Investigating the Effect of Mutual Coupling on SVD Based Beamforming over MIMO Channels

Feng Wang fwang@itee.uq.edu.au
School of Information Technology & Electrical Engineering
University of Queensland
Brisbane, 4072, Australia

Marek E. Bialkowski meb@ itee.uq.edu.au
School of Information Technology & Electrical Engineering
University of Queensland
Brisbane, 4072, Australia

Xia Liu xialiu@ itee.uq.edu.au
School of Information Technology & Electrical Engineering
University of Queensland
Brisbane, 4072, Australia

Abstract

This paper reports on investigations into a narrowband Multiple-Input Multiple-Output (MIMO) system that applies the Singular Value Decomposition (SVD) beamforming over a Rician channel. Assuming that the two sides of the communication link have a perfect knowledge of Channel State Information (CSI), the system applies the SVD-based beamforming both at the transmitter and receiver. The assessment of the system performance takes into account Mutual Coupling (MC) that is present in the transmitting and receiving array antennas. It is shown that for some particular ranges of Signal to Noise Ratio (SNR) and inter-element spacing, mutual coupling can increase the capacity and thus can be beneficial in terms of decreasing Symbol Error Rate (SER).

Keywords: MIMO, Mutual Coupling, SVD, Beam-forming, LMMSE.

1. INTRODUCTION

Multi-Input Multi-Output (MIMO) systems employing multiple element antennas (MEAs) both at the transmitter and receiver have been proved to offer a significant capacity gain over traditional Single-Input Single-Output (SISO) systems in rich scattering environments [1] [2]. Because of this attribute, the MIMO technique is regarded as one of the most promising techniques for future wireless communications and as such it has received a significant attention from academia and industry in the last decade.

Initial investigations into MIMO systems were carried out by researchers working in the field of information theory [1] [2]. They have shown that the MIMO technique can provide an increased system capacity under Non-Line of Sight (NLOS) signal propagation conditions when the channel matrix is formed by Independent Identically Distributed (*i.i.d*) complex Gaussian entries [1]. In practice, the properties of an actual wireless channel differ from the *i.i.d.* conditions, as signals experience both NLOS and Line Of Sight (LOS) signal propagation paths.

In order to exploit the potential of MIMO, different transmission schemes have been proposed. It has been shown that if the Channel State Information (CSI) is available only at the receiver, Space-time coding [3] [4] can offer considerable benefits to the operation of MIMO system. However, when full or partial CSI is available at the transmitter, a directional beamforming can be a more viable signal transmission scheme [5] [6] [7]. This is because it can increase the average received Signal to Noise Ratio (SNR) due to the array antenna gain. Such benefits have been proved for the case of significant LOS path existing between the transmitter and receiver. However, when NLOS conditions dominate the signal propagation environment, the directional beamforming becomes inferior because of absence of the well defined “desired direction”. In this case, a Singular Value Decomposition (SVD) based beamforming is an attractive alternative. This is because it can better exploit the spatial properties of MIMO channel. When CSI is available at the transmitter and receiver sides, the SVD based beamforming accompanied by linear transmit-receive processing enables transmission of symbols over the eigenmodes of the MIMO channel [2]. This type of signal transmission is often referred to as Multichannel Beam-forming (MB) [9]. In [8], the Schur-concave and Schur-convex functions have been used to design the double beamforming system in which beamforming is applied both at the transmitter and receiver. It has been shown that the channel-diagonalized MB is an optimal transmission scheme. In [10], investigations have been carried out into the combined transmission scheme involving the space-time coding and the SVD based Multichannel Beamforming.

It is worthwhile to note that all of these works have been accomplished using the assumption of “ideal array antennas”, in which many practical issues such as the array geometry and orientation, and the effect of mutual coupling due to finite element spacing are not taken into account. It is known that Mutual Coupling (MC) is always present in multi-element antennas and its effect cannot be ignored especially in tightly spaced arrays. The problem of Mutual Coupling in relation to MIMO systems has been reported in [11] [12] [13] [14]. In [12], the effect of MC has been investigated in relation to the interference rejection capabilities of Beamforming arrays. As a follow up of investigations in [11] and [12], the work in [14] has considered the impact of mutual coupling on the information rate (capacity) when the beamforming signal transmission scheme is applied over a Rician MIMO channel. In this paper, the impact of mutual coupling is investigated with regard to the SVD based multichannel beamforming.

The paper is structured as follows. In section 2, the system model is introduced. Section 3 describes the SVD based multichannel beamforming transmission scheme. The mutual coupling effect is modeled in section 4. The analysis concerning the effect of mutual coupling on the SVD based multichannel beamforming, including numerical results, is presented in section 5. Section 6 concludes the paper and gives suggestions for a future work.

2. SYSTEM MODEL

2.1 MIMO system with double beam-forming

The investigation is undertaken for a flat block-fading narrow-band MIMO system equipped with N transmitting antennas and M receiving antennas both applying the beamforming strategy. For this system, the received baseband signal can be represented as

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{r} \in C^{M\times Q}$ is the received signal matrix, $\mathbf{H} \in C^{M\times N}$ is the complex channel matrix, $\mathbf{s} \in C^{N\times Q}$ is the transmitted symbol matrix, and $\mathbf{n} \in C^{M\times Q}$ is the zero-mean circularly symmetric complex Gaussian noise matrix with power spectral density of $\sigma^2/2$ per element.

Assuming that the transmitter applies the beamforming scheme, the transmitted signal vector can be written as

$$\mathbf{s} = \mathbf{W}_T \mathbf{x} \tag{2}$$

where $\mathbf{W}_T \in C^{N \times P}$ is the transmit beamforming matrix and $\mathbf{x} \in C^{P \times Q}$ is the output symbol matrix of the space-time encoder with P≤min(M,N). The input information symbols are of the form $Z = (z_1, z_2, \cdots, z_L)$, $L \le PQ$.

To keep the transmit power constant, the following condition is imposed

$$\mathbf{W}_T^H \mathbf{W}_T = \mathbf{I}_{P \times P} \tag{3}$$

The signal matrix estimated at the receiver is given by

$$\mathbf{y} = \mathbf{W}_R^H \mathbf{r} \tag{4}$$

where $\mathbf{y} \in C^{P \times Q}$ is the estimated matrix of $\mathbf{x} \in C^{P \times Q}$ and $\mathbf{W}_R \in C^{P \times M}$ is the receiving beamforming matrix. Each column of $\mathbf{W}_R$ can be interpreted as a beamvector adapted to each spatial sub-channel.

2.2 Channel model

It is assumed that array antennas at the transmitter and receiver are formed by linear parallel wire dipoles. The distance between the transmitting and receiving arrays is given by D and an angle between their axes is θ. The received signal is assumed to include both LOS and NLOS components. As a result, the elements h_{rt} of the channel matrix **H** representing the transfer function or channel response between the t^{th} transmitting element and the r^{th} receiving element are given by [11][15]

$$h_{rt} = \sqrt{\frac{1}{1+K}} h_{rt}^{NLOS} + \sqrt{\frac{K}{1+K}} h_{rt}^{LOS} \tag{5}$$

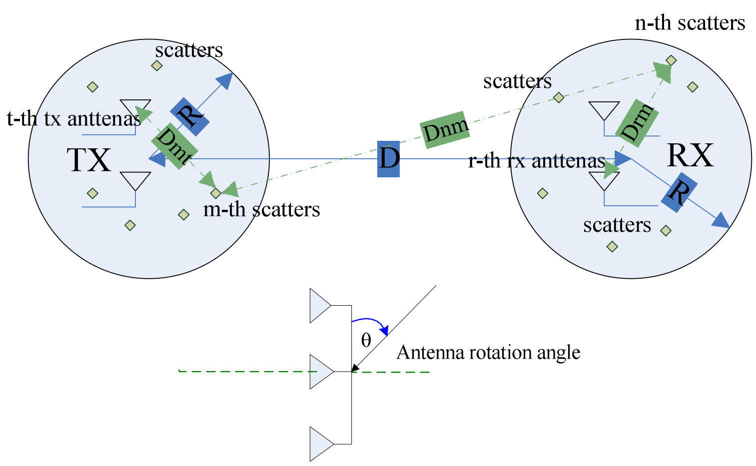

FIGURE 1: Configuration of the investigated MIMO system showing relative orientation of the transmitting and receiving array antennas and a two ring double-bounce scattering signal propagation model.

In the above expression, K is the Rician factor, which is defined as the power ratio between the LOS and NLOS components. The LOS component is represented by

$$h_{rt}^{LOS} = \exp(-j\frac{2\pi}{\lambda}d_{rt}) \tag{6}$$

and the NLOS component is given by

$$h_{rt}^{LOS} = \sqrt{\frac{1}{S_R S_T}}\sum_{m=1}^{S_T}\sum_{n=1}^{S_R}\alpha_{mn}\exp(-j\frac{2\pi}{\lambda}(d_{mt}+d_{nm}+d_{rn})) \tag{7}$$

in which α_{mn} is the scattering coefficient for the path between the m^{th} scatterer at the transmitter side and the n^{th} scatterer at the receiver. This coefficient is assumed to be represented by a normal complex random variable with zero mean and unit variance. d_{rt} is the distance between the t^{th} transmitting antenna and the r^{th} receiving antenna and d_{nm} is the distance between the m^{th} and n^{th} scatterers. d_{mt} is the distance between the m^{th} scatterer and t^{th} transmit antenna. d_{rn} is the distance between the r^{th} receiving antenna and the n^{th} scatterer. It is assumed that S_R scatterers surround the receiver and S_T scatterers encircle the transmitter. Also it is assumed that H is known both to the receiver and transmitter and thus perfect CSI is available at the two sides of the communication link.

3. SVD BASED BEAMFORMING WITH BEAM SELECTIONS

By applying the Singular Value Decomposition (SVD) [16], the MIMO channel matrix $\mathbf{H}\in\mathcal{C}^{M\times N}$ can

be represented in the following form [10]

$$\begin{aligned}\mathbf{H} &= \mathbf{U\Sigma V}^H \\ &= \underbrace{[\mathbf{u}_1\cdots\mathbf{u}_M]}_{\mathbf{U}}\underbrace{\begin{bmatrix}\sigma_1 & & & & & \\ & \ddots & & & & \\ & & \sigma_M & 0 & \cdots & 0\end{bmatrix}}_{\mathbf{\Sigma}}\underbrace{[\mathbf{v}_1\cdots\mathbf{v}_N]}_{\mathbf{V}}^H\end{aligned} \tag{8}$$

where σ_m is the m^{th} non-negative singular value from a set of $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_M$, **U** and **V** are the corresponding singular vector unitary matrices, $\mathbf{U}^H\mathbf{U}\in C^{M\times M}$ and $\mathbf{V}^H\mathbf{V}\in C^{N\times N}$. $(\cdot)^H$ denotes the hermitian operation which implies the transposed complex conjugate of the argument. **V** is the matrix with the columns given by the eigenvectors of $\mathbf{H}^H\mathbf{H}$, which are related to the eigenmodes of the MIMO channel matrix

$$\left(\mathbf{H}^H\mathbf{H}\right)\mathbf{V} = \mathbf{V}\left(\mathbf{\Sigma}^H\mathbf{\Sigma}\right) \tag{9}$$

As assumed earlier, perfect CSI is available at both the receiver and transmitter. Under this assumption, the maximum capacity is achieved by transmitting independent Gaussian codes on the eigenmodes with water-filling power allocation strategy [1] [9].

The SVD-based multichannel beamforming technique exploits the MIMO channels' eigenmodes by employing P ($P\leq M$ and $P\leq N$) columns of matrix **V** which correspond to the P largest eigenmodes (alternatively named singulars). Therefore the transmit beamforming matrix is given by

$$\mathbf{W}_T = \mathbf{V}^P \; and\, \mathbf{s} = \mathbf{V}^p\mathbf{x} \tag{10}$$

In order to represent the received signals in terms of eigenmodes, expression (8) is used and leads to the following

$$\begin{aligned}\mathbf{y} &= \mathbf{U}_P^H\mathbf{r}\\ &= \mathbf{U}_P^H\mathbf{H}\mathbf{V}_P\mathbf{x}+\mathbf{U}_P\mathbf{n}\\ &= \mathbf{\Sigma}_{P\times P}\mathbf{x}\end{aligned} \tag{11}$$

It is important to note that this beamforming method does not always result in a robust transmission scheme, especially when the transmitter has only an approximate knowledge of CSI. This shortfall can be compensated by a beamforming performed at the receiver. The optimal receive beamforming matrix can be obtained by selecting one of the sub-streams' minimum mean square error (MMSE) as the cost function. This results in the *Wiener solution*. Receivers employing the beamforming matrix based on the *Wiener solution* are named LMMSE receivers. The receive beamforming matrix can thus be represented by

$$\mathbf{W}_R = (\mathbf{H}\mathbf{V}_P\mathbf{V}_P^H\mathbf{H}^H+\mathbf{I}_n)^{-1}\mathbf{H}\mathbf{V}_P \tag{12}$$

The MIMO channel matrix **H** can be decomposed into *P* parallel independent sub-channels, each sub-channel bearing one symbol from the transmitted signal matrix. The received signal matrix can be written as

$$\mathbf{y} = \mathbf{W}_R^H\mathbf{H}\mathbf{W}_T+\mathbf{W}_R^H\mathbf{n} \tag{13}$$

The equivalent scalar form of (13) is given by

$$y_{pq} = \varepsilon\sqrt{d_p}\,x_{pq}+n_p, 1\le p\le P \ \ and\ \ 1\le q\le Q \tag{14}$$

where $\varepsilon=1/(K+1)^{0.5}$, y_{pq} is the estimated symbol in the p^{th} row, q^{th} column of the signal matrix, n_p is the p^{th} elements of n, d_p is the p^{th} largest eigen-value of

$$\mathbf{\Gamma} = \left(\varepsilon^2\mathbf{I}_s\right)^{-1}\mathbf{H}^H\mathbf{H} \tag{15}$$

4. SVD based beamforming with mutual coupling

4.1 Modeling of mutual coupling

For the array formed by linear parallel wire dipoles, the mutual coupling matrix can be expressed by [12]

$$\boldsymbol{C} = (Z_A+Z_T)(\mathbf{Z}+Z_T\boldsymbol{I}_M)^{-1} \tag{16}$$

where Z_A is the element impedance in isolation and Z_T is the impedance of the receiver at each element, and is chosen as the complex conjugate of Z_A to obtain the impedance match. Z is the mutual impedance matrix with all the diagonal elements equal to Z_A+Z_T. Its non-diagonal elements Z_{nm} are dependent on the physical parameters of dipoles including length, and distance between them. For a side-by-side array configuration and dipole length equal to 0.5λ, Z_{nm} is given by [11] [12]

$$Z_{mn} = \begin{cases} 30\left[0.5722+\ln(2\beta l)-C_i(2\beta l)\right]+j\left[30S_i(2\beta l)\right], & m=n\\ 30\left[2C_i(u_0)+C_i(u_1)-C_i(u_2)\right]+j\left[302S_i(u_0)-S_i(u_1)-S_i(u_2)\right] & m\ne n\end{cases} \tag{17}$$

where

$$C_i(u) = \int_{\infty}^{u} \frac{\cos(x)}{x} dx$$
$$S_i(u) = \int_{0}^{u} \frac{sin(x)}{x} dx \tag{18}$$

and the constants are given by [13]

$$u_0 = \beta d_h$$
$$u_1 = \beta\left(\sqrt{d_h^2 + l^2} + l\right) \tag{19}$$
$$u_2 = \beta\left(\sqrt{d_h^2 + l^2} - l\right)$$

where d_h is the horizontal distance between the two dipole antennas.

4.2 Effect of mutual coupling on SVD based beamforming

The earlier derived expressions (13) describing the operation of the SVD based beamforming system neglect mutual coupling. The inclusion of mutual coupling is straight forward and can be accomplished by modifying the earlier assumed channel matrix **H** to $\mathbf{H}_{mc}$. As a result, the operation of the SVD based beam-forming system that takes into account the mutual coupling present in the transmitting and receiving array antennas is described by

$$\mathbf{y} = \mathbf{W}_R^H \underbrace{\mathbf{C}_R \mathbf{H} \mathbf{C}_T}_{\mathbf{H}_{mc}} \mathbf{W}_T + \mathbf{W}_R^H \mathbf{n} \tag{20}$$

where $\mathbf{C}_R$ and $\mathbf{C}_T$ represent the mutual coupling at receiver and transmitter respectively.

It is apparent that when the mutual coupling effect is neglected (the case of an ideal antenna array), the mutual coupling matrices are given by unit matrices

$$\mathbf{C}_R = \mathbf{I}_{M\times M} \quad and \quad \mathbf{C}_T = \mathbf{I}_{N\times N} \tag{21}$$

The new channel matrix $\mathbf{H}_{mc}$ that includes mutual coupling effects can be decomposed by the SVD technique to

$$\begin{aligned} \mathbf{H}_{mc} &= \mathbf{C}_R \mathbf{H} \mathbf{C}_T \\ &= \mathbf{U}_{mc} \mathbf{\Sigma}_{mc} \mathbf{V}_{mc}^H \end{aligned} \tag{22}$$

The $\mathbf{H}_{mc}$ matrix dimensions are the same as of the original matrix **H** which neglects the mutual coupling. Due to the presence of mutual coupling, the channel characteristics are changed and the unitary property does not hold any more

$$\begin{aligned} \mathbf{U}_{mc}^H \mathbf{U} &\neq \mathbf{I}_{M\times M} \\ \mathbf{V} \mathbf{V}_{mc}^H &\neq \mathbf{I}_{N\times N} \end{aligned} \tag{23}$$

This means that it is impossible to perfectly separate the received signal into different eigenmodes. This property is named as “cross talk” between eigenmodes [17].

5. NUMERICAL RESULTS

The performance of the SVD based beamforming technique including mutual coupling is assessed via Monte-Carlo simulations. The effect of the Rician factor K and the receive antenna spacing is investigated. Equations (5), (6) and (7) are used to model the channel matrix **H**. The transmitting and receiving array antennas are assumed parallel one to each other and thus the orientation angle θ is equal to 0^o. The distance, D, between the transmitter and the receiver is assumed to be 50λ and the rings enclosing the array antennas are of radius R equal to 16λ, where λ is the carrier wavelength. $S_R=S_T=$ 50 scatterers are assumed to be uniformly distributed in the two rings surrounding the transmitting and receiving array antennas.

Since the SVD based multichannel beamforming utilizes the MIMO channels' eigenmodes (singulars) to transmit signals, it is important to investigate their statistical properties. Figure 2 shows the cumulative distributions for the sum of four singulars of a 4×4 Rician MIMO channel for different values of Rician factor K. Note that in this case, the sum of eigenvalues is related to the capacity of MIMO system. Simulations assume that the array rotation angle is equal to 0^o, which means that the signals are transmitted and received in the broadside direction. The inter-element spacing is 0.5 λ. Two cases are considered when the mutual coupling is neglected (continuous blue lines) and when the mutual coupling is taken into account (dashed red lines).

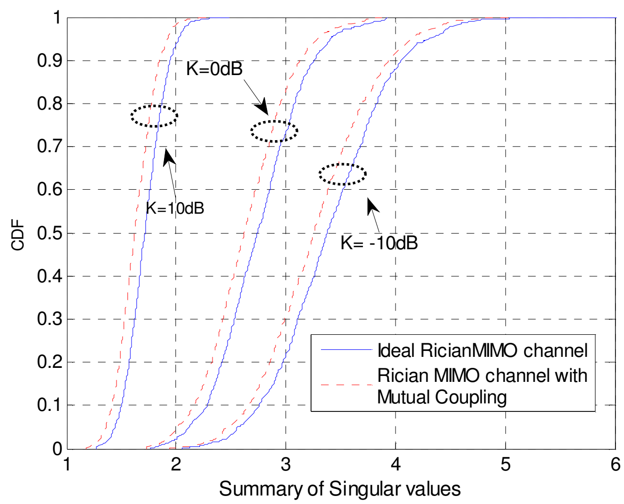

FIGURE 2: CDF of Sum of Singular values for 4×4 MIMO.

It is apparent from the results presented in Figure 2 that irrespective of the value of the Rician factor K, larger sums of singular values indicating a larger capacity are for the case without mutual coupling (blue lines).

Figure 3, Figure 4 and Figure 5 show the probability density of the individual singular values of the 4×4 MIMO channel for different values of Rician factor K=-10dB, 0dB and 10dB.

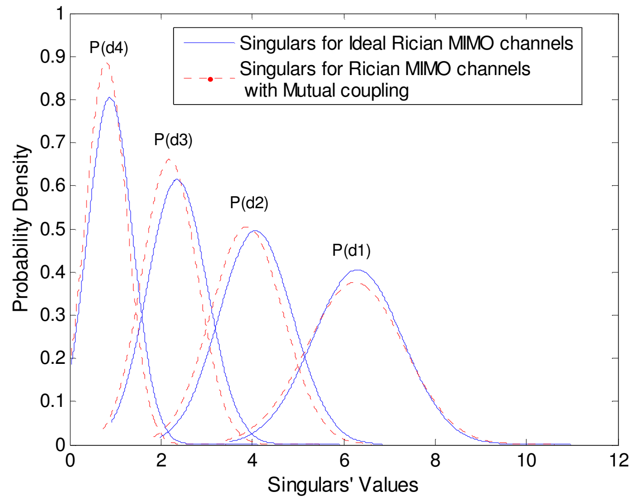

FIGURE 3: PDF of Singular values for 4×4 MIMO (K= -10dB).

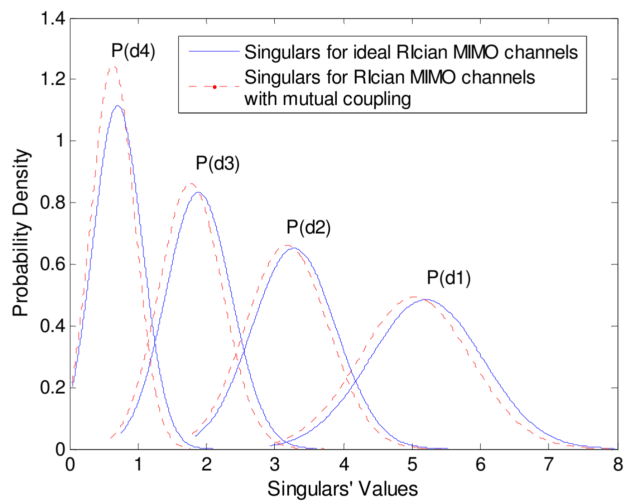

FIGURE 4: PDF of Singular values for 4×4 MIMO (K= 0dB).

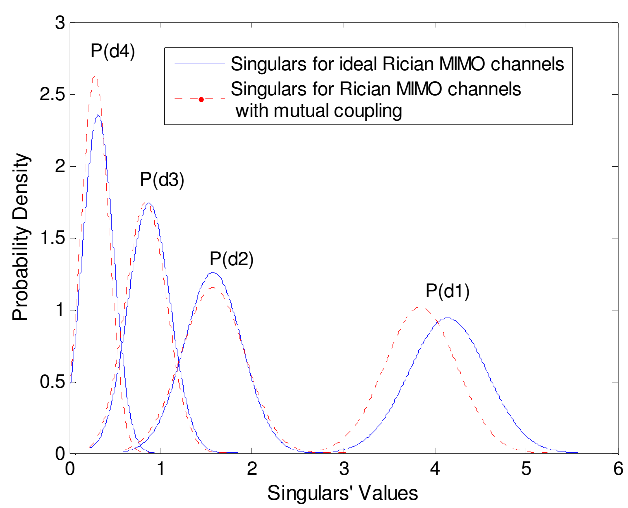

FIGURE 5: PDF of Singular values for 4×4 MIMO (K= 10dB).

It can be seen from the results presented in Figures 3, 4 and 5 that when the mutual coupling effect is taken into consideration (dashed red lines) the singulars are slightly shifted to the left with respect to the singulars when mutual coupling is neglected (continuous blue lines). This shift is caused by the impedance mismatch causing the drop of the singular values. This result is consistent with the results shown in Figure 2. Also seen from the presented figures is that the singulars values show different probability properties for different values of K. When K = -10dB, meaning that NLOS component dominates the Rician channel, all the singular values probability density plots have crossing points with their neighbours. When K = 10dB, meaning that the LOS component dominates the Rician channel, the largest singular value is isolated from the other singular values. This indicates that the largest singular value establishes the dominant position when K is considerably increased.

Figure 6 shows the capacity as a function of SNR for the SVD based beamforming system with a LMMSE receiver operating over a 4×4 MIMO Rician channel. When K = -10dB, meaning that NLOS component dominates the Rician channel, the two capacity plots for the SVD based beamforming with and without mutual coupling cross at 19.35dB. At higher SNR (SNR>19.35dB) the mutual coupling degrades the capacity performance. At lower SNR, (SNR<19.35dB) mutual coupling increases the capacity. When K = 10dB, meaning that LOS component dominates the Rician channel, one can see that the red dashed lines are always below the blue ones, which means that the mutual coupling degrades the capacity.

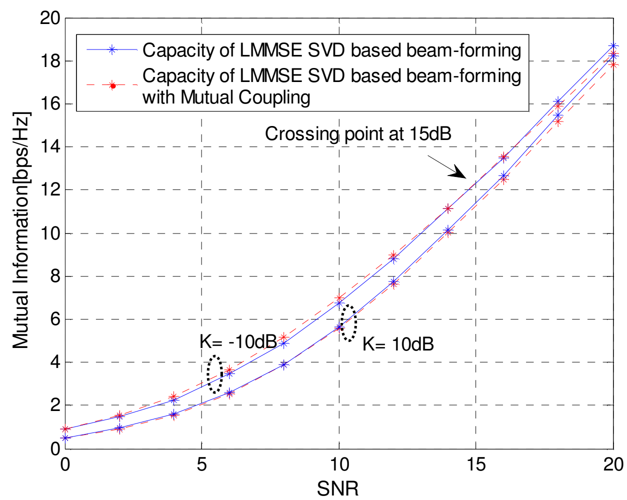

FIGURE 6: Capacity of SVD based beam-forming with LMMSE receiver for

a 4×4 Rician MIMO channels (Inter-element spacing is equal to 0.5λ).

Figure 7 shows the capacity of SVD based beam-forming system with LMMSE receiver operating for over a 4×4 MIMO Rician channel for a varying inter-element spacing.

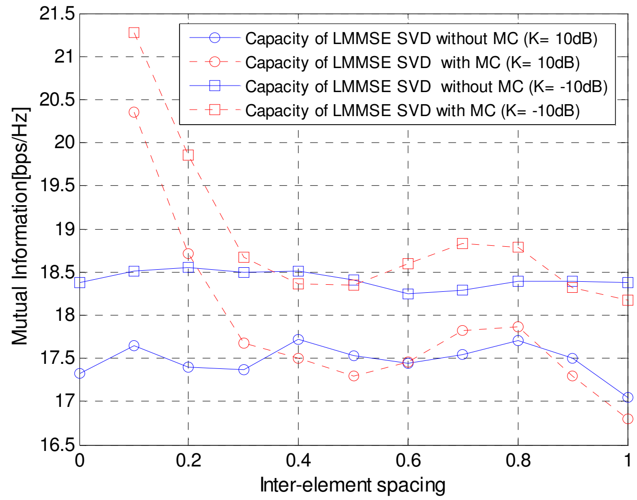

FIGURE 7: Capacity of SVD based beam-forming with LMMSE receiver for a 4×4 Rician MIMO channels

(SNR is equal to 20dB; Inter-element spacing transmitter is1.0λ).

One can see that there are three crossing points for the two plots with and without mutual coupling. For K = 10dB, the plots with and without mutual coupling cross at 0.35λ, 0.6λ and 0.85λ. In the range of inter-element spacing from 0 to 0.35λ and from 0.6λ to 0.85λ, mutual coupling increases the capacity. For k = -10dB, the curves show the same trend, although the crossing points' locations are slightly different. From the results presented in Figure 7 it is apparent that the largest differences in mutual information are observed for the inert-element spacing less than 0.35λ.

Figure 8 shows the SER performance for SVD based beamforming with LMMSE receiver for a 2×2 MIMO Rician channel. The Alamouti coded SVD based beam-forming is applied [4].

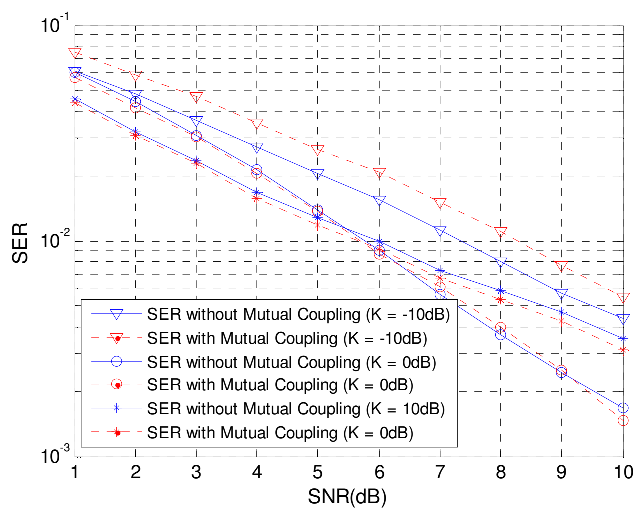

FIGURE 8: SER performance of Alamouti coded SVD based beam-forming with LMMSE receiver for a 2×2

Rician MIMO channels (Inter-element spacing for transmitter and receiver array are 1.0λ and 0.5λ).

The presented plots in Figure 8 show that the Alamouti coded SVD based beamforming with LMMSE receiver has a better SER performance in a scattering-rich environment. This is when the Rician factor takes smaller values. Also it can be seen that the effect of mutual coupling on SER performance depends on the signal propagation environment. When K = 10dB, the LOS component dominates the channel, mutual coupling is beneficial in terms of decreasing SER. This drop can be attributed to the decreased correlation between different antenna elements. When K = -10dB and the NLOS component dominates the channel, the "cross talk" effect becomes non-negligible and the presence of mutual coupling leads to an increased SER. When NLOS and LOS components are comparable (K= 0dB), the SER plots for SVD based beamforming with and without mutual coupling have a cross-point at some value of SNR. At higher SNR, mutual coupling contributes to a better SER performance while at lower SNR mutual coupling is responsible for decreasing the SER performance.

6. CONCLUSIONS

In this paper, investigations have been performed into the SVD based multichannel beamforming technique which takes into account the effects of mutual coupling present in transmitting and receiving array antennas. In these investigations, the transmitting and receiving antennas have been assumed to be in the form of uniform arrays of parallel wire dipoles that are surrounded by circular rings of uniformly distributed scattering objects. In the analysis, it has been postulated that the communication link includes both Line-of-Sight and Non-line of Sight signal propagation paths. The mutual coupling has been taken into account by modifying the channel matrix by the transmitting and receiving antenna coupling matrices.

The assessment of this Multiple-Input Multiple-Output system has been performed via computer simulations assuming different values of the Rician factor and different values of inter-element antenna spacing. The presented simulation results have shown that for some particular ranges of SNR and inter-element spacing, mutual coupling can increase the capacity. Also it can contribute to a better SER performance.

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