A New Enhanced Method of Non Parametric power spectrum Estimation.

K.Suresh Reddy reddysureshk375@rediffmail.com
Associate Professor, ECE Department,
G.Pulla Reddy Engineering College,
Kurnool,518 002 AP, India.

Dr.S.Venkata Chalam sv_chalam2005@yahoo.com
Professor, ECE Department,
Ace college of Engineering,
Hyderabad, 500 003, AP, India..

Dr.B.C.Jinaga bcjinaga@jntu.ac.in
OSD, JNTU,
Hyderabad, AP, India.

Abstract

The spectral analysis of non uniform sampled data sequences using Fourier Periodogram method is the classical approach.In view of data fitting and computational standpoints why the Least squares periodogram (LSP) method is preferable than the “classical” Fourier periodogram and as well as to the frequently-used form of LSP due to Lomb and Scargle is explained. Then a new method of spectral analysis of nonuniform data sequences can be interpreted as an iteratively weighted LSP that makes use of a data-dependent weighting matrix built from the most recent spectral estimate. It is iterative and it makes use of an adaptive (i.e., data-dependent) weighting, we refer to it as the iterative adaptive approach (IAA).LSP and IAA are nonparametric methods that can be used for the spectral analysis of general data sequences with both continuous and discrete spectra. However, they are most suitable for data sequences with discrete spectra (i.e., sinusoidal data), which is the case we emphasize in this paper. Of the existing methods for nonuniform sinusoidal data, Welch, MUSIC and ESPRIT methods appear to be the closest in spirit to the IAA proposed here. Indeed, all these methods make use of the estimated covariance matrix that is computed in the first iteration of IAA from LSP. Comparative study of LSP with MUSIC and ESPRIT methods are discussed.

Keywords: A Nonuniform sampled data, periodogram, least-squares method, iterative adaptive approach, Welch, Music and Esprit spectral analysis.

1. INTRODUCTION

Let the data sequence $\{y(t_n)\}_{n=1}^{N}$ consists of N number of samples whose spectral analysis is our goal. We assume that the observations $\{t_n\}_{n=1}^{N}$ are given, $y(t_n)\varepsilon R(n=1,...N)$ and that a possible nonzero mean has been removed from $\{y(t_n)\}_{n=1}^{N}$, so that $\sum_{n=1}^{N} y(t_n) = 0$. We will also assume throughout this paper that the data sequence consists of a finite number of sinusoidal components and of noise, which is a case of interest in many applications. Note that, while this assumption is not strictly necessary for the nonparametric spectral analysis methods discussed in this paper, these methods perform most satisfactorily when it is satisfied.

2. MOTIVATION FOR THE NEW ESTIMATOR

There are two different non parametric approaches to find the spectral analysis of nonuniform data sequences. First is the classical periodogram approach and the second is Least Squares periodogram approach. The proposed enhanced method of Iterative adaptive approach is explained.

2.1 Classical Periodogram Approach*:* The classical periodogram estimate for the power spectrum of non uniformly sampled data sequence $\{y(t_n)\}_{n=1}^{N}$ of length N can be interpreted by

$$P_{FP}(\omega) = \frac{1}{N}\left|y(t_n)e^{-jwt_n}\right|^2 \qquad (1)$$

Where ω is the frequency variable and where, depending on the application, the normalization factor might be different from 1/N (such as $1/N^2$, see, e.g., [1] and [2]). It can be readily verified that can be obtained from the solution to the following least-squares (LS) data fitting problem:

$$p_F(\omega) = \left|\hat{\beta}(\omega)\right|^2 ,$$

$$\hat{\beta}(\omega) = \min_{\beta(\omega)} \sum_{n=1}^{N} \left| y(t_n - \beta(\omega)e^{-j\omega_n} \right|^2 \qquad (2)$$

In the above (2), if we keep $\beta(\omega) = |\beta(\omega)|e^{j\phi(\omega)}$, the LS criterion can be written as

$$\sum_{n=1}^{N} [y(t_n) - |\beta(\omega)|\cos(\omega t_n + \phi(\omega))]^2$$
$$+ |\beta(\omega)|^2 \sum_{n=1}^{N} \sin^2(\omega t_n + \phi(\omega)) \qquad (3)$$

Minimization of the first term in (3) makes sense, given the sinusoidal data assumption made previously. However, the same cannot be said about the second term in (3), which has no data fitting interpretation and hence only acts as an additive data independent perturbation on the first term.

2.2 The LS Periodogram: It follows from the discussion in the previous subsection that in the case of real-valued (sinusoidal) data, considered in this paper, the use of Fourier Periodogram is not completely suitable, and that a more satisfactory spectral estimate should be obtained by solving the following LS fitting problem:

$$\min_{\alpha} \sum_{n=1}^{N} [y(t_n) - \alpha\cos(\omega t_n + \phi)]^2 \qquad (4)$$

The dependence of α and ω can be eliminated using a = α cos (φ) ; b= -α sin(φ) (5)
so that LS criterion can be written as

$$\min_{a,b} \sum_{n=1}^{N} [y(t_n) - a\cos(\omega t_n) - b\sin(\omega t_n)]^2 \qquad (6)$$

The solution to the minimization problem in (6) is well known to be $\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = R^{-1} r$ (7)

Where
$$R = \sum_{n=1}^{N} \begin{bmatrix} \cos(\omega t_n) \\ \sin(\omega t_n) \end{bmatrix} \begin{bmatrix} \cos(\omega t_n) & \sin(\omega t_n) \end{bmatrix} \quad (8)$$

and
$$r = \sum_{n=1}^{N} \begin{bmatrix} \cos(\omega t_n) \\ \sin(\omega t_n) \end{bmatrix} y(t_n) \quad (9)$$

The power of the sinusoidal frequency component ω Can be given as

$$\frac{1}{N} \sum_{n=1}^{N} \left(\begin{bmatrix} \hat{a} & \hat{b} \end{bmatrix} \begin{bmatrix} \cos(\omega t_n) \\ \sin(\omega t_n) \end{bmatrix} \right)^2$$
$$= \frac{1}{N} \begin{bmatrix} \hat{a} & \hat{b} \end{bmatrix} R \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}$$
$$= \frac{1}{N} r^T R^{-1} r \quad (10)$$

Hence the periodogram for Least Squares Criterion can be given as

$$p_{LSP}(\omega) = \frac{1}{N} r^T(\omega) R(\omega) r(\omega) \quad (11)$$

The LSP has been discussed, for example, in [3]–[8], under different forms and including various generalized versions. In particular, the papers [6] and [8] introduced a special case of LSP that has received significant attention in the subsequent literature.

2.3 Iterative Adaptive Approach: The algorithm for the proposed estimate is discussed as with the notations. Let $\Delta\omega$ denote the step size of the grid considered for the frequency variable, and let $K = \frac{\omega_{\max}}{\Delta_\omega}$ denote the number of the grid points needed to cover the frequency interval $[0, \omega_{\max}]$, where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x ; also, let $\omega_k = k\Delta_\omega$ for k=1,…,K.

$$Y = \begin{bmatrix} y(t_1) \\ y(t_2) \\ . \\ . \\ y(tn) \end{bmatrix}, \ \theta_k = \begin{bmatrix} a(\omega_k) \\ b(\omega_k) \end{bmatrix}, \ A_K = \begin{bmatrix} c_k & s_k \end{bmatrix},$$

$$c_k = \begin{bmatrix} \cos(\omega_k t_1) \\ . \\ \cos(\omega_n t_n) \end{bmatrix}, s_k = \begin{bmatrix} \sin(\omega_k t_1) \\ . \\ \sin(\omega_n t_n) \end{bmatrix} \quad (12)$$

Using this notation we can write the Least squares criterion in (6) as follows in the vector form at, $\omega = \omega_k$

$$\left\| Y - A_K \theta_K \right\|^2 \quad (13)$$

Where $\|\cdot\|$ denotes the Euclidean norm. The LS estimate of θ_k in (7) can be rewritten as

$$\hat{\theta}_k = \left(A_k^T A_k \right)^{-1} A_k^T Y. \quad (14)$$

In addition to the sinusoidal component with frequency ω_k ,the data of Y also consists of other sinusoidal components with frequencies different from ω_k. as well as noise. Regarding the latter, we do not consider a noise component of explicitly, but rather implicitly via its contributions to the data spectrum at $\{\omega_k\}_{k=1}^{K}$; for typical values of the signal-to-noise ratio, these noise contributions to the spectrum are comparatively small. Let us define

$$Q_k = \sum_{P=1, P\neq K}^{K} \left(A_P D_P A_P{}^T\right); \tag{15}$$

$$D_P = \frac{a^2(\omega_p) + a^2(\omega_p)}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{16}$$

which can be thought of as the covariance matrix of the other possible components in Y, besides the sinusoidal component with frequency ω_k considered in (13).

In some applications, the covariance matrix of the noise component of Y is known (or, rather, can be assumed with a good approximation) to be

$$\sum = \begin{bmatrix} \sigma_1^2 & . & . & . & 0 \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ 0 & . & . & . & \sigma_N^2 \end{bmatrix} \text{, with given } \left\{\sigma_n^2\right\}_{n=1}^{N} \tag{17}$$

In such cases, we can simply add $\sum$ to the matrix Q_k in (16).Assuming Q_k that is available, and that it is invertible, it would make sense to consider the following weighted LS (WLS) criterion, instead of (13),

$$[Y - A_k \theta_k]^T Q_k^{-1} [Y - A_k \theta_k] \tag{18}$$

It is well known that the estimate of $\boldsymbol{\theta_k}$ obtained by minimizing (18) is more accurate, under quite general conditions, than the LS estimate obtained from (13).Note that a necessary condition for $\mathbf{Q}_k^{-1}$ to exist is that (2K-1)>N, which is easily satisfied in general.

The vector that minimizes (18) can be given by

$$\hat{Q}_k = \left(A_k^T Q_k^{-1} A_k\right)^{-1} \left(A_k^T Q_k^{-1} Y\right). \tag{19}$$

Similar to that of (11) the IAA estimate which makes us of Weighted Least Squares an be given by

$$P_{IAA} = \frac{1}{N} \hat{\theta}_k^T (A_k^T A_k) \hat{\theta}_k \tag{20}$$

The IAA estimate in (20) requires the inversion of NXN matrix Q_k for k=1, 2,…, K and also N≥1 which is computationally an intensive task.

To show how we can simply reduce the computational complexity of (19), let us introduce the matrix

$$\psi = \sum_{p=1}^{K} \left(A_p D_P A_P^T\right) = Q_k + A_k D_k A_k^T \tag{21}$$

A simple calculation shows that

$$Q_k^{-1} A_k = \psi^{-1} A_k \left(I + D_k A_k^T Q_k^{-1} A_k\right). \tag{22}$$

To verify this equation, premultiply it with

The ψ in (21) and observe that $\psi Q_k^{-1} A_k = A_k + A_k D_k A_k^T Q_k^{-1} A_k$

$$= A_k \left(I + A_k D_k A_k^T Q_k^{-1} A_k\right) \tag{23}$$

Inserting (22) in (19) yields the another expression for the IAA estimate

$$\hat{\theta}_k = \left(A_k^T \psi^{-1} A_k\right)^{-1} \left(A_k^T \psi^{-1} Y\right) \tag{24}$$

This is more efficient than in (19) computationally.

2.4 Demerits of Fourier Periodogram and LSP:

The spectral estimates obtained with either FP or LSP suffer from both local and global (or distant) leakage problems. Local leakage is due to the width of the main beam of the spectral window, and it is what limits the resolution capability of the periodogram. Global leakage is due to the side lobes of the spectral window, and is what causes spurious peaks to occur (which leads to "false alarms") and small peaks to drown in the leakage from large peaks (which leads to "misses"). Additionally, there is no satisfactory procedure for testing the significance of the periodogram peaks. In the uniformly sampled data case, there is a relatively well-established test for the significance of the most dominant peak of the periodogram; see [1], [2], and [13] and the references therein. In the nonuniform sampled data case, [8] (see also [14] for a more recent account) has proposed a test that mimics the uniform data case test mentioned above. However, it appears that the said test is not readily applicable to the nonuniform data case; see [13] and the references therein. As a matter of fact, even if the test were applicable, it would only be able to decide whether $\{y(t_n)\}$ are white noise samples, and not whether the data sequence contains one or several sinusoidal components (we remark in passing on the fact that, even in the uniform data case, testing the existence of multiple sinusoidal components, i.e., the significance of the second largest peak of the periodogram, and so forth, is rather intricate [1], [2]). The only way of correcting the test, to make it applicable to nonuniform data, appears to be via Monte Carlo simulations, which may be a rather computationally intensive task (see [13]) The main contribution of the present paper is the introduction of a new method for spectral estimation and detection in the nonuniform sampled data case, that does not suffer from the above drawbacks of the periodogram (i.e., poor resolution due to local leakage through the main lobe of the spectral window, significant global leakage through the side lobes, and lack of satisfactory tests for the significance of the dominant peaks). A pre- view of what the paper contains is as follows.

Both LSP and IAA provide nonparametric spectral estimates in the form of an estimated amplitude spectrum (or periodogram$P(\omega)$). We use the frequencies and amplitudes corresponding to the dominant peaks of $P(\omega)$ (first the largest one, then the second largest, and so on) in a Bayesian information criterion see, e.g., [19] and the references therein, to decide which peaks we should retain and which ones we can discard. The combined methods, viz. LSP BIC and IAA BIC, provide parametric spectral estimates in the form of a number of estimated sinusoidal components that are deemed to fit the data well. Therefore, the use of BIC in the outlined manner not only bypasses the need for testing the significance of the periodogram peaks in the manner of [8] (which would be an intractable problem for RIAA, and almost an intractable one for LSP as well—see [13]), but it also provides additional information in the form of an estimated number of sinusoidal components, which no periodogram test of the type discussed in the cited references can really provide.

Finally, we present a method for designing an optimal sampling pattern that minimizes an objective function based on the spectral window. In doing so, we assume that a sufficient number of observations are already available, from which we can get a reasonably accurate spectral estimate. We make use of this spectral estimate to design the sampling times when future measurements should be per- formed. The literature is relatively scarce in papers that ap- proach the sampling pattern design problem (see, e.g., [8] and [20]). One reason for this may be that, as explained later on, spectral window-based criteria are relatively in- sensitive to the sampling pattern, unless prior information (such as a spectral estimate) is assumed to be available—as in this paper. Another reason may be the fact that measure- ment plans might be difficult to realize in some applications, due to factors that are beyond the control of the experimenter. However, this is not a serious problem for the sampling pattern design strategy proposed here which is flexible enough to tackle cases with missed measurements by revising the measurement plan on the fly.

The amplitude and phase estimation (APES) method, proposed in [15] for uniformly sampled data, has significantly less leakage (both local and global) than the periodogram. We follow here the ideas in [16]–[18] to extend APES to the nonuniformly sampled data case. The so-obtained generalized method is referred to as RIAA for reasons explained in the Abstract.

2.5 The Iterative Adaptive Algorithm: The proposed algorithm for power spectrum estimation can be explained as follows

- Initialization: Using the Least Squares method in (13) obtain the initial estimates of $\{\theta_k\}$ which are denoted by $\left\{\hat{\theta}_k^0\right\}$.
- Iteration: Let $\left\{\hat{\theta}_k^i\right\}$ denote the estimates of $\{\theta_k\}$ at the i[th] iteration (i=0, 1, 2…), and let $\left\{\hat{\psi}^i\right\}$ denote the estimate of ψ obtained from $\left\{\hat{\theta}_k^i\right\}$.
- For i=0, 1, 2.., Compute: $\hat{\theta}_k^{i+1} = \left[A_k^T (\hat{\psi}^i)^{-1} A_k\right]^{-1} \left[A_k^T (\hat{\psi}^i)^{-1} y\right]$ for k=1,…K.

  Until a given number of iterations are performed.
- Periodogram calculations:

  Let $\left\{\hat{\theta}_k^I\right\}$ denotes the estimation of $\{\theta_k\}$ Obtained by the iterative process (I denote iteration number at which iteration is stopped). Using $\left\{\hat{\theta}_k^I\right\}$ compute the IAA periodogram as

$$P_{IAA}(\omega_k) = \frac{1}{N}\left(\hat{\theta}_k^I\right)^T \left(A_k^T A_k\right)\left(\hat{\theta}_k^I\right), \quad \text{for k=1,… K.}$$

3. PROPOSED SYSTEM AND SIMULATED DATA:

The system model for the proposed algorithm is shown in Figure 1.

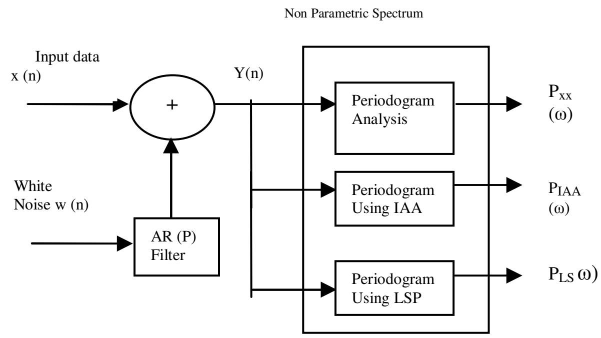


FIGURE 1: Proposed system model for the simulated data.

The system model for the proposed algorithm is shown in Figure 1. We consider a data sequence consisting of M=3 sinusoidal components with frequencies 0.1, 0.4 and 0.41 Hz, and amplitudes 2,4 and 5, respectively. The phases of the three sinusoids are independently and uniformly distributed over $[0,2\pi]$ and the additive noise is white normally distributed with mean of 0 and variance of σ^2=0.01. We define the signal-to-noise ratio (SNR) of each sinusoid as

$$SNR_m = 10\log_{10}\left(\frac{\alpha_m^2/2}{\sigma^2}\right) \text{ dB} \quad m=1,2,3. \tag{25}$$

Where α_m is the amplitude of the mth sinusoidal component hence SNR1=23 dB, SNR2=29 dB and SNR3= 31 dB in this simulation example. The input data sequence for the system model is as follows

$$x(t) = 2\cos(2\pi 0.1t) + 3\cos(2\pi 0.4t) + 4\cos(2\pi 0.4t) + w(t) \tag{26}$$

Where $w(t)$ zero mean Gaussian is distributed white noise with variance of 0.01 and the sampling pattern follows a Poisson process with parameter $\lambda = 0.1s^{-1}$, that is, the sampling intervals are exponentially distributed with mean $\mu = \frac{1}{\lambda} = 10$ s. We choose N=64 and show the sampling pattern in Fig. 3(a). Note the highly irregular sampling intervals, which range from 0.2 to 51.2 s with mean value 9.3 s. Fig. 3(b) shows the spectral window corresponding to Fig. 3(a). The smallest frequency at which the spectral $f_0 \rangle 0$ at which the spectral window has a peak close to N^2 is approximately 10 Hz. Hence $f_{\max}=f_0/2$=5Hz. The step Δf of the frequency grid is chosen as 0.005 Hz. However, they are most suitable for data sequences with discrete spectra (i.e., sinusoidal data), which is the case we emphasize in this paper. Of the existing methods for nonuniform sinusoidal data, Welch, MUSIC and ESPRIT methods appear to be the closest in spirit to the IAA.

4. RESULT ANALYSIS:

The results in Fig. 2 presents the spectral estimates averaged over 100 independent realizations of Monte-Carlo trials of periodogram and Welch estimates. Fig. 4 presents the spectral estimates averaged over 100 independent realizations of LSP and IAA estimates. Fig. 5 presents the spectral estimates averaged over 100 independent realizations of Monte- Carlo trials of Music and Esprit estimates. LSP nearly misses the smallest sinusoid while IAA successfully resolves all three sinusoids. Note that IAA suffers from much less variability than LSP from one trial to another. The plots were taken with the help MATLAB programming by the authors. LSP and IAA are nonparametric methods that can be used for the spectral analysis of general data sequences with both continuous and discrete spectra. However, they are most suitable for data sequences with discrete spectra (i.e., sinusoidal data), which is the case we emphasize in this paper. Of the existing methods for nonuniform sinusoidal data, Welch, MUSIC and ESPRIT methods appear to be the closest in spirit to the IAA proposed here. Indeed, all these methods make use of the estimated covariance matrix that is computed in the first iteration of IAA from LSP. MUSIC and ESPRIT, on the other hand, are parametric methods that require a guess of the number of sinusoidal components present in the data, otherwise they cannot be used furthermore.

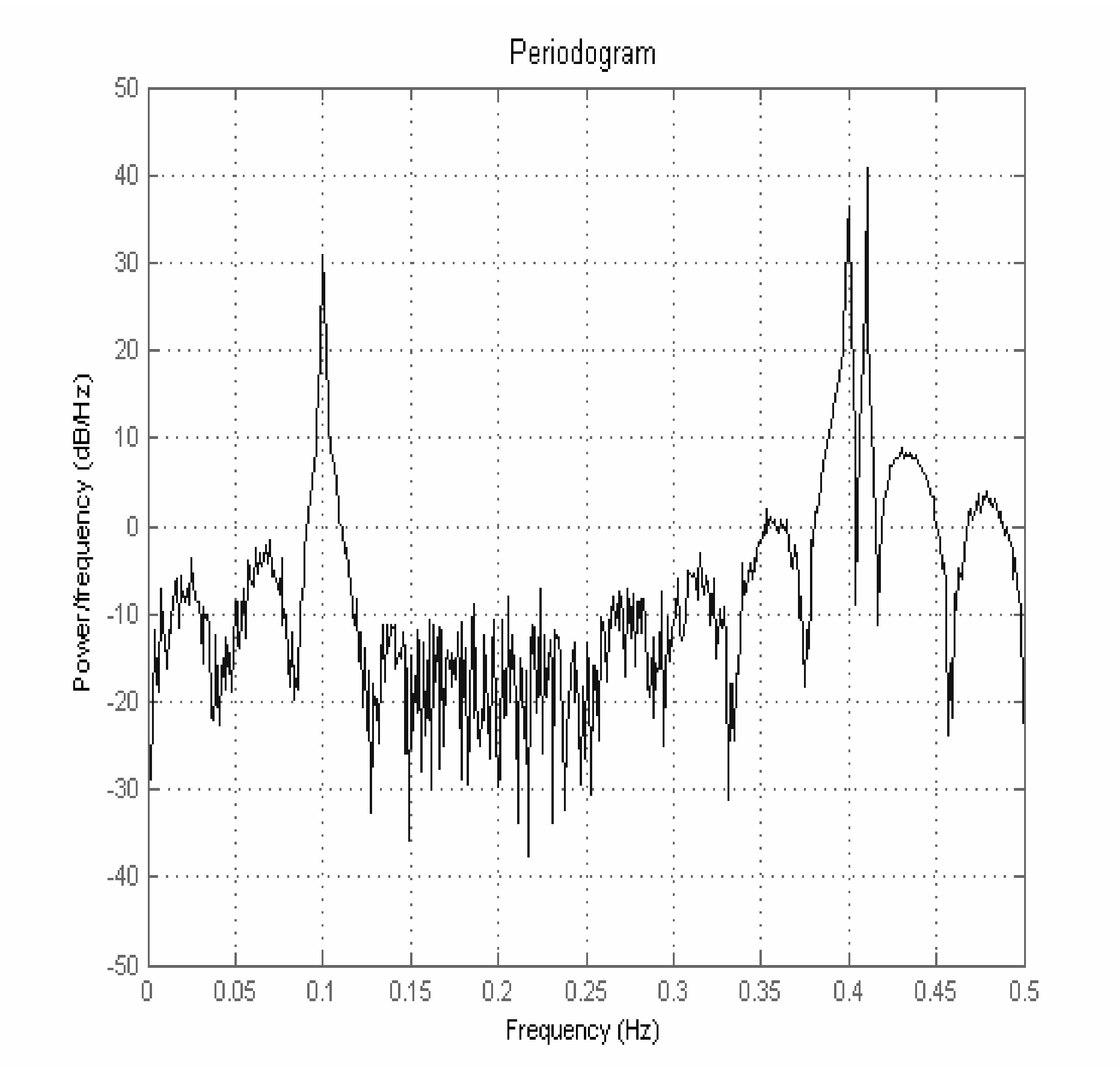


FIGURE 2: Average spectral estimates from 100 Monte Carlo trials of Fourier periodogram

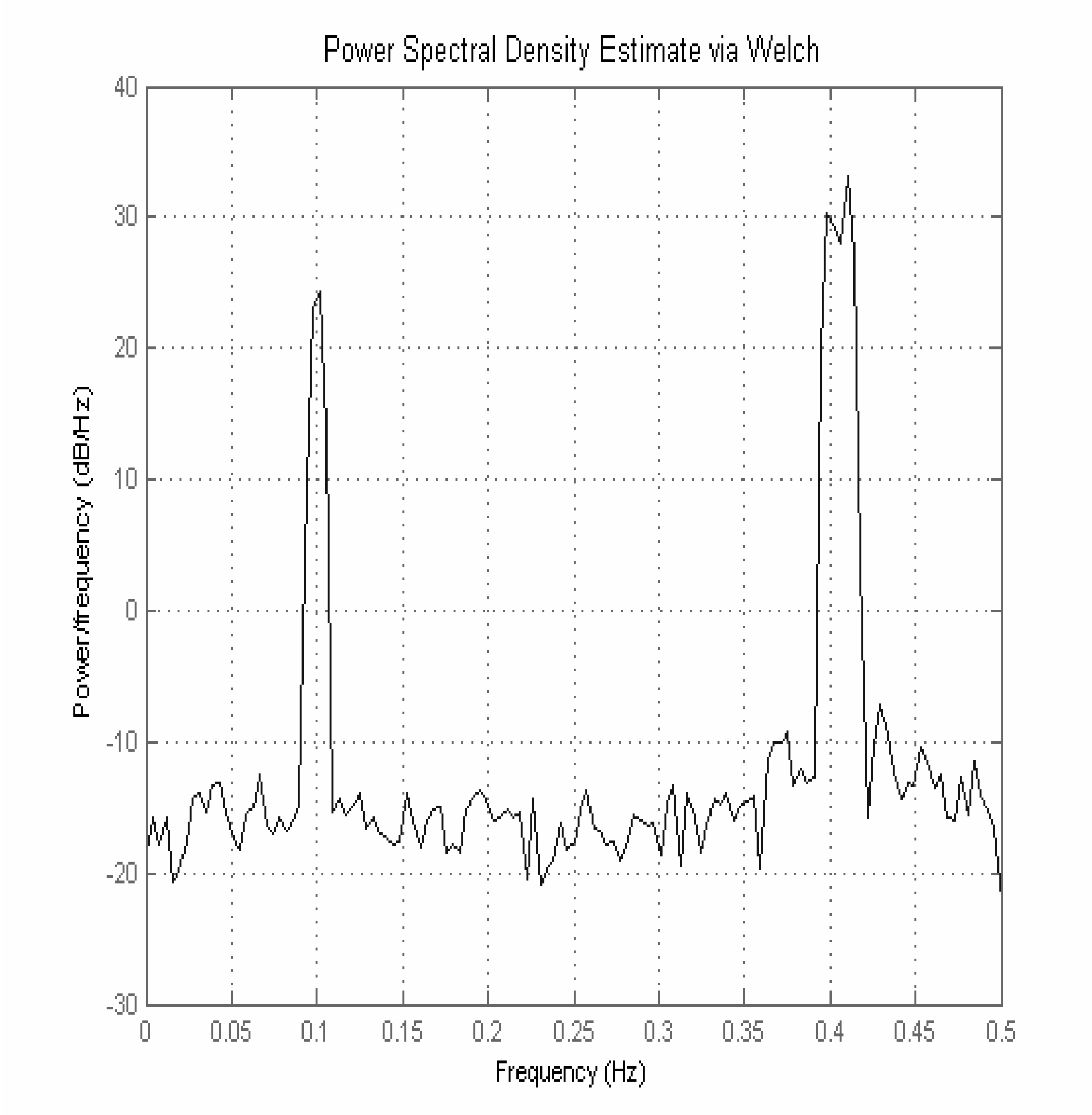


FIGURE 3: Average spectral estimates from 100 Monte Carlo trials of Welch estimates.

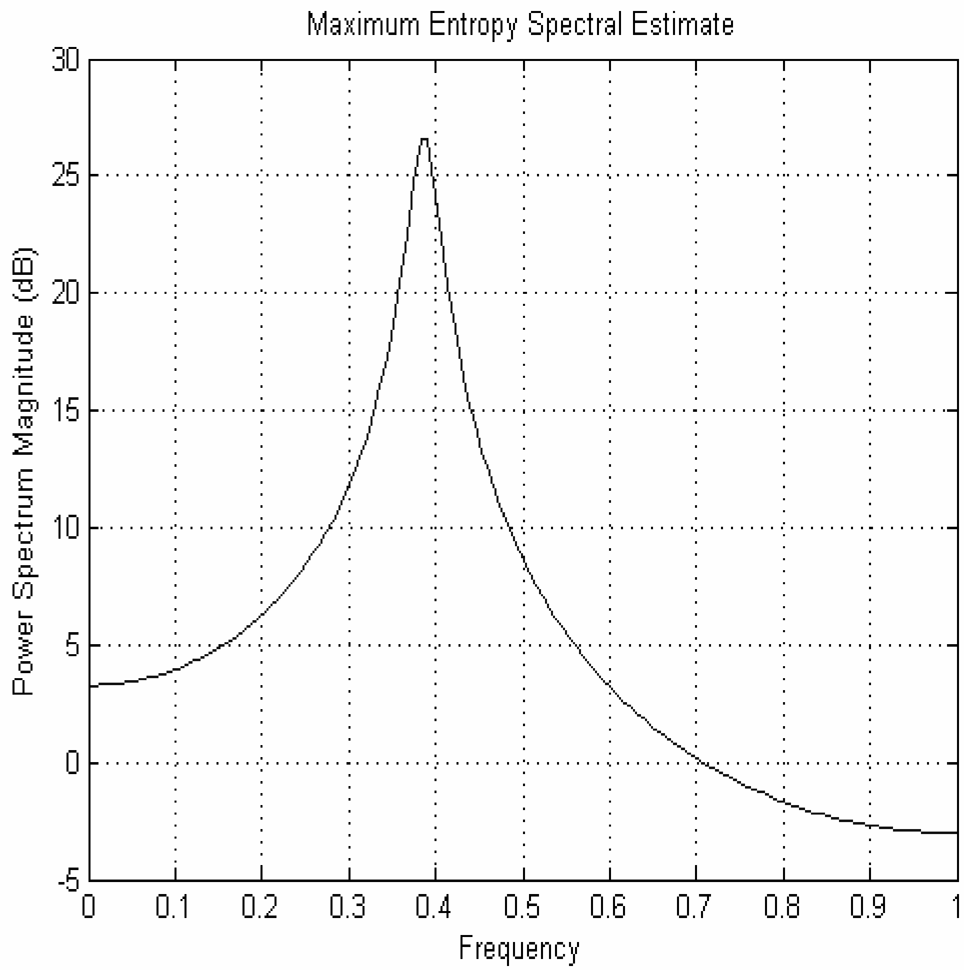
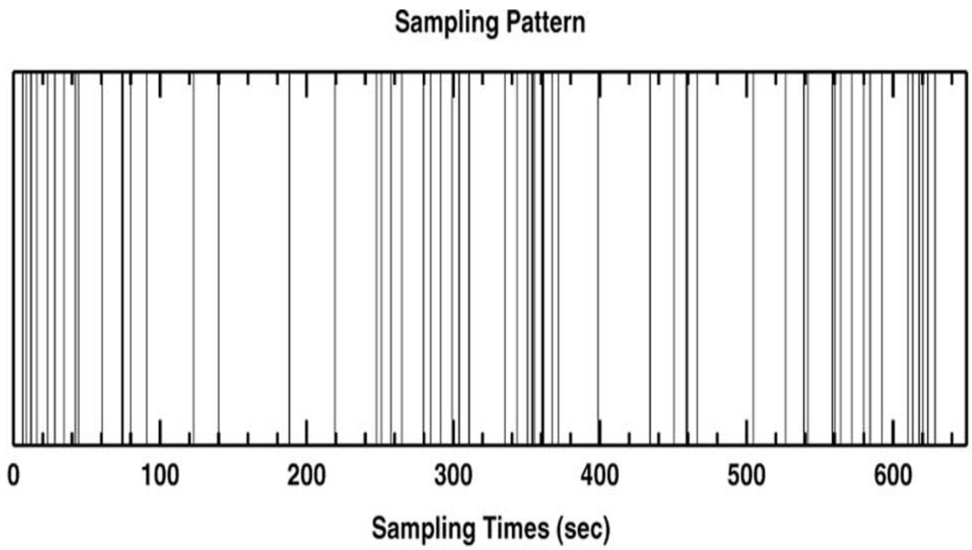
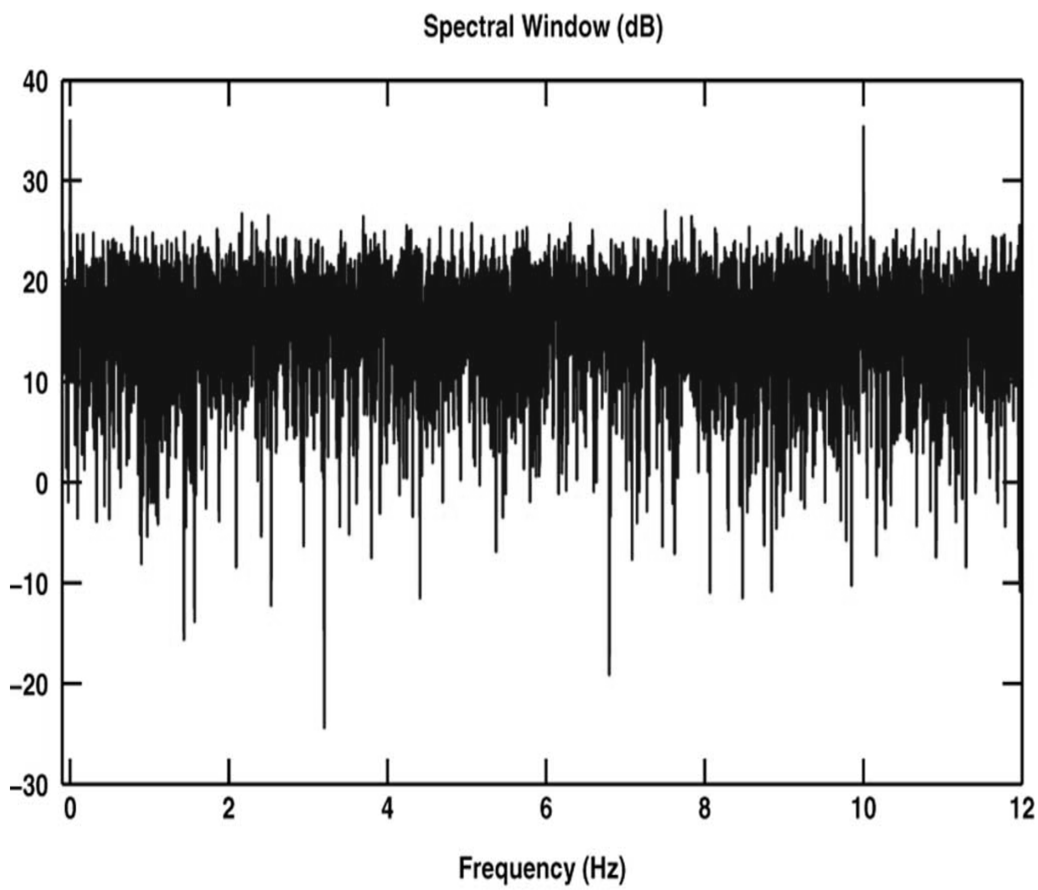


FIGURE 4: Average spectral estimates from 100 Monte Carlo trials of MEM estimates.



(a)



(b)

FIGURE6: Sampling pattern and spectral window for the simulated data case. (a) The sampling pattern used for all Monte Carlo trials in Figs. 2–4. The distance between two consecutive bars represents the sampling interval. (b) The corresponding spectral window

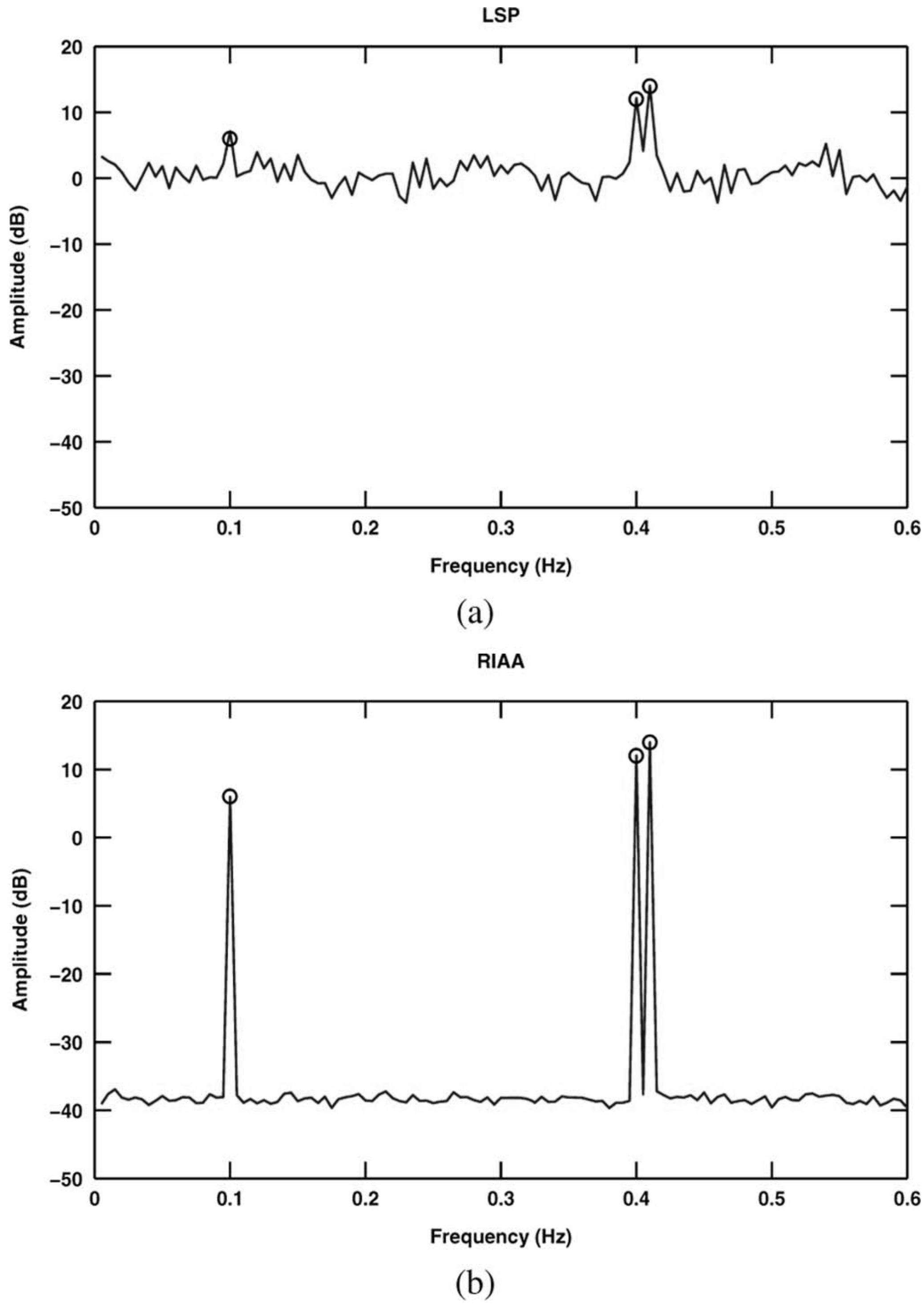
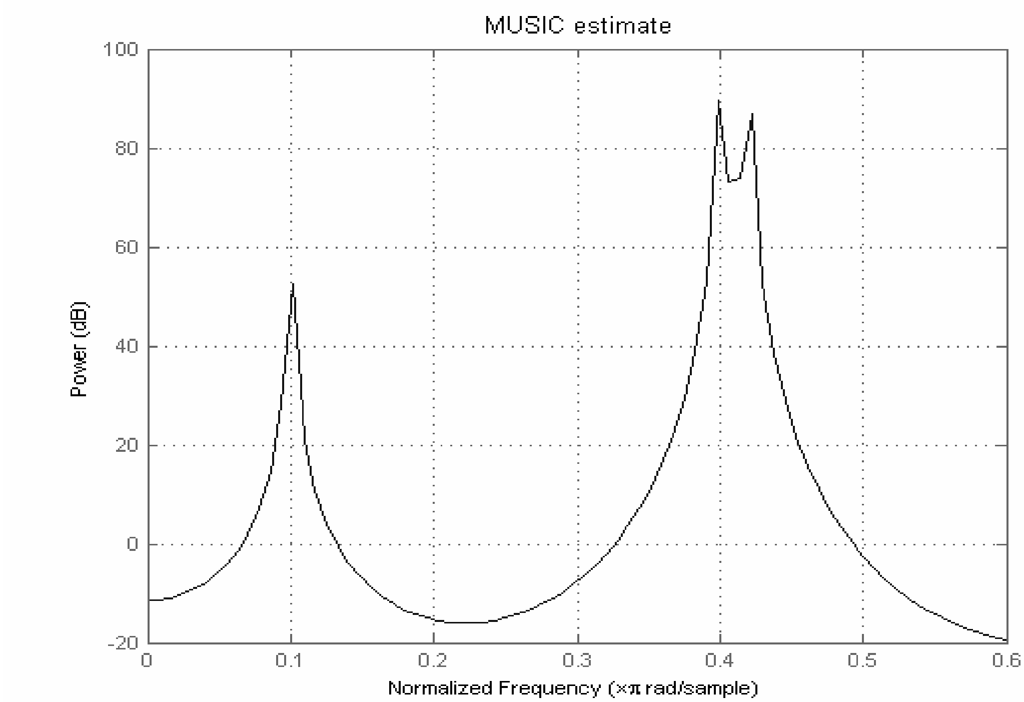
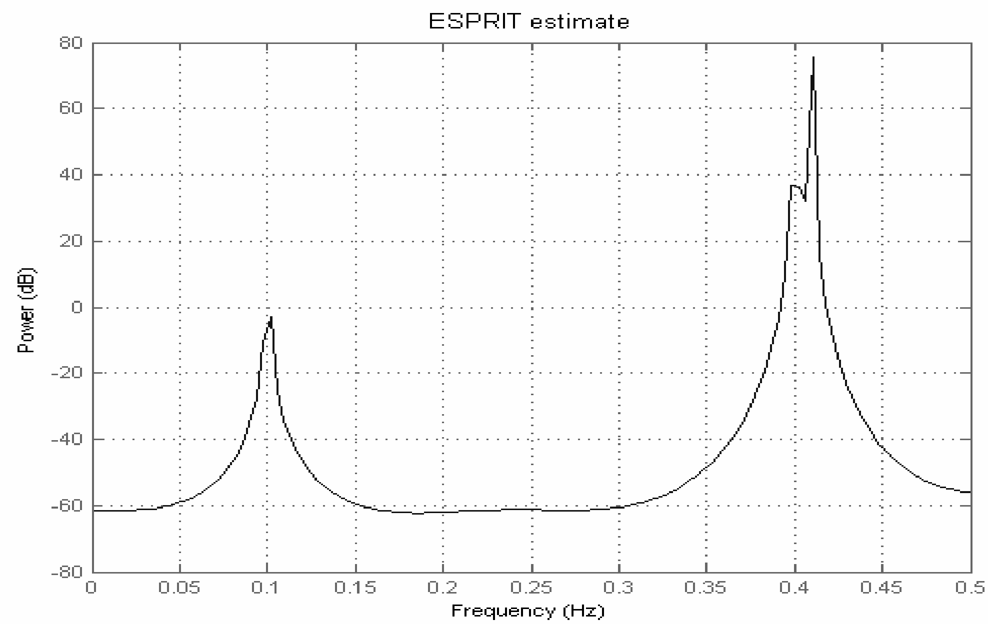


FIGURE7: Average spectral estimates from 100 Monte Carlo trials. The solid line is the estimated spectrum and the circles represent the true frequencies and amplitudes of the three sinusoids. (a) LSP (b) IAA.





(a)

(b)

FIGURE8: Average spectral estimates from 100 Monte Carlo trials. (a) Music estimate and (b) Esprit estimate.

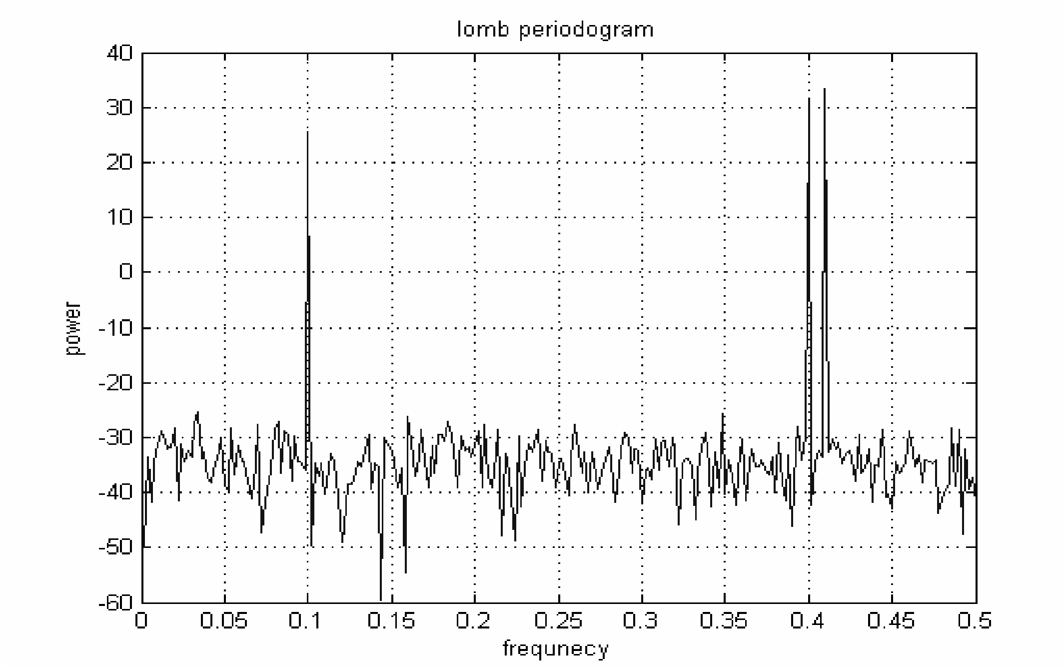


FIGURE9: Average spectral estimates from 100 Monte Carlo trials of Lomb periodogram.

. 4. CONSLUSIONS:

Of the existing methods for nonuniform sinusoidal data, the MUSIC and ESPRIT methods appear to be the closest in spirit to the IAA proposed here (see the cited paper for explanations of the acronyms used to designate these methods). Indeed, all these methods make use of the estimated covariance matrix that is computed in the first iteration IAA from LSP. In fact Welch (when used with the same covariance matrix dimension as IAA) is essentially identical to the first iteration of IAA. MUSIC and ESPRIT.In the case of a single sinusoidal signal in white Gaussian noise, the LSP is equivalent to the method of maximum likelihood and therefore it is asymptotically statistically efficient. Consequently, in this case LSP can be expected to outperform IAA. In numerical computations we have observed that LSP tends to be somewhat better than IAA for relatively large values of N or SNR; however, we have also observed that, even under these conditions that are ideal for LSP, the performance of IAA in terms of MSE (mean squared error) is slightly better (by a fraction of a dB) than that of LSP when or SNR becomes smaller than a certain threshold.

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