A Retail Category Inventory Management Model Integrating Entropic Order Quantity and Trade Credit Financing

P.K. Tripathy msccompsc@gmail.com
P.G. Dept. of Statistics, Utkal University,
Bhubaneswar-751004, India.

S. Pradhan suchipradhan@yahoo.com
Dept. of Mathematics, Orissa Engineering College,
Bhubaneswar-751007, India.

Abstract

A retail category inventory management model that considers the interplay of entropic product assortment and trade credit financing is presented. The proposed model takes into consideration of key factors like discounted-cash-flow. We establish a stylized model to determine the optimal strategy for an integrated supplier-retailer inventory system under the condition of trade credit financing and system entropy. This paper applies the concept of entropy cost estimated using the principles of thermodynamics.
The classical thermodynamics reasoning is applied to modelling such systems. The present paper postulates that the behaviour of market systems very much resembles those of physical systems. Such an analogy suggests that improvements to market systems might be achievable by applying the first and second laws of thermodynamics to reduce system entropy(disorder).
This paper synergises the above process of entropic order quantity and trade credit financing in an increasing competitive market where disorder and trade credit have become the prevailing characteristics of modern market system. Mathematical models are developed and numerical examples illustrating the solution procedure are provided.

Key words: Discounted Cash-Flow, Trade Credit, Entropy Cost.

1. INTRODUCTION

In the classical inventory economic order quantity (EOQ) model, it was tacitly assumed that the customer must pay for the items as soon as the items are received. However, in practice or when the economy turns sour, the supplier allows credit for some fixed time period in settling the payment for the product and does not charge any interest from the customer on the amount owned during this period.Goyal(1985)developed an EOQ model under the conditions of permissible delay in payments.Aggarwal and Jaggi(1995) extended Goyal's(1985) model to consider the deteriorating items .Chung(1999) presented an EOQ model by considering trade credit with DCF approach.Chang(2004) considered the inventory model having deterioration under inflation when supplier credits linked to order quantity. Jaber et al.(2008) established an entropic order quantity (EnOQ) model for deteriorating items by applying the laws of thermodynamics.Chung and Liao(2009) investigated an EOQ model by using a discounted-cash-flows(DCF) approach and trade credit depending on the quantity ordered.

The specific purpose of this paper is to trace the development of entropy related thought from its thermodynamic origins through its organizational and economic application to its relationship to discounted-cash-flow approach.

2. Model Development

2.1 Basis of the Model:

The classical economic order quantity (EOQ) or lot sizing model chooses a batch size that minimises the total cost calculated as the sum of two conflicting cost functions, the order/setup cost and the inventory holding costs. The entropic order quantity (EnOQ) is derived by determining a batch size that minimizes the sum of the above two cost and entropic cost.

Notations

T	:	the inventory cycle time, which is a decision variable;
C,A	:	the purchase cost and ordering cost respectively
h	:	the unit holding cost per year excluding interest change;
D	:	the demand rate per unit time.
D(T)	:	the demand rate per unit time where cycle length is T.
r	:	Discount rate (opportunity cost) per time unit.
Q	:	Procurement quantity;
M	:	the credit period;
W	:	quantity at which the delay payment is permitted;
σ(t)	:	total entropy generated by time t.
S	:	rate of change of entropy generated at time t.
E(t)	:	Entropy cost per cycle;
PV1(T)	:	Present value of cash-out-flows for the basic EnOQ model;
PV2(T)	:	present value of cash-out-flows for credit only on units in stock when $T \leq M$.
PV3(T)	:	Present value of cash-out-flows for credit only on units in stock when $T \geq M$.
$P(t), P_0)t)$	:	unit price and market equilibrium price at time t respectively.
$\mathrm{PV}_{\infty}(\mathrm{T})$	:	the present value of all future cash-flows.
T^*	:	the optimal cycle time of $\mathrm{PV}_{\infty}(\mathrm{T})$ when T>0.

ASSUMPTIONS

(1) The demand is constant.
(2) The ordering lead time is zero.
(3) Shortages are not allowed.
(4) Time period is infinite.
(5) If Q<W, the delay in payment is not permitted, otherwise, certain fixed trade credit period M is permitted. That is, Q<W holds if and only if T<W/D.
(6) During the credit period, the firm makes payment to the supplier immediately after use of the materials. On the last day of the credit period, the firms pays the remaining balance.

2.2 Commodity flow and the entropy cost:

The commodity flow or demand/unit time is of the form

D= -k(P(t)-P0(t))(1)

The concept represented by equation (1) is analogous to energy flow (heat or work) between a thermodynamics system and its environment where k (analogous to a thermal capacity) represents the change in the flow for the change in the price of a commodity and is measured in additional units sold per year per change in unit price e.g. units/year/$.

Let P(t) be the unit price at time t and P0(t) the market equilibrium price at time t, where P(t)<P0(t) for every $t \in [0,T]$. At constant demand rate P(t)=P and P0(t)=P0 noting that when P<P0 , the direction of the commodity flow is from the system to the surroundings. The entropy generation rate must satisfy

$$S = \frac{d\sigma(t)}{dt} = k\left(\frac{P}{P_0} + \frac{P_0}{P} - 2\right)$$

To illustrate, assume that the price of a commodity decreases according to the following relationship

$P(t) = P(0) - \frac{a}{T}t$ where, $a = P(0) - P(T)$ as linear form of price being time dependent.

$E(t) = \frac{D}{\sigma(T)} = \frac{TP_0^2}{a}\ln\left[1 - \frac{aT}{TP(0) - TP_0}\right] - P_0T =$ Entropy cost per cycle.

Case-1:Instantaneous cash-flows (the case of the basic EnOQ model)

The components of total inventory cost of the system per cycle time are as follows:

(a) Ordering cost = $A + Ae^{-rT} + Ae^{-2rT} + ... = \frac{A}{1-e^{-rT}}$

(b) Present value of the purchase cost can be shown as

$$CT\int Ddt + CT\int De^{-rt}dt + CT\int De^{-2rt}dt + ... = \frac{CT^2k}{1-e^{-rT}}\left[\frac{a}{2} + P_0 - p(0)\right]$$

(c) The present value of the out-of-pocket inventory carrying cost can be shown as

$$hc\left[\int_0^T D(T-t)e^{-rt}dt + \int_0^T D(T-t)e^{-r(T+t)}dt + \int_0^T D(T-t)e^{-r(2T+r)}dt + ...\right]$$

$$= \frac{hck}{r^2}\left(P(0) - P_0 - \frac{2a}{rT}\right) + \frac{hcka\left(1+e^{-rT}\right)}{r^2\left(1+e^{-rT}\right)} + \frac{hc}{1+e^{-rT}}\left(\frac{kT}{r}\left(p_0 - P(0)\right)\right)$$

So, the present value of all future cash-flow in this case is

$$PV_1(T) = \frac{A}{1-e^{-rT}} + \frac{cT^2k}{1-e^{-rT}}\left(\frac{a}{2} + p_0 - P(0)\right) + \frac{hck}{r^2}\left(P(0) - P_0 - \frac{2a}{rT}\right)$$

$$+ \frac{hcka\left(1+e^{-rT}\right)}{r^2\left(1-e^{-rT}\right)} + \frac{hc}{1-e^{-rT}}\left(\frac{kT}{r}\left(p_0 - P(0)\right)\right) + \frac{TP_0^2}{a}\ln\left(1 - \frac{a}{p(0) - P_0}\right) - P_0T \quad (1)$$

Case -2:Credit only on units in stock when $T \le M$.

During the credit period M, the firm makes payment to the supplier immediately after the use of the stock. On the last day of the credit period, the firm pays the remaining balance. Furthermore, the credit period is greater than the inventory cycle length. The present value of the purchase cost can be shown as

$$C\left[\int_0^T De^{-rT}dt + \int_0^T De^{-r(T+t)}dt + \int_0^T De^{-r(2T+t)}dt + ...\right]$$

$$= \frac{C}{1-e^{-rT}}\left\{\frac{K(p(0) - P_0)}{r}\left(1-e^{-rT}\right) + \frac{Ka}{T}\left(\frac{1}{r^2}\left(1-e^{-rT}\right) - \frac{Te^{-rT}}{r}\right)\right\}$$

$$= \frac{-ck\left(P(0) - P_0\right)}{r} + \frac{cka}{Rr^2} - \frac{CTe^{-rT}ka}{rt\left(1-e^{-rT}\right)}$$

The present value of all future cash flows in this case is

$$PV_2(T) = \frac{A}{1-e^{-rT}} - \frac{ck\left(P(0) - P_0\right)}{r} + \frac{cka}{Tr^2} - \frac{ckae^{-rT}}{r\left(1-e^{-rT}\right)} + \frac{hck}{r^2}\left[P(0) - P_0 - \frac{2a}{rT}\right]$$

$$+ \frac{hcka\left(1+e^{-rT}\right)}{r^2\left(1-e^{-rT}\right)} + \frac{hc}{1-e^{-rT}}\left(\frac{kT}{r}\left(P_0 - P(0)\right)\right) + \frac{TP_0^2}{a}\ln\left[1 - \frac{aT}{T(P(0) - P_0T}\right] \quad (2)$$

Case-3: Credit only on units in stock when $T \ge M$

The present value only on units in stock can be shown as

$$C\left[\int_0^M De^{-rt}dt + D(T-M)e^{-rM} + \int_0^M De^{-r(T+t)}dt + D(T-M)e^{-r(M+T)} + \ldots\right]$$
$$= \frac{\left(e^{-rM}-1\right)}{r^2T\left(1-e^{-rT}\right)}\left[rTc\left(kP(0)-P_0k\right)\right]$$
$$-\frac{1}{rT\left(1-e^{-rT}\right)}\left[caMe^{-rM} + rkc(T-M)e^{-rM}\left(TP(0)-at-P_0T\right)\right]$$
.

Therefore the present value of all future cash-flows in this case is

$$PV_3(T) = \frac{A}{1-e^{-rT}} + \frac{hck}{r^2}\left(P(0)-P_0-\frac{2a}{rT}\right) + \frac{hcka\left(1+e^{-rT}\right)}{r^2\left(1-e^{-rT}\right)} + \frac{hc}{1-e^{-rT}}\left(\frac{kT}{r}\left(P_0-P(0)\right)\right)$$
$$+\frac{\left(e^{-rM}-1\right)}{r^2T\left(1-e^{-rT}\right)}\left[rTc\left(kP(0)-P_0k-cak\right)\right]$$
$$-\frac{1}{rT\left(1-e^{-rT}\right)}\left[caMe^{-rM} + rkc(T-M)e^{-rM}\left(TP(0)-at-P_0T\right)\right]$$
$$+\frac{TP_0^2}{a}\ln\left(1-\frac{aT}{p(0)-P_0}\right) - P_0T \qquad (3)$$

Now our main aim is to minimize the present value of all future cash-flow cost $PV_\infty(T)$. That is

Minimize $PV_\infty(T)$

subject to T>0.

We will discuss the situations of the two cases,

(A) Suppose M>W/D

In this case we have

$$PV_\infty(T) = \begin{cases} PV_1(T) & if \quad 0<T<W/D \\ PV_2(T) & if \quad W/D \le T < M \\ PV_3(T) & if \quad M \le T. \end{cases}$$

It was found that

$PV_1(T) - PV_2(T) > 0$ and $PV_1(T) - PV_3(T) > 0$

for T>0 and $T \ge M$ respectively.

which implies

$PV_1(T) > PV_2(T)$ and $PV_1(T) > PV_3(T)$

for T>0 and $T \ge M$ respectively.

Now we shall determine the optimal replenishment cycle time that minimizes present value of cash-out-flows. The first order necessary condition for $PV_1(T)$ in (1) to be minimized is expressed as $\frac{\partial PV_1(T)}{\partial T} = 0$ which implies

$$-\frac{re^{-rT}A}{\left(1-e^{-rT}\right)^2} + \frac{2CT\left(1-e^{-rT}\right) - re^{-rT}CT^2}{\left(1-e^{-rT}\right)^2}\left(\frac{ak}{2}+P_0k-kP(0)\right)$$
$$+\frac{hcka}{r^2}\left[\frac{-re^{-rT}\left(1-e^{-rT}\right)-\left(1+e^{-rT}\right)re^{-rT}}{\left(1-e^{-rT}\right)^2}\right]$$

$$+\frac{hck}{r}\left(P_0 - P(0)\right)\left[\frac{\left(1-e^{-rT}\right)-re^{-rT}T}{\left(1-e^{-rT}\right)^2}\right]+\frac{2hcka}{r^3T^2}+\frac{P_0^2}{a}\ln\left[1-\frac{a}{P(0)-P_0}\right]-p_0=0 \quad (4)$$

Similarly, $PV_2(T)$ in equation(2) to be minimized is $\frac{\partial PV_2(T)}{\partial T}=0$ which implies

$$-\frac{re^{-rT}A}{\left(1-e^{-rT}\right)^2}-\frac{cka}{r}\left[\frac{-re^{-rT}\left(1-e^{-rT}\right)-e^{-2rT}r}{\left(1-e^{-rT}\right)^2}+\frac{hcka}{r^2}\left[\frac{-re^{-rT}\left(1-e^{-rT}\right)-\left(1+e^{-rT}\right)re^{-rT}}{\left(1-e^{-rT}\right)^2}\right]\right]$$

$$+\frac{hck\left(P_0-P(0)\right)}{r}\left[\frac{\left(1-e^{-rT}\right)-rT\left(re^{-rT}\right)}{\left(1-e^{-rT}\right)}\right]+\frac{cka}{r^2T^2}+\frac{2hcka}{r^3T^2}+\frac{P_0^2}{a}\ln\left[1-\frac{a}{P(0)-P_0}\right]-p_0=0 \quad (5)$$

Likewise, the first order necessary condition for $PV_3(T)$ in equation(3) to be minimized is $\frac{\partial PV_3(T)}{\partial T}=0$ which leads

$$-\frac{re^{-rT}A}{\left(1-e^{-rT}\right)^2}-\frac{hcka\left(-re^{-rT}\left(1-e^{-rT}\right)-\left(1+e^{-rT}\right)re^{-rT}\right)}{r^2\left(1-e^{-rT}\right)^2}$$

$$+\frac{hcrk\left(P_0-P(0)\right)\left(1-e^{-rT}-rTe^{-rT}\right)}{r^2\left(1-e^{-rT}\right)}+rc\left(e^{-rM}-1\right)\left(KP(0)-P_0k-ca\right)$$

$$\frac{P_0^2}{a}\ln\left[1-\frac{a}{P(0)-P_0}\right]-p_0+\frac{2hcka}{r^3T^2}=0 \quad (6)$$

Furthermore, we let

$$\Delta_1=\left.\frac{\partial PV_1(T)}{\partial T}\right|_{T=W/D} \quad (7)$$

$$\Delta_2=\left.\frac{\partial PV_2(T)}{\partial T}\right|_{T=W/D} \quad (8)$$

$$\Delta_3=\left.\frac{\partial PV_3(T)}{\partial T}\right|_{T=M} \quad (9)$$

Lemma-1

(a) If $\Delta_1 \geq 0$, then the total present value of PV1(T) has the unique minimum value at the point T=T1 where $T_1 \in (0, W/D)$ and satisfies $\frac{\partial PV_1}{\partial T}=0$.

(b) If $\Delta_1 < 0$, then the value of $T_1 \in (0, W/D)$ which minimizes PV1(T) does not exist.

Proof:

Now taking the second derivative of PV1(T) with respect to $T_1 \in (0, W/D)$, we have

$$\frac{r^2e^{-rT}\left(1-e^{-rT}\right)^2+2r^2e^{-rT}\left(1-e^{-rT}\right)}{\left(1-e^{-rT}\right)^4}$$

$$+\left\{\frac{2c\left(1-e^{-rT}\right)-4rcT}{\left(1-e^{-rT}\right)^{2}}-\frac{r^{2}c^{2}T^{2}e^{-rT}\left(1+e^{-rT}\right)}{\left(1-e^{-rT}\right)^{3}}\right\}\left(\frac{a}{2}+P_{0}k-kP(0)\right)+\frac{hcka}{r^{2}}\left(\frac{2e^{-rT}}{\left(1-e^{-rT}\right)^{2}}\right)$$

$$+\frac{2hckae^{-2rT}}{r\left(1-e^{-rT}\right)^{2}}+\frac{4hcka}{r^{3}T^{3}}+\frac{hck(P_{0}-P(0))}{\left(1-e^{-rT}\right)^{2}}\left[e^{-rT}(1+rT)-2r\right]+\frac{2r^{2}hck(P_{0}-P(0))T.e^{-rT}}{\left(1-e^{-rT}\right)^{3}}$$

We obtain from the above expression $\frac{\partial^{2}PV_{1}(T)}{\partial T^{2}}>0$, which implies $\frac{\partial PV_{1}(T)}{\partial T}>0$ is strictly increasing function of T in the interval (0,W/D).

Also we know that,

$\lim_{T\to 0}\left(\frac{\partial PV_{1}(T)}{\partial T}\right)=-rA<0$ and $\lim_{T\to W/D}\left(\frac{\partial PV_{1}(T)}{\partial T}\right)=\Delta_{1}$

Therefore, if $\lim_{T\to W/D}\left(\frac{\partial PV_{1}(T)}{\partial T}\right)=\Delta_{1}\geq 0$, then by applying the intermediate value theorem, there exists a unique value $T_{1}\in(0,W/D)$ such that $\frac{\partial PV_{1}(T)}{\partial T}=0$ and $\frac{\partial^{2}PV_{1}(T)}{\partial T^{2}}$ at the point T1 is greater than zero.

Thus $T_{1}\in(0,W/D)$ is the unique minimum solution to $PV_{1}(T)$.

However, if $\lim_{T\to W/D}\frac{\partial PV_{1}(T)}{\partial T}=\Delta_{1}<0$ for all $T\in\left(0,W/D\right)$ and also we find $\frac{\partial^{2}PV_{1}(T)}{\partial T^{2}}<0$ for all $T\in\left(0,W/D\right)$. Thus $PV_{1}(T)$ is a strictly decreasing function of T in the interval $\left(0,W/D\right)$. Therefore, we can not find a value of T in the open interval $\left(0,W/D\right)$ that minimizes $PV_{1}(T)$. This completes the proof.

Lemma-2

(a) If $\Delta_{2}\leq 0\leq\Delta_{1}$, then the total present value of PV2(T) has the unique minimum value at the point T=T2 where $T_{2}\in(W/D,M)$ and satisfies $\frac{\partial PV_{2}}{\partial T}=0$.

(b) If $\Delta_{2}>0$, then the present value PV2(T) has a minimum value at the lower boundary point $T=W/D$.

(c) If $\Delta_{2}<0$, then the present value PV2(T) has a minimum value at the upper boundary point T=M.

Proof:

$$\frac{\partial^{2}PV_{2}(T)}{\partial T^{2}}=\frac{r^{2}e^{-rT}\left(1-e^{-rT}\right)^{2}+2r^{2}e^{-rT}\left(1-e^{-rT}\right)}{\left(1-e^{-rT}\right)^{4}}$$

$$-\frac{ckare^{-rT}\left(1+e^{-rT}\right)}{\left(1-e^{-rT}\right)^3}+\frac{hckae^{-2rT}}{r\left(1+e^{-rT}\right)^4}+\frac{hck(P_0-P(0))}{\left(1+e^{-rT}\right)^2}\left\{e^{-rT}\left(1+rT\right)-2r\right\}$$
$$+\frac{2hr^2ck((P_0-P(0))Te^{-rT}}{\left(1-e^{-rT}\right)^3}+\frac{2cka}{r^2T^3}-\frac{4hcka}{r^3T^3}$$

which is >0 where $T\in\left[W/D,M\right]$.

Which implies $\frac{\partial PV_2}{\partial T}$ is strictly increasing function of T in the interval $(W/D,M)$.

Also we have $\left.\frac{\partial PV_2(T)}{\partial T}\right|_{T=W/D}=\Delta_2$.

If $\Delta_2\le 0$, then by applying the intermediate value theorem there exists a unique value $T_2\in\left[W/D,M\right]$ so that $\left.\frac{\partial PV_2(T)}{\partial T}\right|_{T=T_2}=0$. Moreover, by taking the second derivative of PV2(T) with respect to T at the point T2 we have $\frac{\partial^2 PV_2}{\partial T^2}>0$.

Thus $T_2\in\left[W/D,M\right]$ is the unique solution to $PV_2(T)$.

Now if $\left.\frac{\partial PV_2(T)}{\partial T}\right|_{T=W/D}=\Delta_2>0$ which implies $\frac{\partial PV_2}{\partial T}>0$ for all $T_2\in\left(W/D,M\right)$. Therefore, $PV_2(T)$ is strictly increasing function of T in the interval $\left(W/D,M\right)$. Therefore $PV_2(T)$ has a minimum value at the lower boundary point $T=W/D$ and similarly we can prove $PV_2(T)$ has a minimum value at upper boundary point T=M.

Lemma-3

(a) If $\Delta_3\le 0$, then the total present value of $PV_3(T)$ has the unique minimum value at the point T=T3 on $T_{31}\in(M,\infty)$ and satisfies $\frac{\partial PV_3}{\partial T}=0$.

(b) If $\Delta_3>0$, then the present value of $PV_3(T)$ has a minimum value at the boundary T=M.

Proof:

The proof is same to the lemma-2.

Proposition 1

(i) $2e^{rt}-2-rT>0$

(ii) $e^{2rT}+1-3e^{rT}>0$

Proof:

(i) $$2e^{-rT}-2-rT=2\left(e^{rT}-1\right)-rT$$
$$=2\left(1+rt+\frac{(rT)^2}{2!}+.....+1\right)-rT$$
$$=rt+2\left(\frac{(rT)^2}{2!}+\frac{(rT)^3}{3!}+.....\right)$$
which is always +ve as value of r and T are always positive.

(ii) $e^{2rT}+1-3e^{rT}$

$$=1+2rT+\frac{(2rT)^2}{2!}+\frac{(3rT)^3}{3!}+.....+1-3\left(1+rT+\frac{(rT)^2}{2!}+\frac{(rT)^3}{3!}+...\right)$$

$$=-1-rT+\frac{(rT)^2}{2}+\frac{6(rT)^3}{6}+....$$

$$=-1-rT+\frac{(rT)^2}{2}+(rT)^3+10.5(rT)^4$$

which is also give a positive value a r and T are always positive.

By this two position it is easy to say that $\Delta_1 > \Delta_2$.

Then the equations (7) – (9) yield

$\Delta_1 < 0$ iff $PV_1^{/}(W/D) < 0$ iff $T_1^* > W/D$ (10)

$\Delta_2 < 0$ iff $PV_2^{/}(W/D) < 0$ iff $T_2^* > W/D$ (11)

$\Delta_3 < 0$ iff $PV_2^{/}(M) < 0$ iff $T_2^* > M$ (12)

$\Delta_3 < 0$ iff $PV_3^{/}(M) < 0$ iff $T_3^* > M$ (13)

From the above equations we have the following results.

Theorem-1

(1) If $\Delta_1 > 0, \Delta_2 \geq 0$ and $\Delta_3 > 0$, then $PV_\infty(T^*) = \min\{PV_\infty(T_1^*), PV_\infty(W/D)\}$. Hence T^* is T_1^* or W/D associated with the least cost.

(2) If $\Delta_1 > 0, \Delta_2 < 0$ and $\Delta_3 > 0$, then $PV_\infty(T^*) = PV_\infty(T_2^*)$. Hence T^* is T_2^*.

(3) If $\Delta_1 > 0, \Delta_2 < 0$ and $\Delta_3 \leq 0$, then $PV_\infty(T^*) = PV_\infty(T_3^*)$. Hence T^* is T_3^*.

(4) If $\Delta_1 \leq 0, \Delta_2 < 0$ and $\Delta_3 > 0$, then $PV_\infty(T^*) = PV_\infty(T_2^*)$. Hence T^* is T_2^*.

(5) If $\Delta_1 \leq 0, \Delta_2 < 0$ and $\Delta_3 \leq 0$, then $PV_\infty(T^*) = PV_\infty(T_3^*)$. Hence T^* is T_3^*.

Proof:

(1) If $\Delta_1 > 0, \Delta_2 \geq 0$ and $\Delta_3 > 0$, which imply that $PV_1^{/}(W/D) > 0$, $PV_2^{/}(W/D) \geq 0$, $PV_2^{/}(M) > 0$ and $PV_3^{/}(M) > 0$. From the above lemma we implies that

(i) $PV_3(T)$ is increasing on $[M, \infty)$

(ii) $PV_2(T)$ is increasing on $[W/D, M)$

(iii) $PV_1(T)$ is increasing on $[T_1^*, W/D)$ and decreasing on $(0, T_1^*]$

Combining above three, we conclude that $PV_\infty(T)$ has the minimum value at $T = T_1^*$ on $(0, W/D)$ and $PV_\infty(T)$ has the minimum value at $T = W/D$. Hence, $PV_\infty(T^*) = \min\{PV_\infty(T_1^*), PV_\infty(W/D)\}$. Consequently, T^* is T_1^* or W/D associated with the least cost.

(2) If $\Delta_1 > 0, \Delta_2 < 0$ and $\Delta_3 > 0$, which imply that $PV_1^{/}(W/D) > 0$, $PV_2^{/}(W/D) < 0$, $PV_2^{/}(M) > 0$ and $PV_3^{/}(M) > 0$ which implies that $T_1^* \leq W/D$, $T_2^* > W/D$, $T_2^* < M$ and $T_3^* < M$ respectively. Furthermore from the lemma

(i) $PV_3(T)$ is increasing on $[M, \infty)$

(ii) $PV_2(T)$ is decreasing on $[W/D, T_2^*]$ and increasing on $(T_2^*, M]$

(iii) $PV_1(T)$ is decreasing on $(0, T_1^*]$ and increasing on $(T_1^*, W/D]$.

From the above we conclude that $PV_\infty(T)$ has the minimum value at $T = T_1^*$ on $(0, W/D)$ and $PV_\infty(T)$ has the minimum value at $T = T_2^*$ on $[W/D, \infty)$. Since $PV_1(T)$ > $PV_2(T)$ and T>0. Then $PV_\infty(T^*) = PV_\infty(T_2^*)$ and T^* is T_2^*

(3) If $\Delta_1 > 0, \Delta_2 < 0$ and $\Delta_3 \le 0$, which implies that $PV_1^{/}(W/D) > 0$, $PV_2^{/}(W/D) < 0$, $PV_2^{/}(M) \le 0$ and $PV_3^{/}(M) \le 0$ which imply that $T_1^* < W/D$, $T_2^* > W/D$, $T_2^* \ge M$ and $T_3^* > M$ respectively. Furthermore, from the lemma it implies that

(i) $PV_3(T)$ is decreasing on (M, T_3^*) and increasing on (T_3^*, ∞)

(ii) $PV_2(T)$ is decreasing on $(W/D, M)$

(iii) $PV_1(T)$ is decreasing on $(0, T_1^*)$ and increasing on $(T_1^*, W/D]$.

Combining all above, we conclude that $PV_\infty(T)$ has the minimum value at $T = T_1^*$ on $(0, W/D)$ and $PV_\infty(T)$ has the minimum value at $T = T_3^*$ on $[W/D, \infty)$. Since, $PV_2(T)$ is decreasing on $(0, T_2^*)$, $T_1^* < W/D$ and $T_2^* \ge M > W/D$ we have $PV_1(T_1^*) > PV_2(T_2^*)$, $PV_2(T_1^*) > PV_2(M)$ and $PV_3(M) > PV_3(T_3^*)$. Hence we conclude that $PV_\infty(T)$ has the minimum value at $T = T_3^*$ on $(0, \infty)$. Consequently, T^* is T_3^*.

(4) If $\Delta_1 \le 0, \Delta_2 < 0$ and $\Delta_3 > 0$, which implies that $PV_1^{/}(W/D) \le 0$, $PV_1^{/}(W/D) \le 0$, $PV_2^{/}(M) > 0$ and $PV_3^{/}(M) > 0$ which imply that $T_1^* \ge W/D$, $T_2^* > W/D$, $T_2^* < M$ and $T_3^* < M$. Furthermore, we have

(i) $PV_3(T)$ is increasing on $[M, \infty)$

(ii) $PV_2(T)$ is decreasing on $[W/D, T_2^*]$ and increasing on $(T_2^*, M]$

(iii) $PV_1(T)$ is decreasing on $(0, W/D)$.

Since $PV_1(W/D) > PV_2(W/D)$, and $PV_2(W/D) > PV_2(T^*)$

So we conclude that $PV_\infty(T)$ has the minimum value at $T = T_2^*$ on $(0, \infty)$. Consequently, T^* is T_2^*.

(5) If $\Delta_1 \le 0, \Delta_2 < 0$ and $\Delta_3 \le 0$, which gives that $PV_1^{/}(W/D) \le 0$, $PV_2^{/}(W/D) < 0$, $PV_2^{/}(M) \le 0$ and $PV_3^{/}(M) \le 0$ and which imply that $T_1^* \ge W/D$, $T_2^* > W/D$, $T_2^* \ge M$ and $T_3^* \ge M$ respectively. Furthermore, from the lemma it implies that

(i) $PV_3(T)$ is decreasing on (M, T_3^*) and increasing on $[T_3^*, \infty)$

(ii) $PV_2(T)$ is decreasing on $[W/D, M]$

(iii) $PV_1(T)$ is decreasing on $(0, W/D)$.

Since $PV_1(W/D) > PV_2(W/D)$, combining the above we conclude that $PV_\infty(T)$ has the minimum value at $T = T_3^*$ on $(0,\infty)$. Consequently, T^* is T_3^*. This completes the proof.

(B) Suppose $M \le W/D$.

Here $PV_\infty(T)$ can be expressed as follows:

$$PV_\infty(T) = \begin{cases} PV_1(T) & if \quad 0 < T < W/D \\ PV_2(T) & if \quad W/D \le T \end{cases}$$

and let $\left.\dfrac{\partial P_3^{/}(T)}{\partial T}\right|_{T=W/D} = \Delta_4$ (14)

By using proposition 1, we have

$\Delta_1 - \Delta_4 > 0$, which leads to $\Delta_1 > \Delta_4$.

From (14), we also find that

$\Delta_4 < 0$ iff $PV_3^{/}(W/D) < 0$ iff $T_3^* > W/D$ (15)

Lemma-4

(a) If $\Delta_4 \le 0$, then the present value of $PV_3(T)$ possesses the unique minimum value at the point $T = T_3$, where $T_3 \in [W/D, \infty)$ and satisfies $\dfrac{\partial P_3\,(T)}{\partial T} = 0$.

(b) If $\Delta_4 > 0$, then the present value of $PV_3(T)$ possesses a minimum value at the boundary point $T = W/D$.

Proof: The proof is similar to that of Lemma-2.

Theorem-2

(1) If $\Delta_1 > 0$ and $\Delta_4 \ge 0$, then $PV_\infty(T^*) = \min\{PV_\infty(T_1^*), PV_\infty(W/D)\}$. Hence T^* is T_1^* or W/D is associated with the least cost.

(2) If $\Delta_1 > 0$ and $\Delta_4 < 0$, then $PV_\infty(T^*) = \min\{PV_\infty(T_1^*), PV_\infty(T_3^*)\}$. Hence T^* is T_1^* or T_3^* is associated with the least cost.

(3) If $\Delta_1 \le 0$ and $\Delta_4 < 0$, then $PV_\infty(T^*) = PV_\infty(T_3^*)$. Hence T^* is T_3^*.

Proof:

(1) If $\Delta_1 > 0$ and $\Delta_4 > 0$, which imply that $PV_1^{/}(W/D) > 0$ and $PV_3^{/}(W/D) \ge 0$, and also $T_1^* < W/D$ and $T_3^* \le W/D$. Furthermore, we have

(i) $PV_3(T)$ is increasing on $[W/D, \infty)$

(ii) $PV_1(T)$ is decreasing on $(0, T_1^*]$ and increasing on $[T_1^* < W/D)$. Combining the above we conclude that $PV_\infty(T)$ has the minimum value at $T = T_1^*$ on $(0, W/D)$ and $PV_\infty(T)$ has the minimum value at $T = W/D$ on $[W/D, \infty)$. Hence, $PV_\infty(T^*) = \min\{PV_\infty(T_1^*), PV_\infty(W/D)\}$. Consequently, T^* is T_1^* or W/D associated with the least cost.

(2) If $\Delta_1 > 0$ and $\Delta_4 < 0$, which imply that $PV_1^{'}(W/D) > 0$ and $PV_3^{/}(M) < 0$ which implies that $T_1^* < W/D$ and $T_3^* > W/D$ and also

(i) $PV_3(T)$ is decreasing on $[W/D, T_3^*]$ and increasing on $[T_3^*, \infty)$

(ii) $PV_1(T)$ is decreasing on $(0, T_1^*]$ and increasing on $(T_1^*, W/D]$. Combining (i) and (ii) we conclude that $PV_\infty(T)$ has the minimum value at $T = T_1^*$ on $(0, W/D)$ and $PV_\infty(T)$ has the minimum value at $T = T_3^*$ on $[W/D, \infty)$. Hence, $PV_\infty(T^*) = \min\left\{PV_\infty(T_1^*), PV_\infty(T_3^*)\right\}$. Consequently, T^* is T_1^* or T_3^* associated with the least cost.

(3) If $\Delta_1 \le 0$ and $\Delta_4 \le 0$, which implies that $PV_1^{/}(W/D) \le 0$ and $PV_3^{/}(M) < 0$ and $T_1^* \ge W/D$ and $T_3^* > W/D$ and also

(i) $PV_3(T)$ is decreasing on. $[W/D, T_3^*]$ and increasing on $[T_3^*, \infty)$.

(ii) $PV_1(T)$ is decreasing on $(0, W/D)$

From which we conclude that $PV_\infty(T)$ is decreasing on $(0, W/D)$ and $PV_\infty(T)$ hs the minimum value at $T = T_3^*$ on $[W/D, \infty)$. Since, $PV_1(W/D) > PV_3(W/D)$, and conclude that $PV_\infty(T)$ has minimum value at $T = T_3^*$ on $(0, \infty)$. Consequently T^* is T_3^*.
This completes the proof.

NUMERICAL EXAMPLES
The followings are considered to be its base parameters A=$5/order, r=0.3/$, C=$1. a=1, k=2.4, P0(Market Price)=$3, Price at the beginning of a cycle P(0)=$1, D=-k(P(0)-P0)=-2.4(1-3)=4.8

Example-1
If M=2, W=2, W/D<M
Δ_1=31.47>0, Δ_2=-2.5020<0, Δ_3=-1.520559<0
T^*=T3=2.55, $PV_3(T)$=36.910259

Example-2
If M=2, W=3, W/D<M
Δ_1=48.416874>0, Δ_2=-0.112<0, Δ_3=-1.52<0
T^*=T3=2.55, $PV_3(T)$=36.910259

Example-3
If M=5, W=3, W/D<M
Δ_1=48.51>0, Δ_2=-0.112<0, Δ_3=2.1715>0
T^*=T2=3.1, $PV_2(T)$=36.302795

Example-4
If M=20, W=6, W/D<M
Δ_1=83.5186>0, Δ_2=1.47>0, Δ_3=4.001>0
T^*=T_1^* =0.7442, $PV_1(T)$=48.134529

Example-5
If M=5, W=2, W/D<M
Δ_1=36.5465>0, Δ_2=-2.5<0, Δ_3=2.738>0

$T^* = T_2^*$ =3.1, $PV_2(T)$ =36.302795

Example-6

If M=2, W=1, W/D<M

Δ_1=-1.546<0, Δ_2=-15.079<0, Δ_3=-1.52<0

$T^* = T_3^*$ =2.55, $PV_3(T)$ =36.910259

Example-7

If M=10, W=1, W/D<M

Δ_1=-1.546<0, Δ_4=-15.079<0, Δ_3=3.728554>0

$T^* = T_2^*$ =3.1, $PV_2(T)$ =36.302795

Example-8

If M=1, W=5, W/D<M

Δ_1=164..83>0, Δ_4=2.946>0

$T^* = T_1$ =0.7442, $PV_1(T)$ =48.134529

Example-9

If M=10, W=5, W/D<M

Δ_1=83.518661>0, Δ_4=-1.837413<0

$T^* = T_3^*$ =2.66, $PV_3(T)$ =34.98682

Based on the above computational result of the numerical examples, the following managerial insights are obtained and following comparative evaluation are observed. If the supplier does not allow the delay payment, cash-out-flow is more but practically taking in view of real-world market, to attract the retailer(customer)credit period should be given and it observed that it should be less than equal to the inventory cycle to achieve the better goal. Furthermore, it is preferable for the supplier to opt a credit period which is marginally small.

CONCLUSION AND FUTURE RESEARCH

This paper suggested that it might be possible to improve the performance of a market system by applying the laws of thermodynamics to reduce system entropy (or disorder). It postulates that the behaviours of market systems very much resembles that of physical system operating within surroundings, which include the market and supply system.

In this paper, the suggested demand is price dependent. Many researchers advocated that the proper estimation of input parameters in EOQ models which is essential to produce reliable results. However, some of those costs may be difficult to quantify. To address such a problem, we propose in this paper accounting for an additional cost (entropy cost) when analysing EOQ systems which allow a permissible delay payments if the retail orders more than or equal to a predetermined quantity. The results from this paper suggest that the optimal cycle time is more sensitive to the change in the quantity at which the fully delay payment is permitted.

An immediate extension is to investigate the proposed model to determine a retailer optimal cycle time and the optimal payment policy when the supplier offers partially or fully permissible delay in payment linking to payment time instead of order quantity.

ACKNOWLEDGEMENTS

The authors sincerely acknowledge the anonymous referees for their constructive suggestion.

REFERENCES

(1) Goyal, S.K., 1985. Economic order quantity under conditions of permissible delay in payments. Journal of the Operational Research Society, 36, 335-338.

(2) Khouja, M., Mehrez, A., 1996. Optimal Inventory policy under different supplier credits. Journal of manufacturing Systems, 15, 334-339.

(3) Aggarwal, S.P., Jaggi, C.K., 1995. Ordering policies of deteriorating items under permissible delay in payments. Journal of the Operational Research Society, 46, 658-662.

(4) Chung, K.H., 1989. Inventory Control and trade credit revisited. Journal of the Operational Research Society, 40, 495-498.

(5) Jaber, M.Y., Borney, M., Rosen, M.A., Moualck, I., 2008. Entropic order quantity (EnOQ) Model for deteriorating items. Journal of Applied Mathematical modelling.

(6)Chung, K.J., 1999. Bounds on the optimum order quantity in a DCF analysis of the EOQ model under trade credit. Journal of Information and Optimization Sciences, 20, 137-148.

(7) Chung, K.J., Huang, C.K., 2000. The convexities of the present value functions of all future cash out flows in inventory and credit. Production planning and control, 11, 133.140.

(8) Chung, K.J., Liao, J.J., 2009. The optimal ordering policy of the EOQ model under trade credit depending on the ordering quantity from the DCF approach. European Journal of Operational Research, 196, 563-568.

(9) Chung, K.J., Huang,Y.F.,2003.Optimalreplenishment and payment policies in the EOQ model under cash discount and trade credit. Asia-Pacific Journal of Operational Research,20,177-190.

(10)Chang,C.T.,2004.An EOQ model with deteriorating items under inflation when supplier credits linked to order quantity. International Journal of Production Economics,88,307-316.

(11)Liao,J.J.,2007. A note on an EOQ model for deteriorating items under supplier credit linked to ordering quantity. Applied Mathematical Modeling,31,1690-1699.

(12)Shah,N.H.,1977. An order-level lot size inventory model for deteriorating items. AIIE Transactions,9,108-112.

(13)Jaber,M.Y.,Nuwayhid,R.Y.,Rosen,M.A.,2006. A thermodynamic approach to modeling the economic order quantity. Applied Mathematical Modelling, 30,867-883.

(14)Jaber, M.Y.,2007. Lot sizing with permissible delay in payments and entropy cost. Computers and Industrial Engineering,52,78-88.

(15)Teng, J.T., Chang, C.T.,Goyal,S.K.,2005.Optimal pricing and ordering policy under permissible delay in payments. International Journal of production Economics,97,121-129.