Use Fuzzy Midrange Transformation Method to Construction Fuzzy Control Charts limits

Kawa M. Jamal Rashid *kawa.jamal_57@yahoo.com*
School of Administrations and Economics/ Department of Statistics
Sulamani University
Sulamani, Iraq

Suzan S. Haydar *sozanmath@yahoo.com*
School of Administrations and Economics/ Department of Statistics
Sulamani University
Sulamani, Iraq

Abstract

Statistical Process Control (SPC) is approach that uses statistical techniques to monitor the process. The techniques of quality control are widely used in controlling any kinds of processes. The widely used control charts are $\overline{X} - R$ and $\overline{X} - S$ charts. These are called traditional variable control chart, which consists of three horizontal lines called Centre Line (CL), Upper Control Limit (UCL) and Lower Control Limit (UCL) are represented by numeric values. The center line in a control chart denotes the average value of the quality characteristic under study. A process is either "in control" or "out of control" depending on numeric observation values. In the consideration of real production process, it is assumed that there are no doubts about observations and their values. But when these observations include human judgments, evaluations and decisions, a continuous random variable (xi) of a production process should include the variability caused by human subjectivity or measurement devices, or environmental conditions. So, linguistic terms can be used instead of an exact value of continuous random variable. In this context fuzzy set theory is useful tool to handle this uncertainty. Numeric control limits can be transformed to fuzzy control limits by using membership function, therefore; the concept of fuzzy control charts with α cuts by using α -level fuzzy midrange with trapezoidal fuzzy number (TRN) is proposed. The fuzzy control charts for arithmetic mean ($\tilde{\overline{X}}$), and range ($\tilde{\overline{R}}$) are developed. Fuzzy control limits provide a more accurate and flexible evaluation. In this paper through a real illustrative data from Sulaimani Company for Cement in the city of Sulaimani, shows the designing of fuzzy control chart for process average of quality.

Keywords: Statistical Process Control, Fuzzy Number, Fuzzy Control Charts, Membership Function, α -cut and α - Level Fuzzy Midrange.

1. INTRODUCTION

Quality control is a process employed to ensure a certain level of quality in a product or service. It may include whatever actions a business deems necessary to provide for the control and verification of certain characteristics of a product or service. The basic goal of quality control is to ensure that the products, services, or processes provided meet specific requirements and are dependable and satisfactory [1], [2]. The fuzzy set theory is a more suitable tool for handling attribute data since these data may be expressed in linguistic terms such as "very good", "good", "medium", "bad", and "very bad" [3]. The fuzzy set theory was first introduced by Zadeh (1965). Many studies were done to combine statistical methods and fuzzy set theory. The fuzzy numbers are a reasonable way to analyze and evaluate the process. some measures of central tendency in descriptive statistics are used in variable control charts. These measures can be used to convert fuzzy sets into scalars which are fuzzy mode, α -level fuzzy midrange, fuzzy median and fuzzy average [1], [3].

2. THE STRUCTURE OF CONTROL LIMITS FOR FUZZY CONTROL CHART

The X-bar chart is most widely used chart for controlling the process mean quality level as well as the process variability can be controlled by either a control chart for the range, called R-chart or a control chart for the standard deviation, called S-chart.

Montgomery [2005] has proposed the control limits for $\overline{X}$ control chart based on sample range is given below:

$$UCL_{\bar{x}} = \overline{\overline{X}} + A_2 \, \overline{R} \qquad (1)$$

$$CL_{\bar{x}} = \overline{\overline{X}} \qquad (2)$$

$$LCL_{\bar{x}} = \overline{\overline{X}} - A_2 \, \overline{R} \qquad (3)$$

where A_2 is a control chart coefficient, and $\overline{R}$ is the average of R_i's that are the ranges of samples [2], [4], [5].

In the fuzzy case, which is used in this paper, each sample is represented by a trapezoidal (or triangular) fuzzy number (a, b, c, d) where $a \le b \le c \le d$, has the membership function by the following equation [5]:

$$\mu_A(x) = \begin{cases} 0 & , if \;\; x \le a \quad or \quad x \ge d \\ \dfrac{x-a}{b-a} & , if \;\; a < x \le b \\ 1 & , if \;\; b \le x \le c \\ \dfrac{d-x}{d-c} & , if \;\; c < x \le d \end{cases}$$

as shown in Figure1.

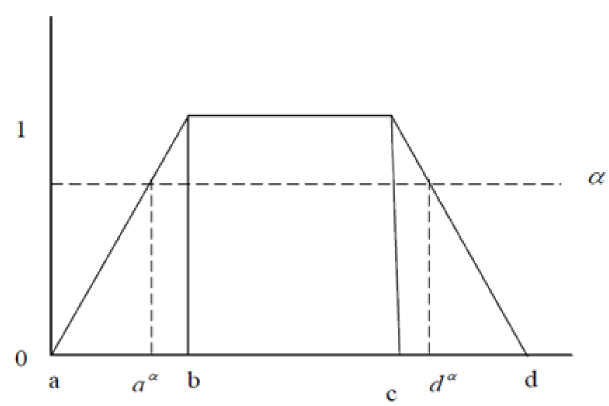

FIGURE 1: Representation of a sample by trapezoidal fuzzy numbers [6].

The main purpose of this study is to define a general architecture of fuzzy control chart with fuzzy control limits, which is provide a more accurate and flexible evaluation by each elementary component. Numeric control limits can be transformed to fuzzy control limits by using membership function. In this study, trapezoidal fuzzy numbers are represented as (X_a, X_b, X_c, X_d) for each observation. Note that a trapezoidal fuzzy number becomes triangular when ($b = c$). The control limits of fuzzy $\tilde{\bar{X}}$ control charts with ranges based on fuzzy trapezoidal number are calculated as follows:

The upper control limit is:

$$U\tilde{C}L_{\bar{x}} = \tilde{C}L_{\bar{x}} + A_2\ \bar{R} = (\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c, \bar{\bar{X}}_d) + A_2\ (\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d)$$

$$= (\bar{\bar{X}}_a + A_2\ \bar{R}_a,\ \ \bar{\bar{X}}_b + A_2\ \bar{R}_b,\ \ \bar{\bar{X}}_c + A_2\ \bar{R}_c,\ \ \bar{\bar{X}}_d + A_2\ \bar{R}_d)$$

$$= (U\tilde{C}L_1, U\tilde{C}L_2, U\tilde{C}L_3, U\tilde{C}L_4)$$

The central limit is:

$$\tilde{C}L_{\bar{x}} = (\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c, \bar{\bar{X}}_d) = (\tilde{C}L_1, \tilde{C}L_2, \tilde{C}L_3, \tilde{C}L_4)$$

The lower control limit is:

$$L\tilde{C}L_{\bar{x}} = \tilde{C}L_{\bar{x}} - A_2\ \bar{R} = (\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c, \bar{\bar{X}}_d) - A_2\ (\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d)$$

$$= (\bar{\bar{X}}_a - A_2\ \bar{R}_a,\ \ \bar{\bar{X}}_b - A_2\ \bar{R}_b,\ \ \bar{\bar{X}}_c - A_2\ \bar{R}_c,\ \ \bar{\bar{X}}_d - A_2\ \bar{R}_d)$$

$$= (L\tilde{C}L_1, L\tilde{C}L_2, L\tilde{C}L_3, L\tilde{C}L_4)$$

Where $R_i = \sum_{j=1}^{m} R_{ij} \quad ,i = a,b,c,d;\ j = 1,2,3,...,m$, the procedure for calculating R_{ij} is as follows:

$$R_{aj} = X_{\max aj} - X_{Min\ dj}$$

$$R_{bj} = X_{\max bj} - X_{Min\ cj}$$

$$R_{cj} = X_{\max cj} - X_{Min\ bj}$$

$$R_{dj} = X_{\max dj} - X_{Min\ aj}$$

Where ($X_{\max aj}, X_{\max bj}, X_{\max cj}, X_{\max dj}$) is the maximum fuzzy number in the j^{th} sample and ($X_{\min aj}, X_{\min bj}, X_{\min cj}, X_{\min dj}$) is the minimum fuzzy number in the j^{th} sample, $j = 1,2,3,...,m$ [7].

Then the trapezoidal fuzzy number is represented as follows figure:

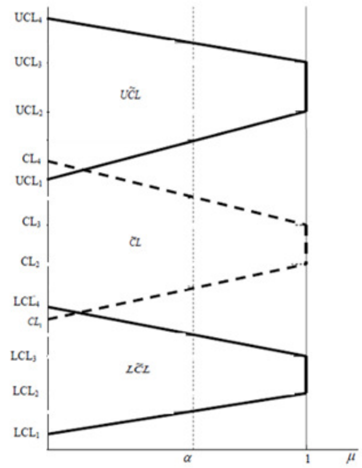

FIGURE 2: Fuzzy X control limits based on ranges using fuzzy trapezoidal number.

3. THE STRUCTURE OF CONTROL LIMLTS FOR α-CUT FUZZY CONTROL CHART

An α-cut comprises all elements whose membership degrees are greater than equal to α. The set $A_\alpha = \{x \in X ; \mu_A(x) \geq \alpha, \ 0 \leq \alpha \leq 1\}$. The α-level sets A_α are also called the α-cut sets [8]. Figure 3 shows a trapezoidal fuzzy number and it's α-cut.

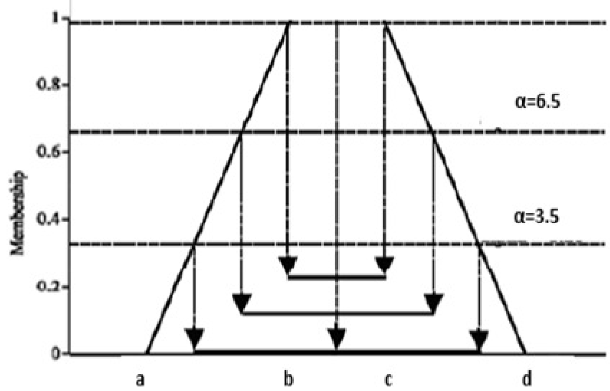

FIGURE 3: Trapezoidal fuzzy number and it's α-cut.

Applying an α-cut to fuzzy $\tilde{\bar{\bar{X}}}$ control chart limits, then control limits based on ranges ($U\tilde{C}L, \tilde{C}L, L\tilde{C}L$) are determined as follows:

The upper control limit is:

$$U\tilde{C}L_{\bar{X}}^{\alpha} = C\tilde{L}_{\bar{X}}^{\alpha} + A_2 \ \bar{R}_{\bar{X}}^{\alpha} = (\bar{\bar{X}}_a^{\alpha}, \bar{\bar{X}}_b, \bar{\bar{X}}_c, \bar{\bar{X}}_d^{\alpha}) + A_2 \ (\bar{R}_a^{\alpha}, \bar{R}_b, \bar{R}_c, \bar{R}_d^{\alpha})$$

$$= (\bar{\bar{X}}_a^{\alpha} + A_2 \ \bar{R}_a^{\alpha}, \ \bar{\bar{X}}_b + A_2 \ \bar{R}_b, \ \bar{\bar{X}}_c + A_2 \ \bar{R}_c, \ \bar{\bar{X}}_d^{\alpha} + A_2 \ \bar{R}_d^{\alpha})$$

$$= (U\tilde{C}L_1^{\alpha}, U\tilde{C}L_2, U\tilde{C}L_3, U\tilde{C}L_4^{\alpha})$$

The central limit is:

$$\tilde{C}L^{\alpha}_{\bar{X}} = (\bar{\bar{X}}^{\alpha}_{a}, \bar{\bar{X}}_{b}, \bar{\bar{X}}_{c}, \bar{\bar{X}}^{\alpha}_{d}) = (\tilde{C}L^{\alpha}_{1}, \tilde{C}L_{2}, \tilde{C}L_{3}, \tilde{C}L^{\alpha}_{4})$$

The lower control limit is:

$$L\tilde{C}L^{\alpha}_{\bar{X}} = C\tilde{L}^{\alpha}_{\bar{X}} - A_2\ \bar{R}^{\alpha}_{\bar{X}} = (\bar{\bar{X}}^{\alpha}_{a}, \bar{\bar{X}}_{b}, \bar{\bar{X}}_{c}, \bar{\bar{X}}^{\alpha}_{d}) - A_2\ (\bar{R}^{\alpha}_{a}, \bar{R}_{b}, \bar{R}_{c}, \bar{R}^{\alpha}_{d})$$

$$= (\bar{\bar{X}}^{\alpha}_{a} - A_2\ \bar{R}^{\alpha}_{a},\ \ \bar{\bar{X}}_{b} - A_2\ \bar{R}_{b},\ \ \bar{\bar{X}}_{c} - A_2\ \bar{R}_{c},\ \ \bar{\bar{X}}^{\alpha}_{d} - A_2\ \bar{R}^{\alpha}_{d})$$

$$= (L\tilde{C}L^{\alpha}_{1}, L\tilde{C}L_{2}, L\tilde{C}L_{3}, L\tilde{C}L^{\alpha}_{4})$$

Where:

$$\bar{\bar{X}}^{\alpha}_{a} = \bar{\bar{X}}_{a} + \alpha(\bar{\bar{X}}_{b} - \bar{\bar{X}}_{a})$$

$$\bar{\bar{X}}^{\alpha}_{d} = \bar{\bar{X}}_{d} + \alpha(\bar{\bar{X}}_{d} - \bar{\bar{X}}_{c})$$

$$\bar{R}^{\alpha}_{a} = \bar{R}_{a} + \alpha(\bar{R}_{b} - \bar{R}_{a})$$

$$\bar{R}^{\alpha}_{d} = \bar{R}_{d} + \alpha(\bar{R}_{d} - \bar{R}_{c})$$

The α - cut fuzzy $\tilde{\bar{\bar{X}}}$ control limits based on ranges are shown in figure 4.

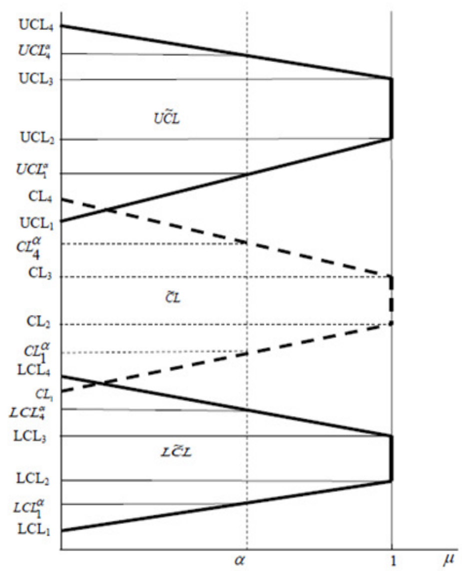

FIGURE 4: α - Cut Fuzzy $\tilde{\bar{\bar{X}}}$ control limits based on ranges using fuzzy trapezoidal number.

4. α-CUT FUZZY $\tilde{\bar{\bar{X}}}$ CONTROL CHART BASED ON RANGES AT α-CUT FUZZY MIDRANGE

The α-cut fuzzy midrange is one of the transformation techniques (among the four) used to transform the fuzzy set into scalar. It is used to check the production process, whether the process is “in-control” or “out-of-control”. The control limits for α-cut fuzzy midrange for α-cut fuzzy $\tilde{\overline{X}}$ control chart based on ranges can be obtained as follows:

$$U\tilde{C}L^{\alpha}_{mr-\overline{X}} = C\tilde{L}^{\alpha}_{mr-\overline{X}} + A_2\ (\frac{\overline{R}^{\alpha}_a + \overline{R}^{\alpha}_d}{2})$$

$$\tilde{C}L^{\alpha}_{mr-\overline{X}} = f^{\alpha}_{mr-\overline{X}}(C\tilde{L}) = \frac{\overline{X}^{\alpha}_a + \overline{X}^{\alpha}_d}{2}$$

$$L\tilde{C}L^{\alpha}_{mr-\overline{X}} = C\tilde{L}^{\alpha}_{mr-\overline{X}} - A_2\ (\frac{\overline{R}^{\alpha}_a + \overline{R}^{\alpha}_d}{2})$$

The definition of α-cut fuzzy midrange of sample j for fuzzy $\tilde{\overline{X}}$ control chart is:

$$S^{\alpha}_{mr-\overline{X},j} = \frac{(X_{aj} + X_{dj}) + \alpha[(X_{bj} - X_{aj}) - (X_{dj} - X_{cj})]}{2}$$

Then, the condition of process control for each sample can be defined as:

$$\Pr ocess\ control = \{incontrol;\ for\ L\tilde{C}L^{\alpha}_{mr-\overline{X}} \leq S^{\alpha}_{mr-\overline{X},j} \leq U\tilde{C}L^{\alpha}_{mr-\overline{X}}\}$$

Based upon the value of $S^{\alpha}_{mr-\overline{X},j}$ for each sample, decision about the process can be made [4], [9].

5. FUZZY $\tilde{R}$ CONTROL CHART

The control limits for Shewhart R control chart is given by:

$$UCL_R = D_4\overline{R}$$

$$CL_R = \overline{R}$$

$$LCL_R = D_3\overline{R}$$

Where D_4 and D_3 are control chart co-efficient, these co-efficient values are obtained by using the co-efficient table given by Montgomery [2005] [4], [6].

By using the traditional R control chart procedure, the control limits for fuzzy $\tilde{R}$ control chart with trapezoidal fuzzy number is obtained as:

$$U\tilde{C}L_R = D_4(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d)$$

$$\tilde{C}L_R = (\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d)$$

$$L\tilde{C}L_R = D_3(\overline{R}_a, \overline{R}_b, \overline{R}_c, \overline{R}_d)$$

6. CONTROL LIMITS FOR α- CUT FUZZY $\tilde{R}$ CONTROL CHART

The control limits of α-cut fuzzy $\tilde{R}$ control chart based on trapezoidal fuzzy numbers are obtained as follows:

$$U\tilde{C}L_R^{\alpha} = D_4(\overline{R}_a^{\alpha}, \overline{R}_b, \overline{R}_c, \overline{R}_d^{\alpha})$$

$$\tilde{C}L_R^{\alpha} = (\overline{R}_a^{\alpha}, \overline{R}_b, \overline{R}_c, \overline{R}_d^{\alpha})$$

$$L\tilde{C}L_R^{\alpha} = D_3(\overline{R}_a^{\alpha}, \overline{R}_b, \overline{R}_c, \overline{R}_d^{\alpha})$$

7. α-CUT FUZZY $\tilde{R}$ CONTROL CHART AT α– LEVEL FUZZY MIDRANGE

The control limits of α- Level fuzzy midrange for α- cut fuzzy $\tilde{R}$ control chart based on fuzzy Trapezoidal numbers are defined by:

$$U\tilde{C}L_{mr-R}^{\alpha} = D_4 \; f_{mr-R}^{\alpha}(\tilde{C}L)$$

$$\tilde{C}L_{mr-R}^{\alpha} = f_{mr-R}^{\alpha}(\tilde{C}L) = \frac{\overline{R}_a^{\alpha} + \overline{R}_d^{\alpha}}{2}$$

$$L\tilde{C}L_{mr-R}^{\alpha} = D_3 \; f_{mr-R}^{\alpha}(\tilde{C}L)$$

Fuzzy transformation techniques are used for deciding if the process is “under-control” or “out-of-control” after calculating the control limits. The α - level fuzzy midrange of sample j for fuzzy $\tilde{R}$ control chart can be transformed to crisp numbers with the fuzzy transformation techniques. In this paper, the fuzzy midrange transformation technique is used. The α-level fuzzy midrange is defined as:

$$S_{mr-R,j}^{\alpha} = \frac{(R_{aj} + R_{dj}) + \alpha[(R_{bj} - R_{aj}) - (R_{dj} - R_{cj})]}{2}$$

Then, the condition of process control for each sample can be defined as:

$$\begin{Bmatrix} \text{Pr}\, ocess \;\; incontrol \;\; ; \, for \;\; L\tilde{C}L_{mr-R}^{\alpha} \leq S_{mr-R,j}^{\alpha} \leq U\tilde{C}L_{mr-R}^{\alpha} \\ Out \;\; of \;\; control \;\; ; otherwise \end{Bmatrix}$$

The values of $S_{mr-R,j}^{\alpha}$ for all the samples are compared and decision about the process variability is made [4], [6], [9].

8. NUMERICAL ILLUSTRATION

In this section an application is considered to highlight the features of the above proposed fuzzy control charts. In this paper through a real illustrative data from Sulaimani Company for cement made in the city of Sulaimani, shows the designing of fuzzy control chart for process average of variable quality. The application was made on controlling the proportion of CO_3 component in the cement. Thirty samples with a sample size of 4 (the total measurement number is 4×30=120) were taken from the production process in Sulaimani Company. These measurements are converted into trapezoidal fuzzy numbers and given in Table 1.

sample No.	Xa				Xb				Xc			
	1	2	3	4	1	2	3	4	1	2	3	4
1	2.42	2.44	2.59	2.46	2.63	2.65	2.8	2.67	2.84	2.86	3.01	2.88
2	2.43	2.26	2.48	2.41	2.64	2.47	2.69	2.62	2.85	2.68	2.9	2.83
3	2.49	2.43	2.42	2.47	2.7	2.64	2.63	2.68	2.91	2.85	2.84	2.89
4	1.94	2.22	2.34	2.49	2.15	2.43	2.55	2.7	2.36	2.64	2.76	2.91
5	2.65	2.48	2.34	2.64	2.86	2.69	2.55	2.85	3.07	2.9	2.76	3.06
6	2.29	2.22	2.39	2.26	2.5	2.43	2.6	2.47	2.71	2.64	2.81	2.68
7	2.45	2.58	2.26	2.22	2.66	2.79	2.47	2.43	2.87	3	2.68	2.64
8	2.63	2.54	2.42	2.68	2.84	2.75	2.63	2.89	3.05	2.96	2.84	3.1
9	2.53	2.47	2.42	2.37	2.74	2.68	2.63	2.58	2.95	2.89	2.84	2.79
10	2.02	1.95	2.47	2.43	2.23	2.16	2.68	2.64	2.44	2.37	2.89	2.85
11	2.42	2.48	2.53	2.72	2.63	2.69	2.74	2.93	2.84	2.9	2.95	3.14
:	:	:	:	:	:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:	:	:	:	:	:
20	2.81	2.69	2.79	2.4	3.02	2.9	3	2.61	3.23	3.11	3.21	2.82
21	3.02	2.62	2.74	2.64	3.23	2.83	2.95	2.85	3.44	3.04	3.16	3.06
22	2.79	2.37	2.34	2	3	2.58	2.55	2.21	3.21	2.79	2.76	2.42
23	3.79	2.56	2.49	2.39	4	2.77	2.7	2.6	4.21	2.98	2.91	2.81
24	2.57	2.14	2.34	2.45	2.78	2.35	2.55	2.66	2.99	2.56	2.76	2.87
25	2.03	2.44	2.01	2.58	2.24	2.65	2.22	2.79	2.45	2.86	2.43	3
26	2.33	2.39	2.32	2.39	2.54	2.6	2.53	2.6	2.75	2.81	2.74	2.81
27	2.4	2.39	2.03	2.63	2.61	2.6	2.24	2.84	2.82	2.81	2.45	3.05
28	2.75	2.98	3.02	2.84	2.96	3.19	3.23	3.05	3.17	3.4	3.44	3.26
29	2.77	2.97	2.79	2.94	2.98	3.18	3	3.15	3.19	3.39	3.21	3.36
30	2.49	2.43	2.42	2.47	2.7	2.64	2.63	2.68	2.91	2.85	2.84	2.89

TABLE 1: proportion of CO_3 in cement for 30 day.

For $n = 4$, the coefficient for different control charts are obtained from the Statistical Tables as $A_2 = 0.729, \quad D_4 = 2.282, \quad D_3 = 0$. These coefficients are used in constructing various control charts. By using fuzzy $\overline{X}$ control chart based on ranges, we obtain the following results that the process is out of control for only 28^{th} and 29^{th} samples, otherwise, the process was under control with respect to $\overline{X}$, figure (5) shows $\overline{X}$ -chart of the average of (X_a, X_b, X_c) with UCL, LCL, only point 27^{th} is out of control, the range between UCL and LCL is (1.07).

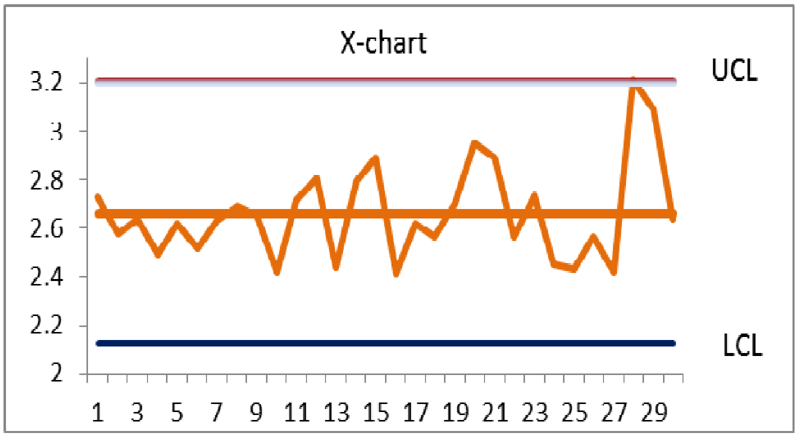

Figure 5: Shewhart $\overline{X}$ control chart based on rang.

Using equations of, $\tilde{C}L, U\tilde{C}L, L\tilde{C}L$ for $\overline{X}$ control chart are determined as follows:
The fuzzy center line of the control data is:

$$\tilde{C}L_{\overline{X}} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c) = (2.496,\ 2.706,\ 2.916)$$

$$\overline{R} = (\overline{R}_a, \overline{R}_b, \overline{R}_c) = (0.008,\ 0.428,\ 0.848)$$

The fuzzy control limits are:

$$U\tilde{C}L_{\overline{X}} = \tilde{C}L_{\overline{X}} + A_2\ \overline{R} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c) + A_2(\overline{R}_a, \overline{R}_b, \overline{R}_c)$$

$$= (2.496,\ 2.706, 2.916) + 0.729(0.008,\ 0.428, 0.848)$$

$$= (2.501,\ 3.018, 3.534)$$

$$L\tilde{C}L_{\overline{X}} = \tilde{C}L_{\overline{X}} - A_2\ \overline{R} = (\overline{\overline{X}}_a, \overline{\overline{X}}_b, \overline{\overline{X}}_c) - A_2(\overline{R}_a, \overline{R}_b, \overline{R}_c)$$

$$= (2.496,\ 2.706, 2.916) - 0.729(0.008,\ 0.428, 0.848)$$

$$= (2.490,\ 2.393, 2.297)$$

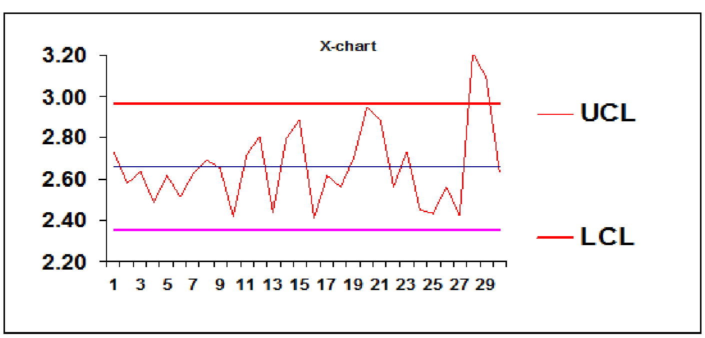

FIGURE 6: The fuzzy control limits of $\overline{X}$ chart based on rang.

From Figure (6) it shows that the point (27^{th} and 28^{th}) out of control it clear that the rang between UCL, and LCL of X-chart less than fuzzy control chart.

By using α -Cut we get:

$$\bar{\bar{X}}_a^{0.65} = \bar{\bar{X}}_a + 0.65(\bar{\bar{X}}_b - \bar{\bar{X}}_a) = 2.632$$

$$\bar{\bar{X}}_c^{0.65} = \bar{\bar{X}}_c + 0.65(\bar{\bar{X}}_c - \bar{\bar{X}}_b) = 2.779$$

$$\bar{R}_a^{0.65} = \bar{R}_a + 0.65(\bar{R}_b - \bar{R}_a) = 0.281$$

$$\bar{R}_c^{0.65} = \bar{R}_c + 0.65(\bar{R}_c - \bar{R}_b) = 0.575$$

Then, $\tilde{C}L, U\tilde{C}L, L\tilde{C}L$ of α - cut fuzzy $\bar{X}$ control chart becomes:

The fuzzy center line of the control data is:

$$\tilde{C}L_{\bar{X}}^{0.65} = (\bar{\bar{X}}_a^{0.65}, \bar{\bar{X}}_b, \bar{\bar{X}}_c^{0.65}) = (2.632,\ 2.706,\ 2.779)$$

$$\bar{R} = (\bar{R}_a^{0.65}, \bar{R}_b, \bar{R}_c^{0.65}) = (0.281,\ 0.428,\ 0.575)$$

The fuzzy control limits are:

$$U\tilde{C}L_{\bar{X}}^{0.65} = \tilde{C}L + A_2\bar{R} = (\bar{\bar{X}}_a^{0.65}, \bar{\bar{X}}_b, \bar{\bar{X}}_c^{0.65}) + A_2\ (\bar{R}_a^{0.65}, \bar{R}_b, \bar{R}_c^{0.65})$$

$$= (2.837,\ 3.018,\ 3.198)$$

$$L\tilde{C}L_{\bar{X}}^{0.65} = \tilde{C}L - A_2\bar{R} = (\bar{\bar{X}}_a^{0.65}, \bar{\bar{X}}_b, \bar{\bar{X}}_c^{0.65}) - A_2\ (\bar{R}_a^{0.65}, \bar{R}_b, \bar{R}_c^{0.65})$$

$$= (2.427,\ 2.393,\ 2.3598)$$

Now using α - level fuzzy midrange techniques for fuzzy $\bar{X}$ control chart to transform to crisp numbers as:

$$U\tilde{C}L_{mr-\bar{X}}^{0.65} = \tilde{C}L_{mr-\bar{X}}^{0.65} + A_2\ (\frac{\bar{R}_a^{0.65} + \bar{R}_c^{0.65}}{2}) = 3.018$$

$$\tilde{C}L_{mr-\bar{X}}^{0.65} = f_{mr-\bar{X}}^{0.65}(\tilde{C}L) = (\frac{\bar{\bar{X}}_a^{0.65} + \bar{\bar{X}}_c^{0.65}}{2})$$

$$L\tilde{C}L_{mr-\bar{X}}^{0.65} = \tilde{C}L_{mr-\bar{X}}^{0.65} - A_2\ (\frac{\bar{R}_a^{0.65} + \bar{R}_c^{0.65}}{2}) = 2.39$$

The definition of α -cut fuzzy midrange of sample j for fuzzy $\tilde{\tilde{X}}$ control chart is:

$$S^{\alpha}_{mr-\bar{X},j} = \frac{(X_{aj} + X_{dj}) + \alpha[(X_{bj} - X_{aj}) - (X_{dj} - X_{cj})]}{2}$$

We get our decision as follows in table (2):

sample	$S^{\alpha}_{mr-\bar{X},j}$	2.393< $S^{\alpha}_{mr-\bar{X},j}$ < 3.018	sample	$S^{\alpha}_{mr-\bar{X},j}$	2.393< $S^{\alpha}_{mr-\bar{X},j}$ < 3.018
1	2.6875	In control	16	2.5625	In control
2	2.605	In control	17	2.7	In control
3	2.6625	In control	18	2.61	In control
4	2.4575	In control	19	3.05	out control
5	2.7375	In control	20	2.8825	In control
6	2.5	In control	21	2.965	In control
7	2.5875	In control	22	2.585	In control
8	2.7775	In control	23	3.0175	In control
9	2.6575	In control	24	2.585	In control
10	2.4275	In control	25	2.475	In control
11	2.7475	In control	26	2.5675	In control
12	2.7425	In control	27	2.5725	In control
13	2.37	out control	28	3.1075	out control
14	2.82	In control	29	3.0775	out control
15	2.965	In control	30	2.6625	In control

TABLE (2): The decision using α-level fuzzy midrange of α-cut fuzzy $\tilde{\tilde{X}}$.

As shown in the above table, the process is out of control for $13^{th}, 19^{th}, 28^{th}, and\ 29^{th}$ samples, otherwise, the process was under control with respect to $S^{0.65}_{mr-\bar{X},j}$. This chart is shown in the following figure:

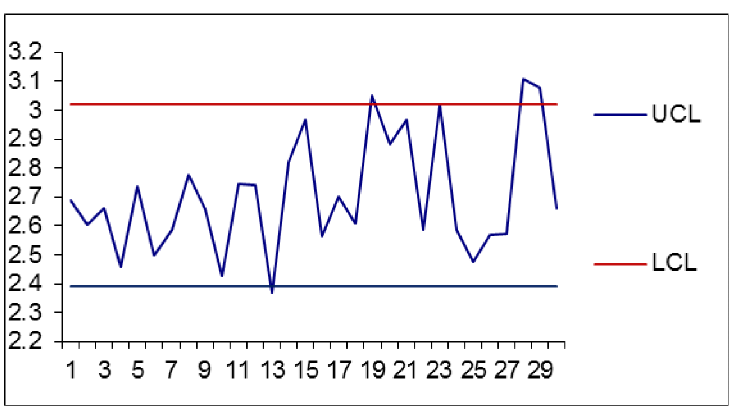

FIGURE 7: α-cut fuzzy $\tilde{\tilde{X}}$ control chart based on ranges at α -cut fuzzy midrange.

Another way to construct the fuzzy control limits is to use the sample range as an estimate of the variability of the process. Remember that the range is simply the difference between the largest and smallest values in the sample. The spread of the range can tell us about the variability of the data.

The fuzzy control limits for Shewharts R control chart are given by:

$$\tilde{C}L_R = (\overline{R}_a, \overline{R}_b, \overline{R}_c) = (0.008,\ 0.428, 0.848)$$

$$U\tilde{C}L_R = D_4(\overline{R}_a, \overline{R}_b, \overline{R}_c)$$

$$= 2.282(0.008,\ 0.428, 0.848)$$

$$= (0.0183,\ 0.9767, 1.9351)$$

$$L\tilde{C}L_R = D_3(\overline{R}_a, \overline{R}_b, \overline{R}_c)$$

$$= 0(0.008,\ 0.428, 0.848)$$

$$= (0,\ 0, 0)$$

By using α-Cut we get:

$$\overline{R}_a^{0.65} = \overline{R}_a + 0.65(\overline{R}_b - \overline{R}_a) = 0.281$$

$$\overline{R}_c^{0.65} = \overline{R}_c + 0.65(\overline{R}_c - \overline{R}_b) = 0.575$$

Then, the control limits of α- cut fuzzy $\tilde{R}$ control chart given by:

$$\tilde{C}L_{\tilde{R}}^{0.65} = (\overline{R}_c^{0.65}, \overline{R}_b, \overline{R}_c^{0.65}) = (0.281,\ 0.428, 0.575)$$

$$U\tilde{C}L_{\tilde{R}}^{0.65} = D_4(\overline{R}_a^{0.65}, \overline{R}_b, \overline{R}_c^{0.65})$$

$$= 2.282(0.281,\ 0.428, 0.575)$$

$$= (0.6412,\ 0.9767, 1.312)$$

$$L\tilde{C}L_{\tilde{R}}^{0.65} = D_3(\overline{R}_a^{0.65}, \overline{R}_b, \overline{R}_c^{0.65})$$

$$= 0(0.281,\ 0.428, 0.575)$$

$$= (0,\ 0, 0)$$

Now using α- level fuzzy midrange techniques for fuzzy $\tilde{R}$ control chart to transform to crisp numbers as:

$$U\tilde{C}L_{mr-R}^{0.65} = D_4\ f_{mr-R}^{0.65}(\tilde{C}L) = D_4\ (\frac{\overline{R}_a^{0.65} + \overline{R}_c^{0.65}}{2}) = 0.9767$$

$$\tilde{C}L_{mr-R}^{0.65} = f_{mr-R}^{0.65}(\tilde{C}L) = \frac{\overline{R}_a^{0.65} + \overline{R}_c^{0.65}}{2} = 0.428$$

$$L\tilde{C}L_{mr-R}^{0.65} = D_3\ f_{mr-R}^{0.65}(\tilde{C}L) = 0$$

The value of α -level fuzzy midrange of α -cut fuzzy $\tilde{R}$ control chart are given in table (3):

sample	$S_{mr-\overline{R},j}^{\alpha}$	**0<** $S_{mr-\overline{R},j}^{\alpha}$ **< 0.98**	**sample**	$S_{mr-\overline{R},j}^{\alpha}$	**0<** $S_{mr-\overline{R},j}^{\alpha}$ **< 0.98**
1	0.17	In control	16	0.59	In control
2	0.22	In control	17	0.3	In control
3	0.07	In control	18	0.18	In control
4	0.55	In control	19	1.3	out control
5	0.31	In control	20	0.41	In control
6	0.17	In control	21	0.4	In control
7	0.36	In control	22	0.79	In control
8	0.26	In control	23	1.4	out control
9	0.16	In control	24	0.43	In control
10	0.52	In control	25	0.57	In control
11	0.3	In control	26	0.07	In control
12	0.97	In control	27	0.6	In control
13	0.37	In control	28	0.27	In control
14	0.43	In control	29	0.2	In control
15	0.4	In control	30	0.07	In control

TABLE (3): The decision using α-level fuzzy midrange of α -cut fuzzy $\tilde{R}$.

Above table shows that the process was in control with respect to $S_{mr-\overline{R},j}^{0.65}$ for each sample except samples 19^{th} and 23^{th} out of control, as shown in figure 8. So these fuzzy control limits can be used to control the production process and detect small deviations.

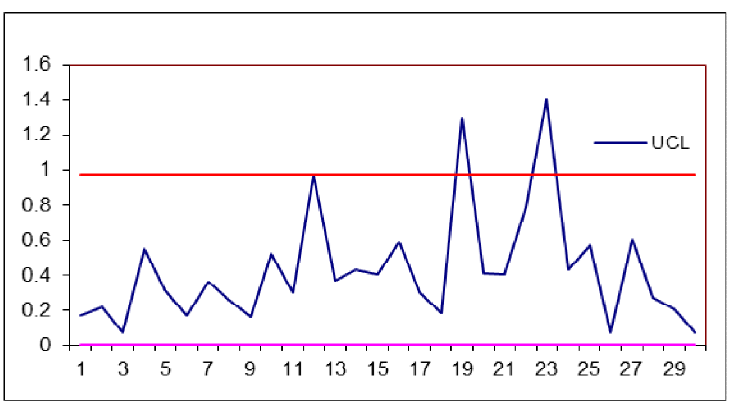

FIGURE 8: α -cut fuzzy $\tilde{R}$ control chart at α-cut fuzzy midrange.

We note that fuzzy observations and fuzzy control limits can provide more flexibility for controlling a process, since reveal small deviations in the production process in addition to large deviations which is important to reducing the deviations between observations.

9. CONCLUSION

This paper shows that fuzzy set theory is useful tool to handle uncertainty and it applicable on traditional variable control charts, such that fuzzy control charts developed for linguistic data that are mainly based on membership and probabilistic approaches and α-cut control charts for limits chart are developed. Fuzzy control charts (Fuzzy control limits) is very effective to identify the signals in the variable control charts, it can provide more flexibility for controlling process and have more appropriate mathematical description frame than control chart approach and give more meaning results than traditional quality control charts. The aim of this study is to present the theoretical structure of the "α-level fuzzy midrange for the α-cut fuzzy control chart", its reveal small deviations in the production process in addition to large deviations which is important to reducing the deviations between observations.

10. REFERECES

[1] A. kanagawa, F.Tamaki, and H. Ohta. "Control charts for process average and variability based on linguistic data". International Journal of Production Research, pp. 913-922, 1993.

[2] J. Oakliand. "Statistical Process Control'. Sixth edition, 2008.

[3] J.H.WANG, and T. RAZ. "On the Construction of Control Charts Using Linguistic Variables". International of Production Research, Pp. 477-487, (1990).

[4] A. Pandurangan, and R. Varadharajan. "Construction of α-cut Fuzzy $\tilde{\bar{X}} - \tilde{R}$ and $\tilde{\bar{X}} - \tilde{S}$ Control Charts Using Fuzzy Trapezoidal Number". Vol. 9, Issue 1, pp. 100-111, 2011.

[5] L.A. Zadeh. "Fuzzy sets, Information and Control". Pp. 338-353, 1965.

[6] A. Saravanan, P.Nagarajan. "α-Cut Fuzzy Control Charts for Bottle Bursting Strength Data". International Journal of Electronics, Communication, Vol. 2, Issue 4, pp. 17-30, 2012.

[7] S. Şentürk. "Fuzzy Regression Control Chart Based on α-cut Approximation". International Journal of Computational Intelligence Systems, Vol.3, No. 1, pp.123-140, 2010.

[8] M. Gülbay, C. Kahraman, and D. Ruan. "α-Cuts fuzzy control charts for linguistic data". International Journal of Intelligent Systems, Vol. 19, Issue 12, pp. 1173-1195, 2004.

[9] M. Moamrni, A. Saghaei, and M.G. Salanghooch. "The Effect of Measurement Error on $\tilde{\bar{X}} - \tilde{R}$ Fuzzy Control Charts". Vol. 2, No. 1, pp.173-176, 2012.