Probabilistic Analysis of an Evaporator of a Desalination Plant with Priority for Repair Over Maintenance

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Abstract

The paper presents a probabilistic analysis of an evaporator of a desalination plant. Multi stage flash desalination process is being used for water purification. The desalination plant operates round the clock with seven evaporators and during normal operation; six of these evaporators will be in service while one is under maintenance and works as standby. Any major failure/annual maintenance brings the evaporator to a complete halt and stops the water production. The priority is given to repair over maintenance. For the present analysis, seven years maintenance data has been extracted from the operations and maintenance reports of the plant.Measures of the plant effectiveness have been obtained probabilistically. Semi-Markov processes and regenerative point techniques are used in the entire analysis.

Keywords: Desalination plant, Maintenance, Failures, Semi- Markov, Regenerative process.

1. NOTATIONS

U_{ms} Under Maintenance during summer

U_{mwb} Under Maintenance during winter before service

U_{mwa} Under Maintenance during winter after service

W_{ms} Waiting for Maintenance during summer

W_{mwb} Waiting for Maintenance during winter before service

W_{mwa} Waiting for Maintenance during winter after service

F_{rs} Failed unit is under repair during summer

F_{rwb}	Failed unit is under repair during winter before service
F_{rwa}	Failed unit is under repair during winter after service
β_1	Rate of summer to winter change
β_2	Rate of winter to summer change
λ	Rate of failure of any component of the unit
γ	Rate of Maintenance
γ_1	Rate of shutting down
γ_2	Rate of recovery after shut down
α	Rate ofrepair
©	Symbol for Laplace Convolution
Ⓢ	Symbol for Stieltje's convolution
*	Symbol for Laplace transforms
**	Symbol for Laplace Stieltje's transforms
$\varphi_i(t)$	c.d.f. of first passage time from a regenerative state i to a failed state j
$p_{ij}(t)$, $Q_{ij}(t)$	p.d.f. and c.d.f. of first passage time from a regenerative state i to a regenerative state j or to a failed state j in (0, t]
$g_m(t)$, $G_m(t)$	p.d.f. and c.d.f. of maintenance rate
$g_{sr}(t)$, $G_{sr}(t)$	p.d.f. and c.d.f. of recovery rate after shutdown
$g(t)$, $G(t)$	p.d.f. and c.d.f. of repair rate

2. INTRODUCTION

Desalination is a water treatment process that removes salt from sea water or brackish water. It is the only option in arid regions, since the rainfall is marginal. This can be achieved by a major process known as Multi-stage Flash distillation Process which is very expensive and involves sophisticatedsystems. Since, desalination plants are designed to fulfill the requirement of water supply for a larger sector in arid regions, they are normally kept in continuous production modeexcept for emergency/forced/planned outages. It is therefore, essential to maintain the efficiency ofthese desalination plantsusing good maintenance practices to avoid big losses.

Many researchers have analyzed systems and obtained various reliability indicesthat are useful for effective equipment/plant maintenance.G. Taneja&V. Naveen [1] studied models with patience time and chances of non-availability of expert repairman,B. Parashar&G. Taneja [2] evaluated the reliability and profit of a PLC hot standby system based on master slave concept and two types of repair facilities;Rizwan et. al. [3], [4] &[5] have analyzed aPLC system, desalination plant system and a CC plant system. Recently, Padmavathi et al. [6] explored a possibility of analyzing desalination plant with online repair and emergency shutdowns situation. In all these papers various measures of system effectiveness are obtained under different failure possibilities. The novelty of the work lies in the application of modeling methodology for reliability analysis of systems as real case studies under different failure possibilities. Interesting variation on the reliability results could be obtained for a situation of a desalination plant when the annual maintenance of the plant is planned during winter season and the plant is shut down for one month; and the priority is given to repair over maintenance.

Thus, the present paper offers a probabilistic analysis of adesalination plantwhere the annual maintenance of the plant is carried out during winter season and the plant is shut down for one month for this purpose; and the priority is given to repair over maintenance on failure of a unit. The desalination plant under discussion operates round the clock for water purification and ensures the continuous production of water for domestic usage. The plant consists of seven evaporators and at any given time; six out of seven evaporators are operative whereas one is always under maintenance and works as standby. Any major failure/annual maintenance brings the evaporator to a complete halt and the water production stops until the fault restored. Seven years maintenance data has been extracted from the operations and maintenance report of a desalination plant in Oman. A robust model embedding the real failure situations, as categorized in the data with priority of repair over maintenance, has been developed (Fig. 1). The real values of various failure rates and probabilities are being used in this analysis for achieving the reliability indicators.

Using the data, the following values are estimated:

Estimated rate of failure of any component of the unit (λ) = 0.00002714 per hour

Estimated rate of the unit moving from winter to summer (β_1) =0.0002315 per hour
Estimated rate of the unit moving from summer to winter (β_2) = 0.0002315 per hour

Estimated rate of Maintenance (γ) = 0.0014881

Estimated rate of shutting down (γ_1) = 0.0001142 per hour

Estimated rate of recovery after shut down during winter (γ_2) = 0.0069444 per hour

Estimated value of repair rate (α) = 0.001577 per hour

The system is analyzed probabilistically by using semi-Markov processes and regenerative point techniques. Various measures of system effectiveness such as mean time to system shut down, system availability, busy period analysis of repairman, busy period analysis for repair, expected busy period during shut down and the expected number of repairs are estimated numerically.

3. MODEL DESCRIPTION AND ASSUMPTIONS

- The desalination plant has seven evaporators out of which six are operative and one is under maintenance.
- If a unit is failed in one season, it gets repaired in that season only.
- Maintenance of no unit is done if the repair of some other unit is going on.
- Not more than two units fail at a time.
- During the maintenance of one unit, more than one of the other units cannot get failed.
- All failure times are assumed to have exponential distribution with failure rate (λ) whereas the repair times have general distributions.

- After each repair the unit works as good as new.
- The unit is brought into operation as soon as possible.

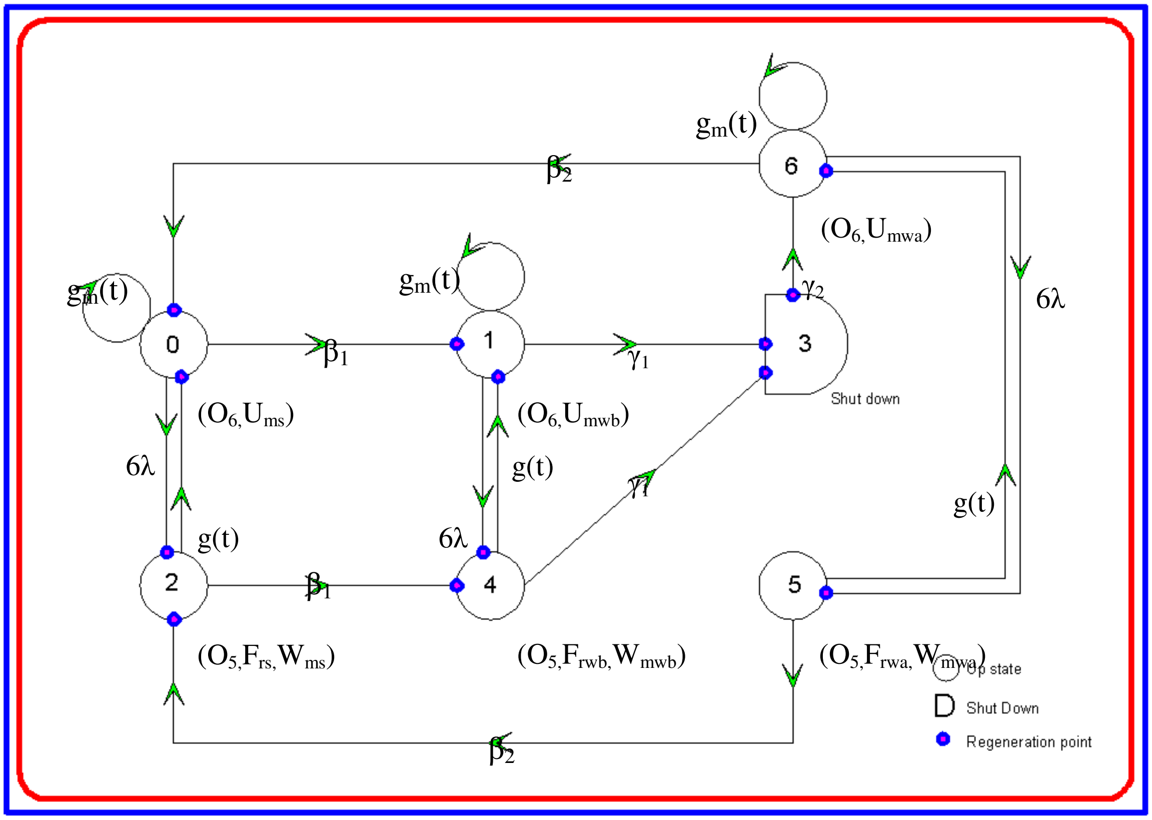

FIGURE 1: State Transition Diagram.

4. TRANSITION PROBABILITIES AND MEAN SOJOURN TIMES

A state transition diagram showing the possible states of transition of the plant is shown in figure 1. The epochs of entry into states 0, 1, 2, 3, 4, 5 and 6 are regeneration points and hence these states are regenerative states. The transition probabilities are as under:

$$dQ_{00} = \gamma e^{-(\gamma+6\lambda+\beta_1)t}dt,\ dQ_{01} = \beta_1 e^{-(6\lambda+\beta_1)t}\,\overline{G_m}(t)dt,\ dQ_{02} = 6\lambda e^{-(6\lambda+\beta_1)t}\,\overline{G_m}(t)dt,$$
$$dQ_{11} = \gamma e^{-(\gamma+6\lambda+\gamma_1)t}dt,\ dQ_{13} = \gamma_1 e^{-(6\lambda+\gamma_1)t}\,\overline{G_m}(t)dt,\ dQ_{14} = 6\lambda e^{-(6\lambda+\gamma_1)t}\,\overline{G_m}(t)dt,$$
$$dQ_{20} = g(t)dt = e^{-(\alpha+\beta_1)t}dt\ \ dQ_{24} = \beta_1 e^{-\beta_1 t}\overline{G}(t) = \beta_1 e^{-(\alpha+\beta_1)t}dt,\ dQ_{36} = \gamma_2\, e^{-\gamma_2 t}dt,$$
$$dQ_{41} = \alpha e^{-(\alpha+\gamma_1)t}dt,\ dQ_{43} = \gamma_1 e^{-(\alpha+\gamma_1)t}dt,\ dQ_{52} = \beta_2 e^{-(\alpha+\beta_2)t}dt,\ dQ_{56} = \alpha e^{-(\alpha+\beta_2)t}dt$$
$$dQ_{60} = \beta_2 e^{-(\gamma+6\lambda+\beta_2)t}dt,\ dQ_{65} = 6\lambda e^{-(\gamma+6\lambda+\beta_2)t}dt,\ dQ_{66} = \gamma e^{-(\gamma+6\lambda+\beta_2)t}dt \qquad (1\text{-}16)$$

Therefore, the non-zero elements p_{ij}can be obtained as $p_{ij} = \lim_{s\to 0}\int_0^\infty q_{ij}(t)dt$ andare given below:

$$p_{00} + p_{01} + p_{02} = 1,\ p_{11} + p_{13} + p_{14} = 1,\ p_{20} + p_{24} = 1,\ p_{36} = 1$$
$$p_{41} + p_{43} = 1,\ p_{52} + p_{56} = 1,\ p_{60} + p_{65} + p_{66} = 1 \qquad (17\text{-}23)$$

The mean sojourn time (μ_i) in the regenerative state '*i*' is defined as the time of stay in that state before transition to any other state. If *T* denotes the sojourn time in the regenerative state '*i*', then:

$$\mu_i = E(T) = \Pr[T > t]$$

$$\mu_0 = \int_0^\infty e^{-\gamma t} e^{-6\lambda t} e^{-\beta_1 t} dt = \frac{1}{\gamma + 6\lambda + \beta_1}; \ \mu_1 = \int_0^\infty e^{-\gamma t} e^{-6\lambda t} e^{-\gamma_1 t} dt = \frac{1}{\gamma + 6\lambda + \gamma_1};$$

$$\mu_2 = \frac{1}{\alpha + \beta_1}, \mu_3 = \frac{1}{\gamma_2}, \ \mu_4 = \frac{1}{\alpha + \gamma_1}, \ \mu_5 = \frac{1}{\alpha + \beta_2}, \ \mu_6 = \frac{1}{6\lambda + \gamma + \beta_2} \qquad (24–30)$$

The unconditional mean time taken by the system to transit to any of the regenerative state '*j*' when time is counted from the epoch of entry into state '*i*' is mathematically stated as:

$$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}{}^{*\prime}(0)$$

$$m_{00} + m_{01} + m_{02} = \mu_0 \ , \ m_{11} + m_{13} + m_{14} = \mu_1$$

$$m_{20} + m_{24} = \mu_2, \ \ m_{36} = \mu_3, \ m_{41} + m_{43} = \mu_4, \ \ m_{52} + m_{56} = \mu_5, \ \ m_{60} + m_{65} + m_{66} = \mu_6 \qquad (31–37)$$

5. THE MATHEMATICAL ANALYSIS

5.1 Mean Time to System Shut Down

The mean time to system shut down can be found by considering the failed states as absorbing states. Let $\Phi_i(t)$ be the c.d.f. of the first passage time from regenerative state ' i ' to a failed state ' j ' Applying simple probabilistic arguments, the following recursive relations for $\phi_i(t)$ are obtained:

$$ø_0(t) = Q_{00}(t) Ⓢ ø_0(t) + Q_{01}(t) Ⓢ ø_1(t) + Q_{02}(t) Ⓢ ø_2(t)$$

$$ø_1(t) = Q_{11}(t) Ⓢ ø_1(t) + Q_{13}(t) + Q_{14}(t) Ⓢ ø_4(t)$$

$$ø_2(t) = Q_{20}(t) Ⓢ ø_0(t) + Q_{24}(t) Ⓢ ø_4(t)$$

$$ø_4(t) = Q_{41}(t) Ⓢ ø_1(t) + Q_{43}(t) \qquad (38–41)$$

Now the mean time to shut down when the unit started at the beginning of state 0, is given by

$$\lim_{s \to 0} \frac{1 - \varphi_0^{**}(s)}{s} = \frac{N(s)}{D(s)} \qquad (42)$$

Where,

$$N(s) = Q_{01}^{**}(s)\, Q_{13}^{**}(s) + Q_{01}^{**}(s)\, Q_{14}^{**}(s)\, Q_{43}^{**}(s) + Q_{02}^{**}(s)\, Q_{24}^{**}(s)\, Q_{43}^{**}(s) - Q_{02}^{**}(s)\, Q_{24}^{**}(s)\, Q_{43}^{**}(s)\, Q_{11}^{**}(s) + Q_{02}^{**}(s)\, Q_{24}^{**}(s)\, Q_{41}^{**}(s)\, Q_{13}^{**}(s)$$

$$D(s) = 1 - Q_{00}^{**}(s) - Q_{11}^{**}(s) + Q_{00}^{**}(s)Q_{11}^{**}(s) - Q_{02}^{**}(s)Q_{20}^{**}(s) + Q_{02}^{**}(s)Q_{11}^{**}(s)\, Q_{20}^{**}(s) - Q_{14}^{**}(s)Q_{41}^{**}(s) + Q_{00}^{**}(s)\, Q_{14}^{**}(s)Q_{41}^{**}(s) + Q_{02}^{**}(s)\, Q_{20}^{**}(s)Q_{14}^{**}(s)Q_{41}^{**}(s)$$

5.2 Availability Analysis of the Unit of the Plant

For repairable systems, an essential significant measure is availability. Using the probabilistic arguments and defining $A_i(t)$ as the probability of unit entering into upstate at instant t, given that the unit entered in regenerative state i at t=0, the following recursive relations are obtained:

$A_0(t) = M_0(t) + q_{00}(t) \copyright A_0(t) + q_{01}(t) \copyright A_1(t) + q_{02}(t) \copyright A_2(t)$

$A_1(t) = M_1(t) + q_{11}(t) \copyright A_1(t) + q_{13}(t) \copyright A_3(t) + q_{14}(t) \copyright A_4(t)$

$A_2(t) = M_2(t) + q_{20}(t) \copyright A_0(t) + q_{24}(t) \copyright A_4(t)$

$A_3(t) = q_{36}(t) \copyright A_6(t)$

$A_4(t) = M_4(t) + q_{41}(t) \copyright A_1(t) + q_{43}(t) \copyright A_3(t)$

$A_5(t) = M_5(t) + q_{52}(t) \copyright A_2(t) + q_{56}(t) \copyright A_6(t)$

$A_6(t) = M_6(t) + q_{60}(t) \copyright A_0(t) + q_{65}(t) \copyright A_5(t) + q_{66}(t) \copyright A_6(t)$ (43–49)

Where $M_0(t) = e^{-(6\lambda+\beta_1+\gamma)t}$, $M_1(t) = e^{-(6\lambda+\gamma_1+\gamma)t}$,
$M_2(t) = e^{-(\alpha+\beta_1)t}$, $M_4(t) = e^{-(\alpha+\gamma_1)t}$, $M_5(t) = e^{-(\alpha+\beta_2)t}$, $M_6(t) = e^{-(6\lambda+\beta_2+\gamma)t}$.

On taking Laplace Transforms of the above equations and solving them for $A_0^*(s)$, the steady state availability is given by,

$$A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_2(0)}{D_2'(0)} \qquad (50)$$

Where,

$N_2(0) = \mu_0 + p_{01}\mu_1 - p_{11}\mu_0 + p_{02}\mu_2 - p_{02}p_{11}\mu_2 + p_{01}p_{14}\mu_4 + p_{02}p_{24}\mu_4 - p_{02}p_{24}\mu_4 p_{11} + p_{01}p_{13}p_{36}\mu_6 + p_{43}p_{36}\mu_6(p_{01}p_{14} + p_{02}p_{24} - p_{11}p_{02}p_{24}) + p_{36}p_{65}\mu_5(p_{01}p_{13} + p_{01}p_{14}p_{43} + p_{02}p_{24}p_{43} - p_{11}p_{02}p_{24}p_{43}) + p_{01}p_{13}p_{36}p_{65}p_{52}(\mu_2 + p_{41}p_{24}) + p_{43}p_{36}p_{65}p_{52}(p_{01}p_{14}\mu_2 - p_{24}\mu_0 - p_{01}\mu_1 p_{24} + p_{11}p_{24}\mu_0) + p_{56}p_{65}(-\mu_0 - p_{01}\mu_1 + p_{11}\mu_0) + p_{02}\mu_2 p_{56}p_{65}(-1 + p_{11}) + p_{56}p_{65}\mu_4(p_{01}p_{14} - p_{02}p_{24} + p_{02}p_{24}p_{11}) + p_{14}p_{41}(-\mu_0 - p_{02}\mu_2) + p_{02}p_{24}p_{41}(\mu_1 + p_{13}p_{36}\mu_6) + p_{24}p_{41}p_{36}p_{65}(p_{02}p_{13}\mu_5 - p_{01}p_{13}p_{52}) + p_{14}p_{41}p_{56}p_{65}(\mu_0 + p_{02}\mu_2) - p_{02}p_{24}p_{41}p_{56}p_{65}\mu_1 - p_{66}\mu_0 - p_{01}p_{66}\mu_1 + p_{11}p_{66}\mu_0 + p_{02}p_{66}\mu_2(1+p_{11}) + p_{66}\mu_4(p_{01}p_{14} - p_{02}p_{24} + p_{02}p_{24}p_{11}) + p_{41}p_{66}(p_{14}\mu_0 + p_{02}p_{14}\mu_2 - p_{02}p_{24}\mu_1)$

$D_2'(0) = p_{00}\mu_0 + p_{11}\mu_1 - p_{11}p_{00}(\mu_0 + \mu_1) + p_{02}p_{20}(\mu_0 + \mu_2) - p_{11}p_{02}p_{20}(\mu_0 + \mu_1 + \mu_2) + p_{01}p_{13}p_{36}p_{60}(\mu_0 + \mu_1 + \mu_3 + \mu_6) + p_{01}p_{14}p_{43}p_{36}p_{60}(\mu_0 + \mu_1 + \mu_3 + \mu_4 + \mu_6) + p_{02}p_{24}p_{43}p_{36}p_{60}(\mu_0 + \mu_2 + \mu_3 + \mu_4 + \mu_6) - p_{11}p_{02}p_{24}p_{43}p_{36}p_{60}(\mu_0 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_6) + p_{01}p_{13}p_{52}p_{20}p_{36}p_{65}(\mu_0 + \mu_1 + \mu_2 + \mu_3 + \mu_5 + \mu_6) + p_{01}p_{14}p_{43}p_{36}p_{65}p_{52}p_{20}(\mu_0 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6) + p_{52}p_{24}p_{43}p_{36}p_{65}(\mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6) - p_{00}p_{52}p_{24}p_{43}p_{36}p_{65}(\mu_0 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6) - p_{11}p_{52}p_{24}p_{43}p_{36}p_{65}(\mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6) + p_{00}p_{11}p_{52}p_{24}p_{43}p_{36}p_{65}(\mu_0 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6) + p_{56}p_{65}(\mu_5 + \mu_6) - p_{00}p_{56}p_{65}(\mu_0 + \mu_5 + \mu_6) - p_{11}p_{56}p_{65}(\mu_1 + \mu_5 + \mu_6) + p_{00}p_{11}p_{56}p_{65}(\mu_0 + \mu_1 + \mu_5 + \mu_6) - p_{02}p_{20}p_{56}p_{65}(\mu_0 + \mu_2 + \mu_5 + \mu_6) + p_{11}p_{02}p_{20}p_{56}p_{65}(\mu_0 + \mu_1 + \mu_2 + \mu_5 + \mu_6) + p_{14}p_{41}(\mu_1 + \mu_4) - p_{00}p_{14}p_{41}(\mu_0 + \mu_1 + \mu_4) - p_{02}p_{20}p_{14}p_{41}(\mu_0 + \mu_1 + \mu_2 + \mu_4) + p_{02}p_{24}p_{41}p_{13}p_{36}p_{60}(\mu_0 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_6) + p_{24}p_{41}p_{13}p_{36}p_{52}p_{65}(\mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6) - p_{00}p_{24}p_{41}p_{13}p_{36}p_{52}p_{65}(\mu_0 + \mu_1 + \mu_2 + \mu_3 + \mu_4 + \mu_5 + \mu_6) - p_{14}p_{41}p_{56}p_{65}(\mu_1 + \mu_4 + \mu_5 + \mu_6) + p_{00}p_{14}p_{41}p_{56}p_{65}(\mu_0 + \mu_1 + \mu_4 + \mu_5 + \mu_6) + p_{02}p_{20}p_{14}p_{41}p_{56}p_{65}(\mu_0 + \mu_1 + \mu_2 + \mu_4 + \mu_5 + \mu_6) + p_{66}\mu_6 - p_{00}p_{66}(\mu_0 + \mu_6) - p_{11}p_{66}(\mu_1 + \mu_6) + p_{00}p_{11}p_{66}(\mu_0 + \mu_1 + \mu_6) - p_{02}p_{20}p_{66}(\mu_0 + \mu_2 + \mu_6) + p_{11}p_{66}p_{02}p_{20}(\mu_0 + \mu_1 + \mu_2 + \mu_6) - p_{14}p_{41}p_{66}(\mu_1 + \mu_4 + \mu_6) + p_{00}p_{14}p_{41}p_{66}(\mu_0 + \mu_1 + \mu_4 + \mu_6) + p_{02}p_{20}p_{14}p_{41}p_{66}(\mu_0 + \mu_1 + \mu_2 + \mu_4 + \mu_6)$ (51 – 52)

5.4 Busy Period Analysis of Repairman

In steady sate, the total fraction of time $B_0^M(t)$ for which the unit is under repair is given by the following recursive relations:

$B_0^M(t) = W_0(t) + q_{00}(t) \copyright B_0^M(t) + q_{01}(t) \copyright B_1^M(t) + q_{02}(t) \copyright B_2^M(t)$

$B_1^M(t) = W_1(t) + q_{11}(t) \copyright B_1^M(t) + q_{13}(t) \copyright B_3^M(t) + q_{14}(t) \copyright B_4^M(t)$,

$B_2^M(t) = q_{20}(t) © B_0^M(t) + q_{24}(t) ©B_4^M(t)$,

$B_3^M(t) = q_{36}(t) © B_6^M(t)$,

$B_4^M(t) = q_{41}(t) © B_1^M(t) + q_{43}(t) © B_3^M(t)$,
$B_5^M(t) = q_{52}(t) © B_2^M(t) + q_{56}(t) © B_6^M(t)$,

$B_6^M(t) = W_6 (t)+q_{60}(t) © B_0^M(t) + q_{65}(t) ©B_5^M(t) + q_{66}(t) ©B_6^M(t)$

Where $W_0 (t)= e^{-(6\lambda+\beta_1+\gamma)t}$, $W_1(t) = e^{-(6\lambda+\gamma_1+\gamma)t}$, $W_6(t) = e^{-(6\lambda+\beta_2+\gamma)t}$ (53 - 60)

Taking Laplace Transforms of the above equations and solving them for $B_0^{M*}(s)$, the following is obtained:

$$B_0^M = \lim_{s\to 0} sB_0^{M*}(s) = \frac{N_3(0)}{D_2'(0)} \tag{61}$$

Where,
$N_3(0) = \mu_0 + p_{01}\mu_1 - p_{11}\mu_0 - p_{14}p_{41}\mu_0 + p_{01}p_{13}p_{36}\mu_6 + p_{02}\, p_{24}\, p_{41}\mu_1 + p_{02}\, p_{24}\, p_{41}\, p_{13}p_{36}\mu_6 + p_{01}\, p_{14}\, p_{43}p_{36}\mu_6 + p_{02}\, p_{24}\, p_{43}p_{36}\mu_6 - p_{11}\, p_{02}\, p_{24}\, p_{43}p_{36}\mu_6 - p_{13}p_{36}\, p_{41}\, p_{24}\, p_{65}\, p_{52}\mu_0 - p_{24}\, p_{43}\, p_{36}p_{65}\, p_{52}\mu_0 - p_{01}\, p_{24}\, p_{43}\, p_{36}p_{65}\, p_{52}\mu_1 + p_{11}\, p_{24}\, p_{43}\, p_{36}p_{65}\, p_{52}\mu_0 - p_{56}p_{65}\mu_0 - p_{01}\, p_{56}p_{65}\mu_1 + p_{11}\, p_{56}p_{65}\mu_0 + p_{14}\, p_{4156}p_{65}\mu_0 - p_{02}\, p_{24}\, p_{41}\, p_{56}p_{65}\mu_1 - p_{66}\mu_0 - p_{01}\, p_{66}\mu_1 + p_{11}\, p_{66}\mu_0 + p_{14}\, p_{41}\, p_{66}\mu_0 - p_{01}\, p_{14}\, p_{41}\, p_{66}\mu_1$ (62)
And $D_2'(0)$ as already mentioned in equation (52).

Proceeding in the same way, the other reliability measures could also be obtained:

- Expected busy period for repair [$B_0^{R*}(s)$]:

$$B_0^S = \lim_{s\to 0} sB_0^{S*}(s) = \frac{N_5(0)}{D_2'(0)} \tag{63}$$

Where,

$N_5(0) = p_{01}\, p_{13}\mu_3 + p_{01}\, p_{14}\mu_3 - p_{02}\, p_{24}\mu_3 - p_{11}\, p_{02}\, p_{24}\mu_3 + p_{02}\, p_{24}\, p_{41}\, p_{13}\mu_3 - p_{01}\, p_{13}\, p_{56}\, p_{65}\mu_3 - p_{01}\, p_{14}\, p_{56}\, p_{65}\mu_3 + p_{11}\, p_{02}\, p_{24}\, p_{56}\, p_{65}\mu_3 - p_{02}\, p_{24}\, p_{56}\, p_{65}\mu_3 - p_{02}\, p_{24}\, p_{41}\, p_{13}\, p_{56}\, p_{65}\mu_3 - p_{01}\, p_{13}\, p_{66}\mu_3 - p_{01}\, p_{14}\, p_{66}\mu_3 - p_{02}\, p_{24}\, p_{66}\mu_3 + p_{11}\, p_{02}\, p_{24}\, p_{66}\mu_3 - p_{02}\, p_{24}\, p_{13}p_{41}p_{66}\mu_3$ (64)

- Expected busy period during shutdown [$B_0^{S*}(s)$]:

$$B_0^S = \lim_{s\to 0} sB_0^{S*}(s) = \frac{N_5(0)}{D_2'(0)} \tag{65}$$

Where,

$N_5(0) = p_{01}\, p_{13}\mu_3 + p_{01}\, p_{14}\mu_3 - p_{02}\, p_{24}\mu_3 - p_{11}\, p_{02}\, p_{24}\mu_3 + p_{02}\, p_{24}\, p_{41}\, p_{13}\mu_3 - p_{01}\, p_{13}\, p_{56}\, p_{65}\mu_3 - p_{01}\, p_{14}\, p_{56}\, p_{65}\mu_3 + p_{11}\, p_{02}\, p_{24}\, p_{56}\, p_{65}\mu_3 - p_{02}\, p_{24}\, p_{56}\, p_{65}\mu_3 - p_{02}\, p_{24}\, p_{41}\, p_{13}\, p_{56}\, p_{65}\mu_3 - p_{01}\, p_{13}\, p_{66}\mu_3 - p_{01}\, p_{14}\, p_{66}\mu_3 - p_{02}\, p_{24}\, p_{66}\mu_3 + p_{11}\, p_{02}\, p_{24}\, p_{66}\mu_3 - p_{02}\, p_{24}\, p_{13}p_{41}p_{66}\mu_3$ (66)

- Expected number of repairs [$R_0^*(s)$]:

$$R_0 = \lim_{s\to 0} sR_0^*(s) = \frac{N_6(0)}{D_2'(0)} \tag{67}$$

Where,

$N_6(0) = p_{02}p_{20} - p_{11}\, p_{02}p_{20} + p_{01}p_{14}\, p_{41} + p_{02}p_{24}\, p_{41} - p_{11}p_{02}p_{24}\, p_{41} - p_{02}p_{20}p_{14}p_{41} + p_{01}p_{20}p_{13}p_{36}p_{65}p_{52} + p_{01}p_{24}p_{41}\, p_{13}p_{36}p_{65}p_{52} + p_{01}\, p_{14}\, p_{43}p_{36}p_{65}p_{52}\, p_{20} - p_{02}p_{20}p_{56}p_{65} + p_{11}$

$p_{02}p_{20}p_{56}p_{65} - p_{01}\, p_{14}p_{41}p_{56}p_{65} - p_{02}p_{24}\, p_{41}\, p_{56}p_{65} + p_{11}p_{02}p_{24}\, p_{41}p_{56}p_{65} + p_{02}p_{14}\, p_{41}\, p_{56}p_{65} + p_{01}\, p_{13}p_{36}\, p_{56}p_{65} + p_{02}p_{24}\, p_{41}\, p_{13}p_{36}\, p_{56}p_{65} + p_{01}\, p_{14}p_{43}p_{36}\, p_{56}p_{65} + p_{02}\, p_{24}p_{43}p_{36}\, p_{56}p_{65} - p_{02}\, p_{24}p_{43}p_{36}\, p_{56}p_{65}\, p_{11} - p_{02}p_{20}\, p_{66} + p_{11}\, p_{02}p_{20}\, p_{66} - p_{01}p_{14}\, p_{41}p_{66} - p_{02}p_{24}\, p_{41}p_{66} + p_{11}\, p_{02}p_{24}\, p_{66} + p_{02}p_{20}\, p_{41}\, p_{14}\, p_{66}$
(68)

and $D_2'(0)$ as already mentioned in equation (52).

6. PARTICULAR CASE

For the particular case, it is assumed that the failure and repair rates are exponentially distributed and therefore the following have been assumed:

$g(t) = \alpha e^{-\alpha t}$, $g_m(t) = \gamma e^{-\gamma t}$, $g_{sr}(t) = \gamma_2 e^{-\gamma_2 t}$

Using (1-16) and (24-30), the following are obtained:

$$p_{00} = g^*_m(6\lambda+\beta_1) = \frac{\gamma}{\gamma+6\lambda+\beta_1};\quad p_{01} = \frac{\beta_1}{6\lambda+\beta_1}[1-g^*_m(6\lambda+\beta_1)] = \frac{\beta_1}{\gamma+6\lambda+\beta_1};$$

$$p_{02} = \frac{6\lambda}{6\lambda+\beta_1}[1-g^*_m(6\lambda+\beta_1)] = \frac{6\lambda}{\gamma+6\lambda+\beta_1}$$

$$p_{11} = g^*_m(6\lambda+\gamma_1) = \frac{\gamma}{\gamma+6\lambda+\gamma_1};\quad p_{13} = \frac{\gamma_1}{6\lambda+\gamma_1}[1-g^*_m(6\lambda+\gamma_1)] = \frac{\gamma_1}{\gamma+6\lambda+\gamma_1}$$

$$p_{14} = \frac{6\lambda}{6\lambda+\gamma_1}[1-g^*_m(6\lambda+\gamma_1)] = \frac{6\lambda}{\gamma+6\lambda+\gamma_1}$$

$$p_{20} = g^*(\beta_1) = \frac{\alpha}{\alpha+\beta_1};\quad p_{24} = 1-g^*(\beta_1) = \frac{\beta_1}{\alpha+\beta_1};\ p_{36} = 1$$

$$p_{41} = g^*(\gamma_1) = \frac{\alpha}{\alpha+\gamma_1};\quad p_{43} = 1-g^*(\gamma_1) = \frac{\gamma_1}{\alpha+\gamma_1};$$

$$p_{52} = 1-g^*(\beta_2) = \frac{\beta_2}{\alpha+\beta_2};\quad p_{56} = g^*(\beta_2) = \frac{\alpha}{\alpha+\beta_2};$$

$$p_{60} = \frac{\beta_2}{6\lambda+\beta_2}[1-g^*_m(6\lambda+\beta_2)] = \frac{\beta_2}{\gamma+6\lambda+\beta_2}$$

$$p_{65} = \frac{6\lambda}{6\lambda+\beta_2}[1-g^*_m(6\lambda+\beta_2)] = \frac{6\lambda}{\gamma+6\lambda+\beta_2}\quad p_{66} = g^*_m(6\lambda+\beta_2) = \frac{\gamma}{\gamma+6\lambda+\beta_2};$$

$$\mu_0 = \int_0^\infty e^{-\gamma t}e^{-6\lambda t}e^{-\beta_1 t}dt = \frac{1}{\gamma+6\lambda+\beta_1};\quad \mu_1 = \int_0^\infty e^{-\gamma t}e^{-6\lambda t}e^{-\gamma_1 t}dt = \frac{1}{\gamma+6\lambda+\gamma_1};$$

$$\mu_2 = \frac{1}{\alpha+\beta_1},\ \mu_3 = \frac{1}{\gamma_2},\ \mu_4 = \frac{1}{\alpha+\gamma_1},\ \mu_5 = \frac{1}{\alpha+\beta_2},\ \mu_6 = \frac{1}{6\lambda+\gamma+\beta_2}$$

Using the above equations and the values estimated from the data, the following are obtained:

μ_0 = 531.22543; μ_1 = 566.5273; μ_2 = 552.94443; μ_3 = 144.00092; μ_4 = 591.29612;

μ_5 = 552.9444; μ_6= 531.22543.

p_{00} =0.790516564, p_{01} = 0.122978687, p_{02}= 0.086504749;

p_{11}=0.843049277, p_{13} = 0.092253306, p_{14} = 0.064697418;

p_{20}= 0.871993365,p_{24}=0.128006635;

$p_{36=}1$;

p_{41}= 0.932473983,p_{43}=0.06752602;

p_{52}=0.128006635, p_{56}=0.871993365;

p_{60}=0.122978687,p_{65}=0.086504749, p_{66}=0.790516564;

Using the summarized data and the expressions of section 5, various measures of system effectiveness are estimated:

Mean time for the unit to shut down = 424 days

Availability of the unit (A_0) = 0.991790084

Expected busy period for repairman (B_0^M) = 0.902028086

Expected busy period for repair (B_0^R) = 0.008209916

Expected busy period during shut down (B_0^S) = 0.089761998

Expected number of repairs (R_0) =0.000141555

As a future direction, it would be interesting to study the variations on these results of the plant when repair or maintenance is done on first come first served basis.

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