and under-actuated features, and flocking is achieved with proven stability via a designed controller by backstepping technique and the graph theory. It is more realistic than the linear or full actuated systems.
2. A systematic backstepping-based control law design method is obtained in this paper. With this design method, the control problems of multiple nonlinear and under-actuated systems can be fixed step by step.
3. The inputs of control are related to force and moment, which is more convenient and direct than velocity and angular velocity in [6][12].

In contrast, our result here fully takes high order nonlinear physics characters of agents into account rather than wheel robots [6] and second integration model [7]. With the challenge to achieve flocking control of multiple agents with under-actuated features, many consensus/flocking results inapplicable [5][6][13]. To address this challenge, we extend the consensus to explicitly incorporate with under-actuated feature via backstepping technique.

The remainder of this paper is organized as follows. In Section II, the flocking control model of VTOL and some preliminary assumptions are given. The main result, namely the flocking control scheme, is proposed via backstepping in Section III. To illustrate the effectiveness of the proposed control, we present in Section IV some simulation results. And finally, Section V concludes the presented results.

2. PROBLEM STATEMENT

2.1 VTOL System

Assume that there are a group of VTOL agents in the same structure. The VTOL aircraft depicted in Fig. 1 is a simplified planar model of a real vertical take-off and landing plane (e.g. the Harrier) [15].

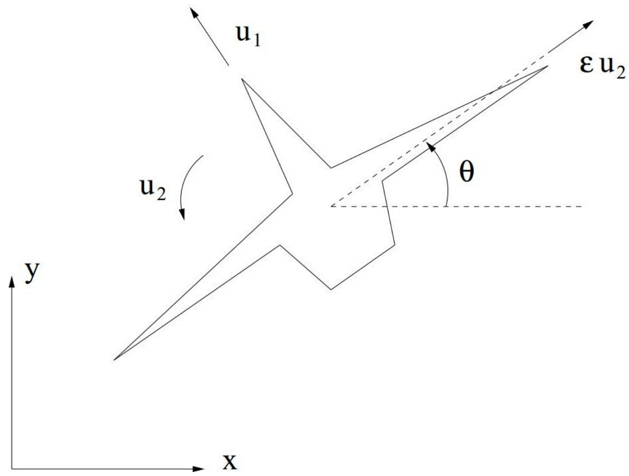

FIGURE 1: The VTOL aircraft.

In the figure, θ is the roll, the xy plane is spanned by the vertical axis and wing axis of aircraft, ε is a coefficient of moment and it is non-dimensional, andu_2 is the roll moment.

The dynamics of one of the VTOL aircraft agents is given in [15] as the following:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= u_1 \sin\theta + \varepsilon u_2 \cos\theta \\
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= u_1 \cos\theta + \varepsilon u_2 \sin\theta - g \\
\dot{\theta} &= \omega \\
\dot{\omega} &= u_2
\end{aligned} \tag{1}$$

where(x_1, y_1) and (x_2, y_2) are the position and velocity of the aircraft, while θ and ω denote its angular and angular velocity, respectively. The VTOL aircraft is an under-actuated system with three degrees of freedom and two control inputs. In [15], it is assumed that $|\varepsilon|$ is relatively small and the VTOL aircraft is treated as a slightly non-minimum phase system.

In order to simplify the issue, a variables transform is introduced.

$$\begin{aligned}
z_1 &= x_1 - \varepsilon \sin\theta \\
z_2 &= x_2 - \varepsilon \cos\theta d\theta \\
\zeta_1 &= y_1 + \varepsilon(\cos\theta - 1) \\
\zeta_2 &= y_2 - \varepsilon \sin\theta d\theta \\
\xi_1 &= \theta \\
\xi_2 &= \omega
\end{aligned} \tag{2}$$

The model of the aircraft agent can be described as:

$$\begin{aligned}\dot{z}_1 &= z_2\\ \dot{z}_2 &= -\tilde{u}_1 \sin\xi_1\\ \dot{\zeta}_1 &= \zeta_2\\ \dot{\zeta}_2 &= \tilde{u}_1 \cos\xi_1 - g\\ \dot{\xi}_1 &= \xi_2\\ \dot{\xi}_2 &= u_2\end{aligned} \tag{3}$$

where $\tilde{u}_1 = u_1 - \varepsilon\xi_2^2$. Under such circumstances, the system can be divided into two subsystems, namely a position subsystem and an angular subsystem.

2.2 Graph and Matrix

The undirected graph $\mathcal{G} = (\mathcal{V}, \varepsilon)$ consists of a vertex set $\mathcal{V} = \{1 \ldots m\}$ and an edge set $\varepsilon \in \mathcal{V} \times \mathcal{V}$, where an edge is an unordered pair of distinct vertices. The definition of adjacency matrix $\mathcal{A} = \mathcal{A}(\mathcal{G}) = [a_{ij}]$, an $m \times m$ matrix, is given by $a_{ij} = 1$, if $(i, j) \in \varepsilon$, and $a_{ij} = 0$ otherwise. In this paper, if $(i, j) \in \varepsilon$, then aircrafts i, j are adjacent. Furthermore, we considered j is a neighbor of i. The neighbors of aircraft i are donated by N_i. If there is a path between any two vertices of $\mathcal{G}$, then $\mathcal{G}$ is connected. The degree of vertex i is obtained by $d_i = \sum_j a_{ij}$.

The incidence matrix, $\mathcal{B} = \mathcal{B}(\mathcal{G}) = [b_{ij}]$ of an undirected graph is a $\{0, 1\}$. Matrix with rows and columns indexed by the vertices and edges of $\mathcal{G}$, respectively, so that $b_{ij} = 1$ if the vertexes i and j are connected by an edge, and $b_{ij} = 0$ otherwise. The laplacian matrix of $\mathcal{G}$ is $L_n = L_n(\mathcal{G}) = diag(d_1, \ldots, d_m) - \mathcal{A}$. For the connected graph $\mathcal{G}$, $L_n = L_n(\mathcal{G}) := \mathcal{B}\mathcal{B}^T$ is symmetric and positive semi-definite [16]. A directed tree is a digraph, where every node has exactly one parent except the root node. A root node is a node that has a directed path to every other node, without any parent. A directed spanning tree is a directed tree that contains all nodes of the digraph. A digraph has a spanning tree if there is a directed spanning tree as a subset of the digraph [8]. For more theories of graph, refer to [16].

In this paper, we consider a group of networked aircraft agents with a connected undirected graph, in which each aircraft is regarded as a vertex, and each existing control interconnection between agents is regarded as an edge.

3. CONTROLLER DESIGN

In order to allow our problem to be more reliable, it is essential to employ the following assumptions.

Assumption 1: For $i = 1, \ldots, m$, the position (x_i, y_i), the velocity (v_{xi}, v_{yi}), the angular information (ξ_{1i}, ξ_{2i}), the positions to its neighbors (x_j, y_j), velocities of its neighbors (v_{xj}, v_{yj}), and angular information of its neighbors (ξ_{1j}, ξ_{2j}) of the ith aircraft for $j \in N_i$ are available for the design of the consensus protocol μ_i.

Assumption 2: For $i = 1, \ldots, m$, $|v_{xd}| + |v_{yd}| > \rho$, $\rho > 0$, where ρ is a constant.

Assumption 3: The graph $\mathcal{G}$ has a spanning tree and the laplacian matrix of $\mathcal{G}$ is balanced.

Remark 1: The physical implication of assumption 1 is that the communications of each neighbor are reliable and delay and noise is not considered here. Assumption 2 is proposed that the velocity of every agent need to be positive and valid.

The objective of controller design is to address a distributed consensus protocol, which uses its own state $(x_i, y_i, v_{xi}, v_{yi}, \xi_{1i}, \xi_{2i})$, its neighbors' state $(x_j, y_j, v_{xj}, v_{yj}, \xi_{1j}, \xi_{2j})$, desired geometric pattern (p_{xi}, p_{yi}), the desired trajectory (x_d, y_d), and the desired velocity (v_{xd}, v_{yd}), so that:

$$\begin{array}{c} \lim_{t\to\inf\infty}((x_i, y_i) - (x_j, y_j)) \to (p_{xi}, p_{yi}) - (p_{xj}, p_{yj}) \\ \lim_{t\to\inf\infty}((v_{xi}, v_{yi}) - (v_{xj}, v_{yj})) \to (0,0) \\ \lim_{t\to\inf\infty}((\frac{\sum x_i}{m}, \frac{\sum y_i}{m}) - (x_d, y_d)) \to (0,0) \end{array} \quad (4)$$

Remark 2: (4) means that groups of aircraft agents reach an agreement that they converge in a given pattern at an identical velocity and the center of their formation moves along the desired trajectory.

We develop a non-linear consensus protocol by backstepping methodology. The system (3) is divided into two subsystems. The first subsystem with state (z_1, ζ_1) is called $z_1\zeta_1$-system, and the second subsystem with state ξ_1 is called ξ-system.

Step 1: $(\tilde{z}_1, \tilde{\zeta}_1)$-subsystem
For every aircraft agent, the desired trajectory (x_d, y_d) in (1) under transform (2) can be obtained:

$$\begin{array}{c} z_{1d} := x_d - \varepsilon \sin\theta \\ \zeta_{1d} := y_d + \varepsilon(\cos\theta - 1) \end{array} \quad (5)$$

Define $\tilde{z}_1(t) = z_{xd}(t) - z_1(t) - p_x$, $\tilde{\zeta}_1(t) = \zeta_{1d}(t) - \zeta_1(t) - p_y$, where $\tilde{z}_1 = [\tilde{z}_{11}(t), \tilde{z}_{12}(t), \ldots, \tilde{z}_{1m}(t)]^T$, $\tilde{\zeta}_1 = [\tilde{\zeta}_{11}(t), \tilde{\zeta}_{12}(t), \ldots, \tilde{\zeta}_{1m}(t)]^T$, $z_{xd} = [z_{xd1}(t), z_{xd2}(t), \ldots, z_{xdm}(t)]^T$, $z_{yd} = [z_{yd1}(t), z_{yd2}(t), \ldots, z_{ydm}(t)]^T$. This way of definition is so as to similar variables in this paper.

Then we can obtain:

$$\begin{array}{c} \dot{\tilde{z}}_1 = \dot{z}_{xd}(t) - \dot{z}_1 = \dot{z}_{xd}(t) - z_2 \\ \dot{\tilde{\zeta}}_1 = \dot{z}_{yd}(t) - \dot{\zeta}_1 = \dot{z}_{yd}(t) - \zeta_2 \end{array} \quad (6)$$

Theorem 1: The $(\tilde{z}_1, \tilde{\zeta}_1)$-subsystem (6) with z_2 is taken as its control input. For any constant $\beta_{1i} \geq 0$,$\sum_{i=0}^{m} \beta_{1i} \geq 0$,$a_{ij} \geq 0$. We define the Lyapunov function candidates as:

$$V_1(\tilde{z}_1) = \frac{1}{2}\tilde{z}_1^T \tilde{z}_1 \quad (7)$$

Distributed control input $z_{2i} = \dot{z}_{xdi} + \beta_{1i}\tilde{z}_{1i} + \sum_{i\in m} a_{ij}(\tilde{z}_{1i}(t) - \tilde{z}_{ij}(t))$ can make the following satisfied.

$$\dot{V}_1(\tilde{z}_1) \leq 0 \quad (8)$$

Proof: All the edges are in graph $\mathcal{G}$ with $i = 1, \ldots, m$. Under assumption 3, the laplacian matrix L_n of $\mathcal{G}$ is balanced. It can be obtained that $(L_n + L_n^T)$ is positive semi definite by using graph theory.

Substituting z_{2i} into (6),

$$\dot{\tilde{z}}_{1i} = -\beta_{1i}\tilde{z}_{1i} - \sum_{i\in m} a_{ij}(\tilde{z}_{1i}(t) - \tilde{z}_{1j}(t)) \quad (9)$$

Differentiate (7) and substituting (8) into (9), then

$$\dot{V}_1 = -\sum_{i=1}^{m} \beta_{1i}\tilde{z}_{1i}^2 - \frac{1}{2}\tilde{z}_1^T(L_n + L_n^T)\tilde{z}_1 \tag{10}$$

Therefore $\dot{V}_1$is negative semi definite.

So as to $\tilde{\zeta}_1$-subsystem (6) with$\zeta_{2i} = \dot{z}_{ydi} + \beta_{2i}\tilde{\zeta}_{1i} + \sum_{i\in m} a_{ij}(\tilde{\zeta}_{1i}(t) - \tilde{\zeta}_{1j}(t))$, we can obtain the same result.

Step 2: $(\tilde{z}_2, \tilde{\zeta}_2)$-subsystem
Define $\tilde{z}_{2i} = z_{2i} - \dot{z}_{xdi} - \beta_{1i}\tilde{z}_{1i} - \sum_{i\in m} a_{ij}(\tilde{z}_{1i}(t) - \tilde{z}_{ij}(t))$, and $\tilde{\zeta}_{2i} = \zeta_{2i} - \dot{z}_{ydi} - \beta_{2i}\tilde{\zeta}_{1i} - \sum_{i\in m} a_{ij}(\tilde{\zeta}_{1i}(t) - \tilde{\zeta}_{ij}(t))$, then

$$\begin{aligned} \dot{\tilde{z}}_{2i} &= \dot{z}_{2i} - \beta_{1i}\dot{\tilde{z}}_{1i} - \textstyle\sum_{i\in m} a_{ij}(\dot{\tilde{z}}_{1i}(t) - \dot{\tilde{z}}_{ij}(t)) \\ \dot{\tilde{\zeta}}_{2i} &= \dot{\zeta}_{2i} - \beta_{2i}\dot{\tilde{\zeta}}_{1i} - \textstyle\sum_{i\in m} a_{ij}(\dot{\tilde{\zeta}}_{1i}(t) - \dot{\tilde{\zeta}}_{ij}(t)) \end{aligned} \tag{11}$$

where, $\ddot{z}_{xd} = 0$, $\ddot{z}_{yd} = 0$.

Define$\delta_1 = -\tilde{u}_1 \sin\xi_1$, $\gamma_1 = \tilde{u}_1 \cos\xi_1$.

Theorem 2: The $(\tilde{z}_2, \tilde{\zeta}_2)$-subsystem with δ_1, γ_1 is taken as its control input. For any constant $\beta_{1i} \geq 0$, $\sum_{i=0}^{m} \beta_{1i} \geq 0$,$\beta_{2i} \geq 0$, $\sum_{i=0}^{m} \beta_{2i} \geq 0$, $a_{ij} \geq 0$, and $r_1(i) > 0$, $r_2(i) > 0$. We define the Lyapunov function candidates as:

$$V_2(\tilde{z}_1, \tilde{z}_2, \tilde{\zeta}_1, \tilde{\zeta}_2) = \frac{1}{2}\tilde{z}_1^T z_1 + \frac{1}{2}\tilde{\zeta}_1^T \zeta_1 + \frac{1}{2}\tilde{z}_2^T z_2 + \frac{1}{2}\tilde{\zeta}_2^T \zeta_2 \tag{12}$$

Distributed control input

$$\delta_{1i} = (\beta_{1i}(\dot{z}_{xdi} - z_{2i}(t)) + \sum_{i\in m} a_{ij}(\dot{\tilde{z}}_{1i}(t) - \dot{\tilde{z}}_{ij}(t)) - r_1(t)\sum_{i\in m} a_{ij}(\dot{\tilde{z}}_{2i}(t) - \dot{\tilde{z}}_{2j}(t))) \tag{13}$$

$$\gamma_{1i} = (\beta_{2i}(\dot{z}_{ydi} - \zeta_{2i}(t)) + \sum_{i\in m} a_{ij}(\dot{\tilde{\zeta}}_{1i}(t) - \dot{\tilde{\zeta}}_{ij}(t)) - r_2(t)\sum_{i\in m} a_{ij}(\dot{\tilde{\zeta}}_{2i}(t) - \dot{\tilde{\zeta}}_{2j}(t))) + g \tag{14}$$

can make the following satisfied.

$$\dot{V}_2(\tilde{z}_1, \tilde{z}_2, \tilde{\zeta}_1, \tilde{\zeta}_2) \leq 0 \tag{15}$$

Proof: Substituting$\dot{z}_2 = \delta_1$, $\dot{\zeta}_2 = \gamma_1$ into (11), we can obtain

$$\begin{aligned} \dot{\tilde{z}}_{2i} &= -r_1(t)\textstyle\sum_{i\in m} a_{ij}(\dot{\tilde{z}}_{2i}(t) - \dot{\tilde{z}}_{2j}(t)) \\ \dot{\tilde{\zeta}}_{2i} &= -r_2(t)\textstyle\sum_{i\in m} a_{ij}(\dot{\tilde{\zeta}}_{2i}(t) - \dot{\tilde{\zeta}}_{2j}(t)) \end{aligned} \tag{16}$$

Differentiate (12) and substituting (15) into (16), then

$$\begin{aligned} V_2 = -\textstyle\sum_{i=1}^{m} \beta_{1i}\tilde{z}_{1i}^2 - \frac{1}{2}\tilde{z}_1^T(L_n + L_n^T)\tilde{z}_1 \\ -\tfrac{1}{2}\tilde{\zeta}_1^T(L_n + L_n^T)\tilde{\zeta}_1 - \tfrac{1}{2}\tilde{z}_2^T\Lambda_1(L_n + L_n^T)\tilde{z}_2 - \tfrac{1}{2}\tilde{\zeta}_2^T\Lambda_2(L_n + L_n^T)\tilde{\zeta}_2 \end{aligned} \tag{17}$$

where, $\Lambda_1 = diag(r_1(1), r_1(2), \ldots, r_1(m))$, and $\Lambda_2 = diag(r_2(1), r_2(2), \ldots, r_2(m))$.

For that $r_1(i) > 0$, $r_2(i) > 0$, Λ_1 and Λ_2 are positive define. With assumption 3 hold, $(L_n + L_n^T)$ is positive semi definite. Then$\Lambda_1(L_n + L_n^T)$ and $\Lambda_2(L_n + L_n^T)$ are positive semi define.

Therefore, $\dot{V}_2$ is negative semi definite.

Then we can obtain

$$\tilde{u}_1 = \sqrt{\delta_1^2 + \gamma_1^2} \tag{18}$$

as the control input in (3).

Define$\bar{\xi}_1 = \arctan\frac{-\delta_1}{\gamma_1 + g} + 2k\pi$, $k = 1, 2, \ldots, n$. In order to obtain the ultimate control input, we use backstepping methodology (refer to [17]). With the control inputs (13), the system in (11) can be written as

$$\begin{aligned}
\begin{pmatrix} \dot{\tilde{z}}_{2i} \\ \dot{\tilde{\zeta}}_{2i} \end{pmatrix} &= \begin{pmatrix} \dot{z}_2 - \beta_{1i}\dot{\tilde{z}}_{1i} - \sum_{i\in m} a_{ij}(\dot{\tilde{z}}_{1i}(t) - \dot{\tilde{z}}_{ij}(t)) \\ \dot{\zeta}_2 - \beta_{2i}\dot{\tilde{\zeta}}_{1i} - \sum_{i\in m} a_{ij}(\dot{\tilde{\zeta}}_{1i}(t) - \dot{\tilde{\zeta}}_{ij}(t)) \end{pmatrix} \\
&= \begin{pmatrix} -\delta_{1i} - \beta_{1i}\dot{\tilde{z}}_{1i} - \sum_{i\in m} a_{ij}(\dot{\tilde{z}}_{1i}(t) - \dot{\tilde{z}}_{ij}(t)) \\ \gamma_{1i} - \beta_{2i}\dot{\tilde{\zeta}}_{1i} - \sum_{i\in m} a_{ij}(\dot{\tilde{\zeta}}_{1i}(t) - \dot{\tilde{\zeta}}_{ij}(t)) - g \end{pmatrix} \\
&\quad + \begin{pmatrix} -\tilde{u}_{1i}\sin\xi_{1i} + \delta_{1i} \\ \tilde{u}_{1i}\cos\xi_{1i} - \gamma_{1i} \end{pmatrix} \\
&= \begin{pmatrix} A \\ B \end{pmatrix} + \tilde{u}_{1i}\begin{pmatrix} \sin\bar{\xi}_{1i} & -\cos\bar{\xi}_{1i} \\ \cos\bar{\xi}_{1i} & \sin\bar{\xi}_{1i} \end{pmatrix}\begin{pmatrix} \frac{\sin\tilde{\xi}_{1i}}{\tilde{\xi}_{1i}} & \frac{\cos\tilde{\xi}_{1i} - 1}{\tilde{\xi}_{1i}} \end{pmatrix}^T \tilde{\xi}_{1i}
\end{aligned} \tag{19}$$

where, $\tilde{\xi}_1 = \xi_1 - \bar{\xi}_1$, $A = -r_1(t)\sum_{i\in m} a_{ij}(\tilde{z}_{2i}(t) - \tilde{z}_{2j}(t))$, and $B = -r_2(t)\sum_{i\in m} a_{ij}(\tilde{\zeta}_{2i}(t) - \tilde{\zeta}_{2j}(t))$.

Then,

$$\dot{\tilde{\xi}}_1 = \dot{\xi}_1 - \dot{\bar{\xi}}_1 = \xi_2 - \dot{\bar{\xi}}_1 \tag{20}$$

Firstly, we calculate$\dot{\bar{\xi}}_1$.

Differentiating $\delta_1$andγ_1, it can be obtained that

$$\begin{aligned}
\dot{\delta}_1 &= \dot{\tilde{u}}_1 \sin\bar{\xi}_1 + \tilde{u}_1\dot{\bar{\xi}}_1\cos\bar{\xi}_1 \\
\dot{\gamma}_1 &= \dot{\tilde{u}}_1 \cos\bar{\xi}_1 - \tilde{u}_1\dot{\bar{\xi}}_1\sin\bar{\xi}_1
\end{aligned} \tag{21}$$

Then we can obtain

$$\dot{\bar{\xi}}_1 = \frac{\dot{\delta}_1\cos\bar{\xi}_1 - \dot{\gamma}_1\sin\bar{\xi}_1}{\sqrt{\delta_1^2 + \gamma_1^2}} \tag{22}$$

Remark 3: Under assumption 2, the desired velocity of every aircraft would not converge to zero at the same time. Furthermore, if (4) is reliable, $\delta_1$and γ_1 will not to be zero at the same time. Step 3: $\tilde{\xi}_1$-subsystem

Theorem 3: The $\tilde{\xi}_1$-subsystem with ξ_2 is taken as its control input. For any constant$\beta_{3i} \geq 0$, $\sum_{i=0}^{m}\beta_{3i} \geq 0$. We define the Lyapunov function candidates as

$$V_3(\tilde{\xi}_1) = \frac{1}{2}\sum_{i\in m}\tilde{z}_{1i}^2 + \frac{1}{2}\sum_{i\in m}\tilde{z}_{2i}^2 + \frac{1}{2}\sum_{i\in m}\tilde{\zeta}_{1i}^2 + \frac{1}{2}\sum_{i\in m}\tilde{\zeta}_{2i}^2 + \frac{1}{2}\sum_{i\in m}\tilde{\xi}_{1i}^2 \tag{23}$$

Distributed control input

$$\xi_{2i} = -\begin{pmatrix} z_{2i} & \zeta_{2i} \end{pmatrix}\begin{pmatrix} \sin\bar{\xi}_{1i} & -\cos\bar{\xi}_{1i} \\ \cos\bar{\xi}_{1i} & \sin\bar{\xi}_{1i} \end{pmatrix}\begin{pmatrix} \frac{\sin\tilde{\xi}_{1i}}{\tilde{\xi}_{1i}} \\ \frac{\cos\tilde{\xi}_{1i}-1}{\tilde{\xi}_{1i}} \end{pmatrix}\tilde{u}_{1i} - \beta_{3i}\tilde{\xi}_{1i} + \dot{\bar{\xi}}_{1i} \tag{24}$$

can make the following to be satisfied.

$$\dot{V}_3(\tilde{\xi}_1) \leq 0 \tag{25}$$

Proof: Substituting (23) into (18) and (19), we have

$$\dot{\tilde{\xi}}_{1i} = -\begin{pmatrix} z_{2i} & \zeta_{2i} \end{pmatrix}\begin{pmatrix} \sin\bar{\xi}_{1i} & -\cos\bar{\xi}_{1i} \\ \cos\bar{\xi}_{1i} & \sin\bar{\xi}_{1i} \end{pmatrix}\begin{pmatrix} \frac{\sin\tilde{\xi}_{1i}}{\tilde{\xi}_{1i}} \\ \frac{\cos\tilde{\xi}_{1i}-1}{\tilde{\xi}_{1i}} \end{pmatrix}\tilde{u}_{1i} - \beta_{3i}\tilde{\xi}_{1i} \tag{26}$$

Differentiate (22) and substituting (25) into (23), then

$$\begin{array}{c} V_3 = -\sum_{i=1}^{m}\beta_{1i}\tilde{z}_{1i}^2 - \frac{1}{2}\tilde{z}_1^T(L_n+L_n^T)\tilde{z}_1 - \frac{1}{2}\tilde{\zeta}_1^T(L_n+L_n^T)\tilde{\zeta}_1 - \frac{1}{2}\tilde{z}_2^T\Lambda_1(L_n+L_n^T)\tilde{z}_2 \\ -\frac{1}{2}\tilde{\zeta}_2^T\Lambda_2(L_n+L_n^T)\tilde{\zeta}_2 - \sum_{i=1}^{m}\beta_{3i}\tilde{\xi}_{1i}^2 \end{array} \tag{27}$$

Therefore $\dot{V}_3$ is negative semi definite.

Step 4: $\tilde{\xi}_2$-subsystem
Define$\tilde{\xi}_{2i} = \xi_{2i} + \beta_{3i}\tilde{\xi}_{1i} - \dot{\bar{\xi}}_{1i}$, then

$$\dot{\tilde{\xi}}_{2i} = \dot{\xi}_{2i} + \beta_{3i}\dot{\tilde{\xi}}_{1i} - \ddot{\bar{\xi}}_{1i} = u_{2i} - \ddot{\bar{\xi}}_{1i} + \beta_{3i}(\xi_{2i} - \dot{\bar{\xi}}_{1i}) \tag{28}$$

Theorem 4: The $\tilde{\xi}_2$-subsystem with u_2 is taken as its control input. For any constant $\beta_{4i} \geq 0$, $\sum_{i=0}^{m}\beta_{4i} \geq 0$. We define the Lyapunov function candidates as

$$V_4(\tilde{\xi}_2) = \frac{1}{2}\sum_{i\in m}\tilde{z}_{1i}^2 + \frac{1}{2}\sum_{i\in m}\tilde{z}_{2i}^2 + \frac{1}{2}\sum_{i\in m}\tilde{\zeta}_{1i}^2 + \frac{1}{2}\sum_{i\in m}\tilde{\zeta}_{2i}^2 + \frac{1}{2}\sum_{i\in m}\tilde{\xi}_{1i}^2 + \frac{1}{2}\sum_{i\in m}\tilde{\xi}_{2i}^2 \tag{29}$$

Distributed control input

$$u_{2i} = \ddot{\bar{\xi}}_{1i} - (\beta_{3i} + \beta_{4i})\xi_{2i} - \beta_{3i}\dot{\bar{\xi}}_{1i} \tag{30}$$

can make the following satisfied.

$$\dot{V}_4(\tilde{\xi}_2) \leq 0 \tag{31}$$

Proof: Substituting (29) into (27), and differentiating (28), we have

$$\begin{array}{c} V_4 = -\sum_{i=1}^{m}\beta_{1i}\tilde{z}_{1i}^2 - \frac{1}{2}\tilde{z}_1^T(L_n+L_n^T)\tilde{z}_1 - \frac{1}{2}\tilde{\zeta}_1^T(L_n+L_n^T)\tilde{\zeta}_1 - \frac{1}{2}\tilde{z}_2^T\Lambda_1(L_n+L_n^T)\tilde{z}_2 \\ -\frac{1}{2}\tilde{\zeta}_2^T\Lambda_2(L_n+L_n^T)\tilde{\zeta}_2 - \sum_{i=1}^{m}\beta_{3i}\tilde{\xi}_{1i}^2 - \sum_{i=1}^{m}\beta_{3i}\tilde{\xi}_{2i}^2 \end{array} \tag{32}$$

Therefore $\dot{V}_4$ is negative semi definite.

Remark 4: With the control input (30) and (18), the system can be globally asymptotically stable and (4) is reliable. In another word, consensus protocol is achieved.

Remark 5: The controller proposed in this paper is general that can also wide extend to other nonlinear under-actuated systems with the same structures, such as under-actuated underwater vehicles, mobile robots [6], UAVs [11].

4. SIMULATION

In order to verify the distributed controller designed in this paper, some numerical simulations are given in this section. There are five aircraft agents and their state information could be obtained without delay. The desired trajectory is

$$(x_d, y_d) = (5, 2.5\sin(t) + 10) \tag{33}$$

and the desired velocity is

$$(v_{xd}, v_{yd}) = (0, 2.5\cos(t)) \tag{34}$$

Pattern is given in Fig.2, the initial positions of them are $p_{10} = [2, 4]$, $p_{20} = [4, 0]$, $p_{30} = [-2, 2]$, $p_{40} = [0, 4]$, and $p_{50} = [-4, -2]$. The given patterns are $p_1 = [-1.1756, -1.618]$, $p_2 = [-1.902, 0.618]$, $p_3 = [1.1756, -1.618]$, $p_4 = [0, 2]$, and $p_5 = [1.9021, 0.618]$.

A graph of five aircraft agents is given in Fig.2.

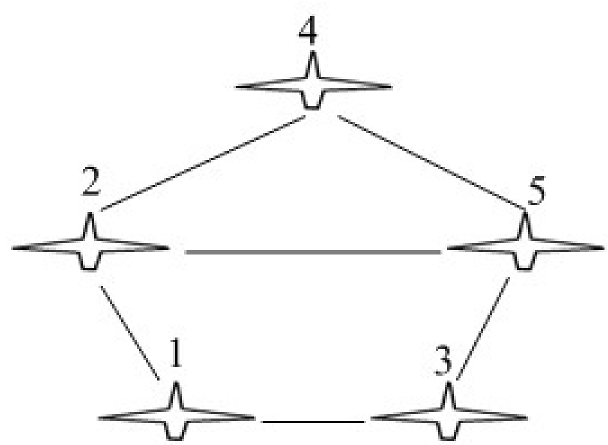

FIGURE 2: Graph of Five Aircraft Agents.

The incident matrix B and the Laplace matrix L_n of the graph in Fig. 2 are

$$B = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & -1 & 1 \end{pmatrix}$$

$$L_n = \begin{pmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 3 & 0 & -1 & -1 \\ -1 & 0 & 2 & 0 & -1 \\ 0 & -1 & 0 & 2 & -1 \\ 0 & -1 & -1 & -1 & 3 \end{pmatrix}$$

The calculation results are $|B| = 2.149$, $\lambda_2(L_n) = 1.382$, $|L_{n1}B| = 4.5826$, $|L_{n2}B| = 7.0711$, $|L_{n3}B| = 4.5826$, $|L_{n4}B| = 4.4721$, and $|L_{n5}B| = 7.0711$. Choose $\beta_{1i} = 1.1$, $\beta_{2i} = 1.1$, $\beta_{3i} = 0.5$, $\beta_{4i} = 0.5$, $\beta_{1i} = 1.1$, $r_1(i) = 1$, $r_2(i) = 1$, $i = 1, \ldots, 5$.

Under the condition that every aircraft agent can obtain the state information as shown in figure 2, and the desired trajectory is available. The distributed controller was designed in section III. The trajectories of agents were presented in figure3.

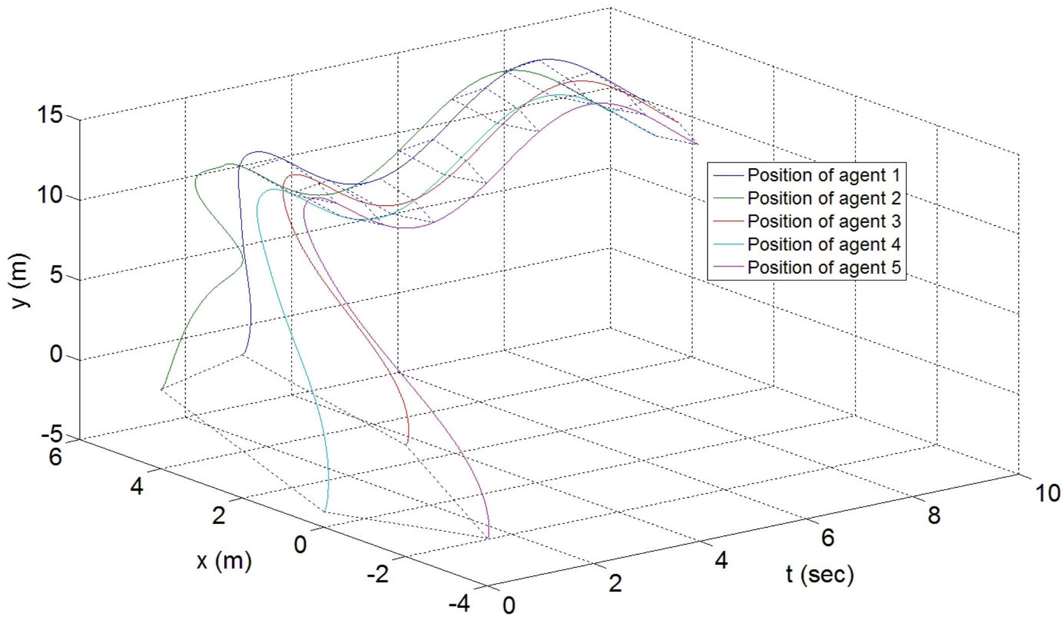

FIGURE 3: The Trajectories of Agents.

Through simulation, two objectives are solved: flocking, and flocking with centroid tracking. The aircraft agents initiate and keep the desired formation, and they follow the desired trajectories after about 5s. Then, the absolute distance between any two aircrafts converges exactly to the desired formation size. As presented in Fig.3, five aircraft agents follow the desired path. Fig.4 shows the center of pattern and the desired trajectory, and Fig.5 shows the formation error between the center of pattern and the desired trajectory. The controls provide convergence of the desired trajectory after a short transience corresponding to the initialization of the formation and the results also prove the effectiveness of the proposed techniques.

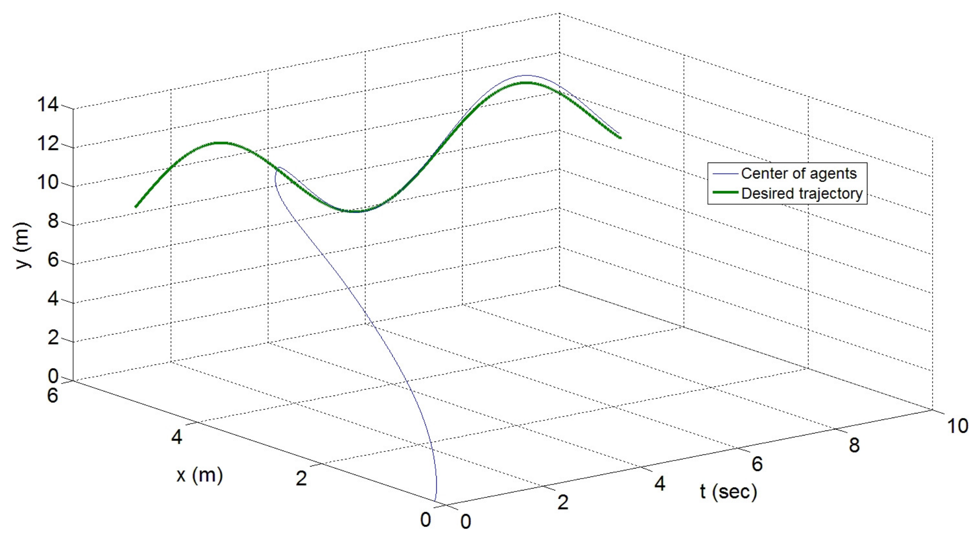

FIGURE 4: The Center of Pattern and Desired Trajectory.

After staying at the initial position for the first second, the aircraft agents initiate the formation following the desired trajectory. The aircrafts achieve perfect positioning after a smooth transience, and the position errors reach a constant in finite time for some tracking error in y axis.

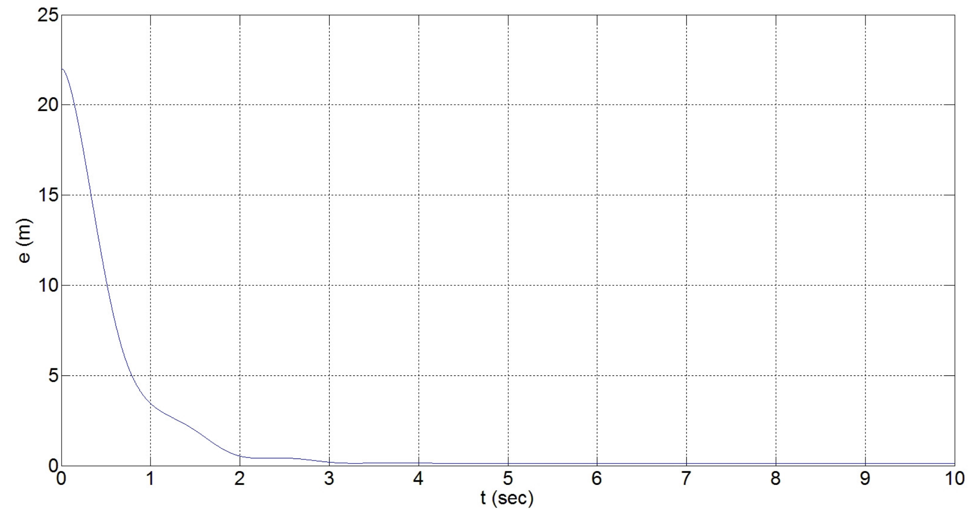

FIGURE 5: The Formation Error.

Fig.6 and Fig.7 show the velocities of agents in axisx, y , respectively. From those figures we can see that the velocities of agents in each axis converge to the center of velocity after 5s, and the positions of agents in each axis maintain a fixed distance at the same time.

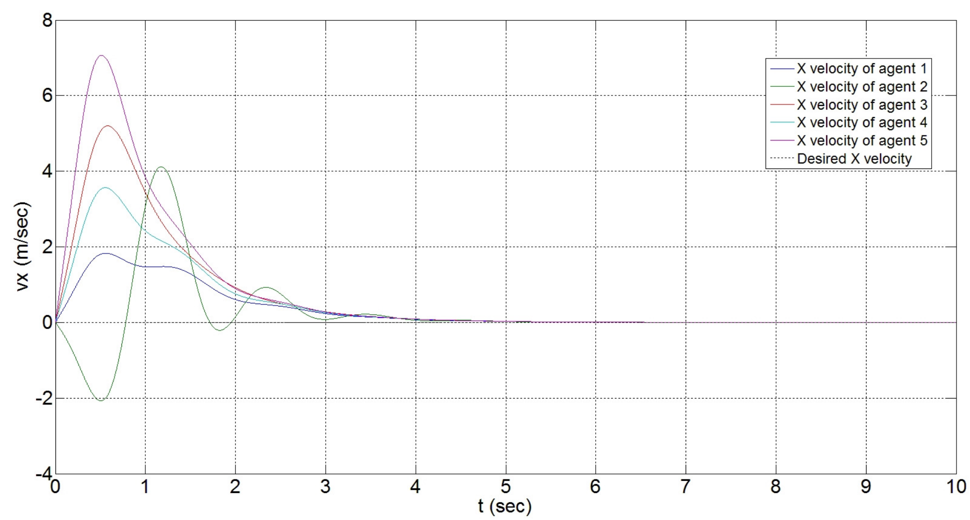

FIGURE 6: Velocities of Agents in axis$x$.

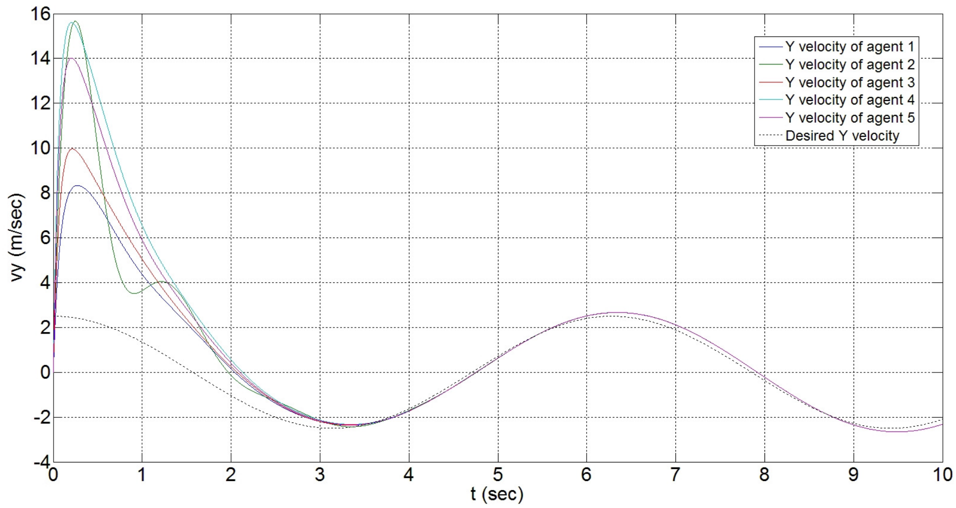

FIGURE 7: Velocities of Agents in axis$y$.

5. CONCLUSION

In this paper, a novel disturbed flocking controller was applied to the formation flight of multiple VTOL agents. Flocking is achieved with designed controller based on the backstepping technique and the graph theory. Simulation results show that the controller exhibits smooth and continuous effects under arbitrary initial conditions and the tracking stage.

The future work include consider the delay between neighbors and apply the controller to actuator malfunction. In addition, the controllers in this paper are full state measurements and it is essential to obtain proper controller using partial states. So it is worthwhile to obtain a control law without the requirement of angular velocity and linear velocity measurements. It is also meaning to extend to other classes of under-actuated systems.

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