Time Domain Analysis and Synthesis Using P^{th} Norm Filter Design

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Abstract

In this paper, a new approach for the design and implementation of FIR filter banks for multirate analysis and synthesis is explored. The method is based on the least P^{th} algorithm and takes into consideration the characteristics of the individual filters. Features of the proposed approach include; it does not need to adapt the weighting function involved and no constraints are imposed during the course of optimization. Mostly, the FIR filter design is concentrated around linear phase characteristics but with the help of minimax solution for FIR filters using the least-P^{th} algorithm, this optimal filter design approach helps us to enhance the properties of LTI systems with good stability. Hence P^{th} norm algorithm will be used in multirate to explore the stability and other properties. We have proposed the band analysis system for analysis and synthesis purpose to explore multirate filter banks. The Matlab toolbox has been used for implementing the filters and its properties are verified with various plots and tables. The results of this paper enable us to achieve good signal to noise ratio (SNR_r) with analysis and synthesis level operations.

Keywords: Analysis, Synthesis, Filter banks, least P^{th} algorithm,

1. INTRODUCTION

Over the last two decades, there has been a steady growth of interest in multirate processing of digital signals. Unlike the single-rate system, the sample spacing in the multirate system can vary from point to point [1]. In multirate filter banks the input signal is divided into channels using band pass filters, and the individual channels are down sampled to rates appropriate to the individual channel bandwidths. The problem of designing filter banks that can provide good frequency resolution while allowing for exact or near perfect reconstruction of the signal is quite challenging because so many dissimilar types of distortion must be minimized and/or eliminated in the same design context. The individual filter issues include the passband size and ripple, stopband size and ripple, transition width and shape, filter phase, filter type (IIR and FIR), filter order, and filter structure. The filter bank issues include the number of bands, frequency coverage, bank efficiency and aliasing, distortion issues include linear distortions (magnitude, phase, and aliasing distortions) and nonlinear distortions (quantization, coding and channel distortions) and the overall processing issues include the system delay and the ability to reject processing distortions [2]. There are several important issues to consider that impact the performance and cost effectiveness of analysis/synthesis filter banks in practical applications. First, the quality of the individual filters in both banks should be good. Typically, this means having high stop band attenuation, good transition band properties and/or good impulse response characteristics. Second, the overall analysis/

synthesis system should reconstruct the input with negligible distortion in the absence of quantization. Third, the filter banks should have an efficient implementation. This impacts the speed of the system as well as the cost. Finally, the overall system delay should be considered. A variety of methodologies based on time domain as well as frequency domain representations are now available [2], [3], [4], but all of these methods have limitations. This paper discusses a new approach using the least P^{th} algorithm to explore the stability and various properties by using the Matlab toolbox for designing various filter banks. The resulting design methodology is uniquely flexible and powerful. By imposing appropriate constraints, a very broad class of FIR analysis /synthesis systems can be designed.

The most efficient tools for the minimax design of FIR digital filters are the Parks-McClellan algorithm and its variants [5]-[7]. However, they only apply to the class of linear phase FIR filters. In many applications, nonlinear phase FIR filters (e.g. those with low group delay) are more desirable. For the minimax design of FIR filters with arbitrary magnitude and phase response several methods are available in the literature. Among others, we mention the weighted least-squares approach [8] in which the weighted function is adapted until a near equiripple filter performance is achieved; the constrained optimization approach [9] in which the design is formulated as a linear or quadratic programming problem; the semidefinite programming approach[10] where the design is accomplished by minimizing an approximation – error bound subject to a set of linear and quadratic constraints that can be converted into linear matrix inequalities. For the 1-D case, minimax design of 1-D FIR filters has been largely focused on the class of linear phase filters. This paper presents a least- P^{th} approach to the design problem. Least – P^{th} optimization as a design tool is not new. It was used quite successfully for the minimax design of IIR filters. However, to date least- P^{th} based algorithms for minimax design of non linear phase 1-D FIR filters have not been reported. In the proposed method, a (near) minimax design is obtained by minimizing a weighted L_p error function without constraints, where the weighting function is fixed during the course of optimization and the power p is taken as an even integer. The proposed method does not need to update the weighting function, and it is an unconstrained convex minimization approach. The approach developed here has some advantages over the method discussed in [2] in terms of computational efficiency, filter quality, implementation structure, mathematical verification of the properties such as causality, stability, etc using the pole zero and magnitude plots.

In section 2, time domain analysis is described. In section 3, basic tools such as decimators, interpolators and multirate filter banks are reviewed. In section 4, the design procedure for implementing the filter banks is presented. Section 5 discusses the design examples to illustrate the effectiveness of the design procedure. Finally some concluding remarks are drawn in Section 6.

2. TIME DOMAIN ANALYSIS

Linear Time Invariant (LTI) systems can be characterized in the time domain by its response to a specific signal called the impulse. This response is called the impulse response of the filter .The impulse response of the filter is the response of the filter at time n to a unit impulse $\delta(n)$ occurring at time 0 and is most often denoted by $h(n)$. The impulse sequence is denoted by $\delta(n)$ and is defined by

$$\delta(n) = 1, \quad n = 0$$
$$= 0, \quad n \neq 0$$

If the input is the arbitrary signal $x(n)$ that is expressed as a sum of weighted impulses, that is,

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k)$$

Then the response of the system to $x(n)$ is the corresponding sum of weighted outputs, that is

$$y(n) = T[x(n)] = T\left[\sum_{k=-\infty}^{\infty} x(k)\delta(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} x(k)T[\delta(n-k)] \qquad (1)$$

$$= \sum_{k=-\infty}^{\infty} x(k)h(n,k)$$

Equation (1) is an expression for the response of a linear system to any arbitrary input sequence $x(n)$. This expression is a function of both $x(n)$ and the responses $h(n,k)$ of the system to the unit impulses $\delta(n-k)$ for $-\infty \prec k \prec \infty$. In fact, if the response of the LTI system to the unit impulse sequence $\delta(n)$ is denoted as $h(n)$, that is

$$h(n) \equiv T[\delta(n)]$$

Then by the time – invariance property, the response of the system to the delayed unit impulse sequence $\delta(n-k)$ is

$$h(n-k) = T[\delta(n-k)]$$

Consequently the formula in equation (1) reduces to

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k) \qquad (2)$$

Now the LTI system is completely characterized by a single function $h(n)$, namely, its response to the unit impulse sequence $\delta(n)$. The formula in equation (2) that gives the response $y(n)$ of the LTI system as a function of the input signal $x(n)$ and the impulse response $h(n)$ is called the convolution sum and we say the input $x(n)$ is convolved with the impulse response $h(n)$ to yield the output $y(n)$.Since LTI systems are characterized by their impulse response $h(n)$, in turn $h(n)$ allows us to determine the output $y(n)$ of the system for any given input sequence $x(n)$ by means of the convolution summation,

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) \qquad (3)$$

In general, any LTI system is characterized by the input –output relationship in Equation (3).Moreover the convolution summation formula in Equation (3) suggests a means for the realization of the system. In case of FIR systems, such a realization involves additions, multiplications and a finite number of memory locations. Consequently, a FIR system is readily implemented directly, as implied by the convolution summation. LTI systems can also be characterized in the time domain by constant coefficient difference equations. The difference equation is a formula for computing an output sample at time n based on past and present input samples and past output samples. In general a causal LTI difference equation is

$$y(n) = b_0 x(n) + b_1 x(n-1) + ... + .b_M x(n-M) + ... - a_1 y(n-1) - a_2 y(n-2) - .... - a_N y(n-N)$$

$$= \sum_{i=0}^{M} b_i x(n-i) - \sum_{j=1}^{N} a_j y(n-j) \qquad (4)$$

where x is the input signal, y is the output signal ,and the constants $b_i, i = 0,1,2,3......,M$ $a_i, i = 1,2,.....,N$ are called the coefficients, integers M and N represent the maximum delay in the input and output respectively. The difference equation (4) is often used as a recipe for numerical implementation in software and hardware.

The basic FIR filter is characterized by the following two equations:

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k) \qquad (5)$$

$$H(z) = \sum_{k=0}^{N-1} h(k)z^{-k} \qquad (6)$$

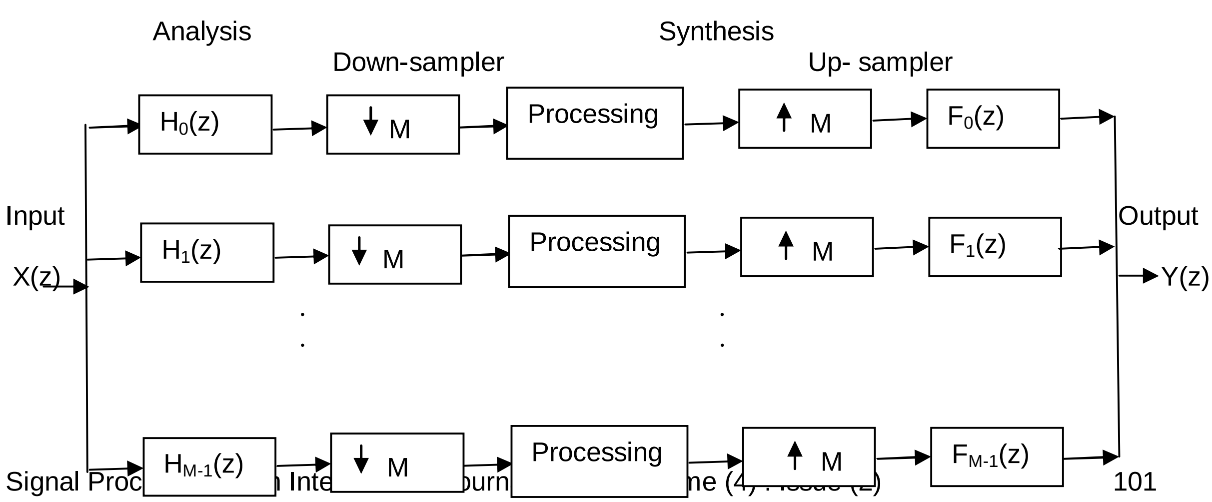
Where $h(k)$, k=0,1,2,….N-1, are the impulse response coefficients of the filter, $H(z)$ is the transfer function of the filter and N is the filter length, that is, the number of filter coefficients. Equation (5) is the FIR difference equation .It is a time domain equation and describes the FIR filter in its non recursive form: the current output sample, $y(n)$, is a function only of past and present values of the input , $x(n)$. When FIR filters are implemented in this form that is by direct evaluation of Equation (5) they are always stable. Equation (6) is the transfer function of the filter. It provides a means of analyzing the filter that is for evaluating the frequency response.

3. FILTER BANKS AND MULTIRATE SYSTEMS

To analyze different systems mathematically, it is useful to have some blocks that are common among the systems and furthermore ease the analysis process. In the analysis of the multirate systems and filter banks, the basic building blocks are the interpolators and decimators that alter the sampling frequency at different parts of the system leading to the name "Multirate". These systems provide new and effective tools to represent signals for understanding, processing and compression purposes. Multirate algorithms provide high computational efficiency and hence increase the number of potential applications. The main advantage of using multirate filter banks is the ability of these systems to separate the signal under consideration in the frequency domain into two or more signals or to compose two or more different signals into a single signal. When splitting the signal into various frequency bands, the signal characteristics are different in each band, and various numbers of bits can be used for coding and decoding the sub-signals. In many applications, the processing unit is used for treating the sub-signal in order to obtain the desired operation for the output signal of the overall system.

A filter bank is a collection of filters that are divided in two groups, the analysis filters and the synthesis filters. Analysis filters divide the incoming signal into sub-bands, while the synthesis filters merge the sub-bands in one signal. When the signal is divided into sub-bands, it is possible to process each sub-band separately. The analysis side also includes downsampling while the synthesis side includes upsampling. In the simplest form, the down-sampler reduces the input sample rate by an integer factor, M, by retaining only every M^{th} sample. On the other hand, the up-sampler increases the input sample rate by an integer factor, M, by inserting M-1 zeros between consecutive samples. Figure (1) shows a typical structure of an M channel Filter bank.

FIGURE1: M CHANNEL FILTER BANK



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4. DESIGN FORMULATION

4.1. The P^{th} Norm and Infinity Norm

Least P^{th} norm provides optimal non linear phase designs that can minimize any norm from 2 (minimum error energy) to infinity (minimax / equiripple error)

The P^{th} norm and infinity norm of a n vector $v = [\mathrm{v}_{1,} v_2 \quad \mathrm{v}_n]^{\mathrm{T}}$ are defined as

$$\|v\|_p = \left(\sum_{i=1}^{n} |\mathrm{v}_{\mathrm{i}}|^p \right)^{\frac{1}{p}}$$

And $\|v\|_\infty = \max_{\mathrm{i}} \left(|\mathrm{v}_{\mathrm{i}}|, \ \text{for} \, 1 \le \mathrm{i} \le \mathrm{n} \right)$. If p is even and the vector components are real numbers, then

$$\|v\|_p = \left(\sum_{i=1}^{n} v_i^{\,p} \right)^{\frac{1}{p}}$$

It is well known that [11] the P^{th} norm and the infinity norm are related by

$$\lim_{p \to \infty} \|\mathrm{v}\|_p = \|\mathrm{v}\|_\infty \qquad (7)$$

To get a sense of how $\|v\|_p$ approaches $\|v\|_\infty$, we compute for $v = [1 \ \ 2 \ \ 3 \ \ldots \qquad 50]^{\mathrm{T}}$

Its p^{th} norm $\|v\|_2 = 207.18$, $\|v\|_{10} = 58.80$, $\|v\|_{50} = 50.44$, $\|v\|_{100} = 49.07$

$\|v\|_{200} = 49.02$, and of course $\|v\|_\infty = 49$

For an even p, the P^{th} norm is a differentiable function of its components but the infinity norm is not. So, when the infinity norm is involved in a (design) problem, one can replace it by P^{th} norm (with p even) so that powerful calculus based tools can be used to help solve the altered problem. Obviously, with respect to the original design problem the results obtained can only be approximate. However as indicated by equation (7), the difference between the approximate and exact solutions become insignificant if power of p is sufficiently large.

4.2. Description Of The Design Procedure

To design a filter that meets the performance needs, such as having the required pass bands, stop bands, or transition regions, and is also the optimal solution, the optimal solution filter minimizes a measure of the error between the desired frequency response and the actual filter response using the least P^{th} norm algorithm. Consider two filter frequency response curves;

$D(w)$ -- The response of ideal filter, as defined by signal processing needs and specifications.

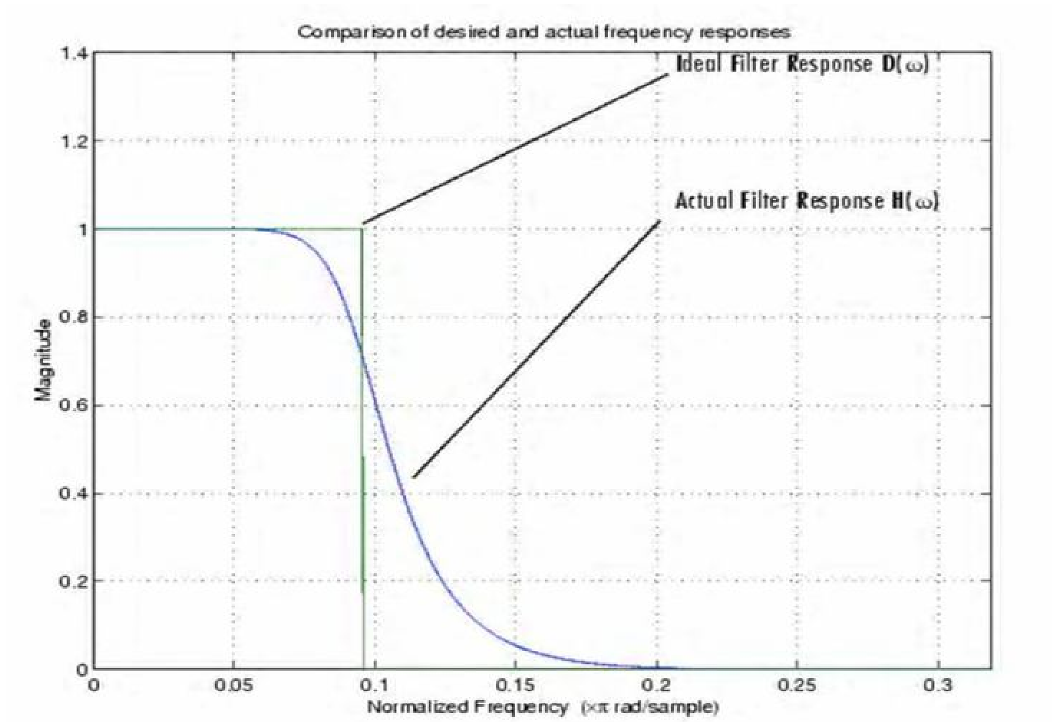
$H(w)$ -- The frequency response of the filter implementation to be selected.

In figure (2), the response curves for $D(w)$ and $H(w)$, both low pass filters are shown. Least P^{th} algorithm seeks to make $H(w)$ match $D(w)$ as closely as possible by a given measure of closeness. More precisely, we define a weighted error

$$E(w) = W(w)[H(w) - D(w)]$$

where $E(w)$ is the error between the ideal and actual filter response values and $W(w)$ is the weighting function. The optimal filter design problem is to determine $H(w)$ that minimizes some measure or norm of $E(w)$ given a particular weighting function $W(w)$ and a desired response $D(w)$. $W(w)$, the weighting function, determines which portions of the actual filter response curve are most important to the filter performance, whether pass band response or attenuation in the stop band. Usually, to measure the error the L_p norm is used. This norm is given by $\int_{\Omega} [E[w]]^p$ and this is the quantity to be minimized.

FIGURE 2: RESPONSE CURVES FOR IDEAL AND ACTUAL LOWPASS FILTERS



Since the minimization in the ∞ norm is complicated, the minimization under the 2 norm is used in the design procedure. L_2 norm minimizes the energy of the ripples, resulting in a small "total" error and attenuates the energy of a signal as much as possible. The usefulness of the L_2 norm in practice is due to the fact that it can easily be found also in the frequency domain (Parseval Theorem). The L_p norm is computed over a region Ω that uses a subset of the positive Nyquist interval [0, π]. Ω covers the positive Nyquist interval except for certain frequency bands deemed to be "don't care" regions or transition bands that are not included in the optimization. The optimal filter design problem is to find the filter whose magnitude response, $|H(w)|$ minimizes

$$E(w) = \int_w [W(w)(|H(w)| - D(w))]^p \, dw \quad (8)$$

for a given Ω, p, $W(w)$ and $D(w)$.

Up to a given tolerance, FIR filter that approximates a rather arbitrary frequency response $D(w)$ in the minimax sense can be obtained by minimizing $E(w)$ in equation (8) with a sufficiently large p.

5. DESIGN EXAMPLES

FIGURE 3 : BLOCK DIAGRAM USING P^TH NORM FILTER DESIGN

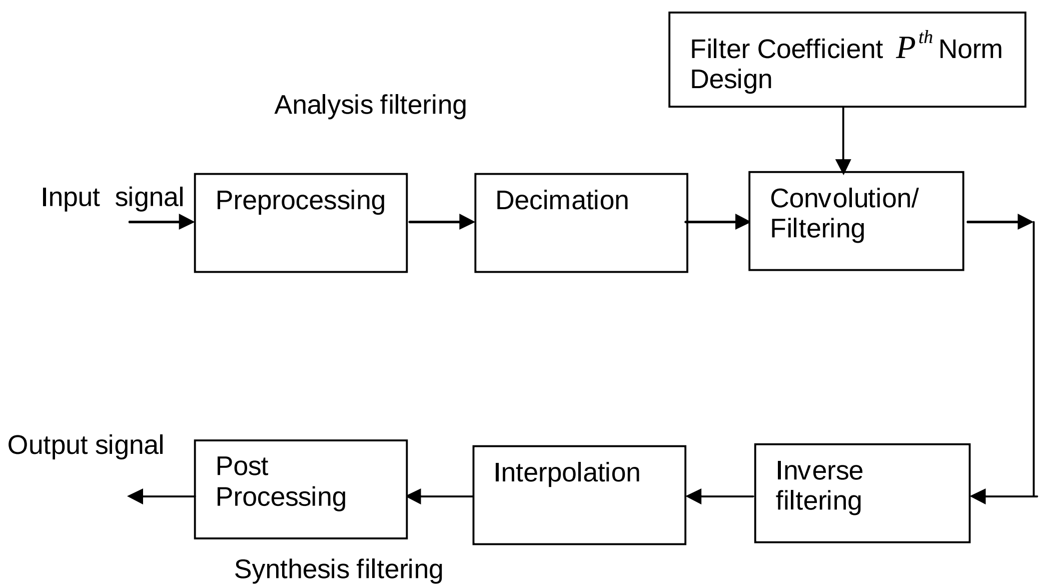


Figure 3 shows proposed system used in application for P^{th} norm filter. A random input signal is preprocessed to remove unwanted parameters, then; decimation is used for multirate analysis purpose. Then P^{th} norm filter coefficients are applied to achieve the analysis parts. For reconstruction, using inverse filtering and interpolation the input signal is reconstructed to calculate reconstruction error to verify the system functionality.

The proposed design procedure can be used to design a wide variety of analysis/ synthesis filter banks with different structural and performance constraints. The imposition of constraints impacts the quality of the resulting overall analysis /synthesis system .The purpose of this section is to illustrate the effectiveness of the design procedure through a series of related examples. In all, four examples are included – three five band systems and one 16-band system. For comparative purposes, the magnitude responses of analysis and synthesis filters are presented. To check the perfect reconstruction quality of the designed filter bank the signal- to- reconstruction noise ratio (SNR_r) which in decibel units is defined as

$$SNR_r = 10\log_{10}\left(\frac{signal\ energy}{reconstruction\ \ noise\ \ energy}\right)$$

$$= 10\log_{10}\left(\frac{\sum_n x^2(n)}{\sum_n (x(n) - \hat{x}(n+k))^2}\right)$$

was used as a measure of reconstruction performance where $x(n)$ and $\hat{x}(n)$are the input and output signals and k is the system delay. The reconstruction performance is examined by calculating the (SNR_r) for two types of input signals, namely, a ramp input (8000 samples) and a random input (8000 samples) which are denoted by $SNR_r 1$and $SNR_r 2$.

1. Basic Five –Band System

In the basic five band system example , a simple structure is imposed on the analysis filters of the system .In this case , the fourth and fifth filters, $h_4(n)$ and $h_5(n)$ are defined to be time reversed and frequency shifted versions of the second and first analysis filters, $h_2(n)$ and $h_1(n)$, respectively. The system filters are 55- tap and the system delay is 54 samples .Thus,

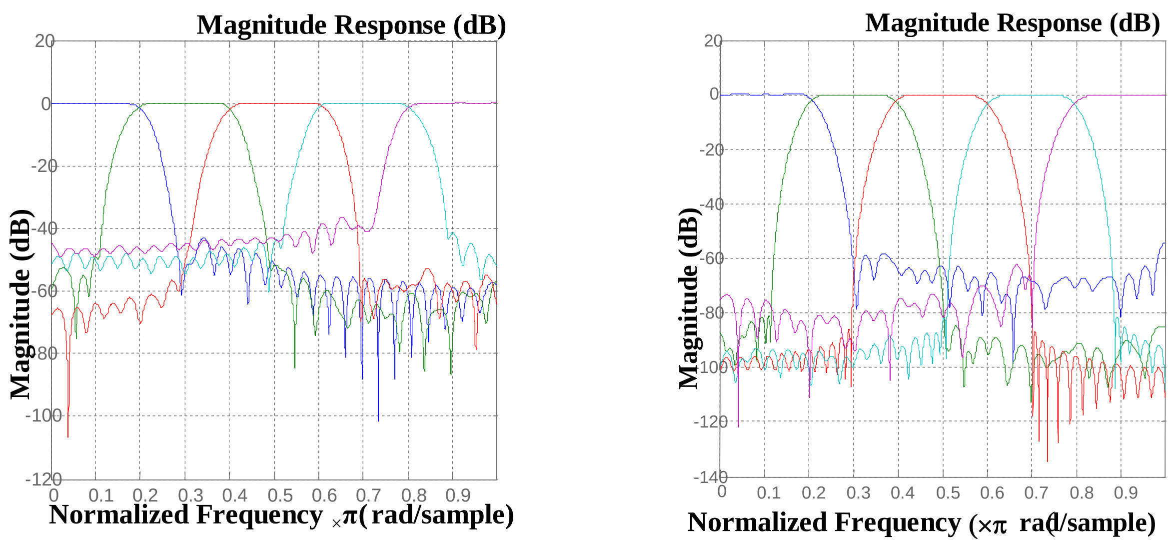
$$h_4(n) = (-1)^n h_2(N-1-n) \tag{10}$$

$$h_5(n) = (-1)^n h_1(N-1-n) \tag{11}$$

In this example, the low pass filter for the analysis section $w_p = 0.1641$and $w_s = 0.2827$ (Normalized 0 to 1) The second band is a band pass filter with $w_{p_1} = 0.2345$and $w_{p_2} = 0.3809$, $w_{s_1} = 0.1086$and $w_{s_2} = 0.5012$.The third band is also a band pass filter with $w_{p_1} = 0.4334$ and $w_{p_2} = 0.5782$, $w_{s_1} = 0.2998$ and $w_{s_2} = 0.6910$. The remaining filters in the bank can be obtained by frequency shifts and time reversals described in (10) and (11). Similarly, for the synthesis section, the low pass filter has $w_p = 0.1527$and $w_s = 0.2944$. The second band is a band pass filter with $w_{p_1} = 0.2349$and $w_{p_2} = 0.3532$, $w_{s_1} = 0.1176$ and $w_{s_2} = 0.5030$. Similarly, for the third and fourth band, $w_{p_1} = 0.4583$and $w_{p_2} = 0.5533$, $w_{p_1} = 0.6517$ and $w_{p_2} = 0.7575$respectively and $w_{s_1} = 0.2966$and $w_{s_2} = 0.7001$, $w_{s_1} = 0.5097$and $w_{s_2} = 0.8834$ respectively. The fifth band is a high pass filter with $w_s = 0.7058$ and $w_p = 0.8846$

Figure(4) shows the frequency response of the analysis and synthesis filters for the resulting five – band system . The reconstruction error and signal to noise ratios of the five band systems are presented in Table 1. Table 2 contains the coefficients of the first three analysis and synthesis filters.

FIGURE 4(a) Analysis and(b) Synthesis filters of the basic five band system with 55 tap analysis and synthesis filters and a system delay of 54 samples(Table 2)



2. Low Delay Five –Band System

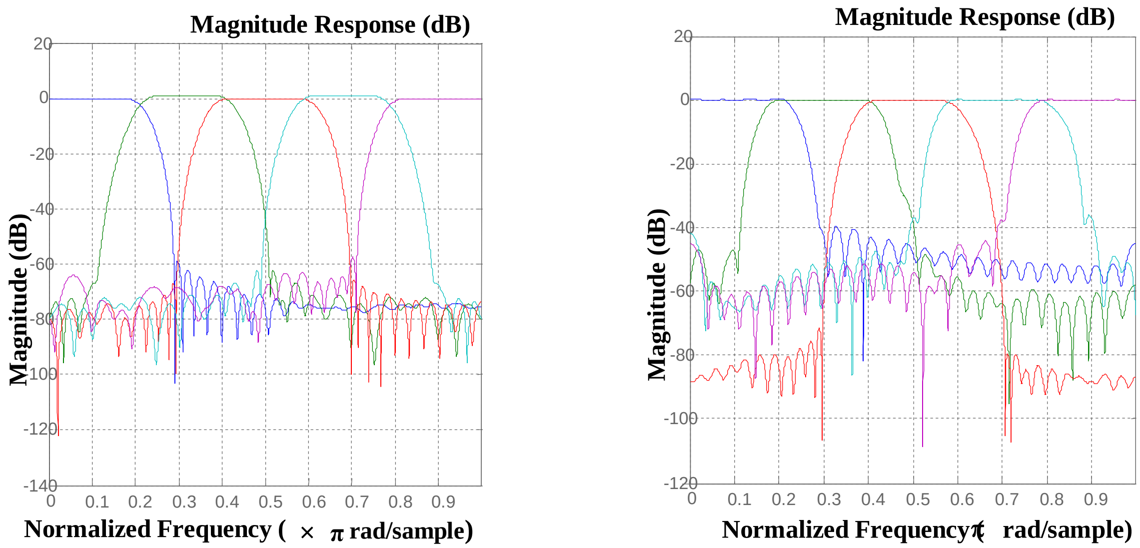
Due to filtering operations that are performed on the signal passing through the filter bank, a delay between the input and the output signals is introduced. In many applications, it is desirable to keep this delay as short as possible. In this example, the design system's flexibility in adjusting the reconstruction delay of the system was utilized and the system delay is chosen to be 28 samples. In this system, the fourth and fifth band analysis filters were selected to be the frequency shifted versions of the first two analysis filters as

$$h_4(n) = (-1)^n h_2(n) \quad (12)$$

$$h_5(n) = (-1)^n h_1(n) \quad (13)$$

For this example, three minimum phase filters were used as starting point filters since they have been found to be good starting point filters for low delay systems. For this case, for the analysis section, the first band which is a low pass filter has $w_p = 0.1767$ and $w_s = 0.2871$. The second and the third band pass filters have $w_{p_1} = 0.4205$ and $w_{p_2} = 0.5752$, $w_{p_1} = 0.2725$ and $w_{p_2} = 0.3657$ respectively and $w_{s_1} = 0.2977$ and $w_{s_2} = 0.6955$, $w_{s_1} = .1059$ and $w_{s_2} = 0.5017$ respectively. Similarly for the synthesis section, the low pass filter has $w_p = 0.1832$ and $w_s = 0.2940$ respectively. The second, third and fourth band filters have $w_{p_1} = 0.2095$ and $w_{p_2} = 0.3881$, $w_{p_1} = 0.4297$, $w_{p_2} = 0.5481$, $w_{p_1} = 0.6128$, $w_{p_2} = 0.7864$ respectively and $w_{s_1} = 0.1136$, $w_{s_2} = 0.5036$, $w_{s_1} = 0.2990$, $w_{s_2} = 0.7039$ $w_{s_1} = 0.5078$, $w_{s_2} = 0.8867$ respectively. The fifth band is a high pass filter with $w_s = 0.7060$ 0.7060 and $w_p = 0.7996$ .Figure (5) shows the frequency response of the resulting analysis and synthesis filters .The reconstruction error and signal to noise ratios of this system are given in Table 1. Table 3 contains the coefficients of the analysis and synthesis filters.

FIGURE 5: (a) Analysis and (b) Synthesis filters of the low delay five-band system with 55-tap filters and a system delay of only 28 samples (Table 3)



3. COSINE MODULATED FIVE –BAND SYSTEM

Cosine modulated filter banks are widely used and known to be highly efficient since each of the analysis and synthesis filters can be implemented with the aid of only one prototype filter and a Discrete Cosine Transform. In the cosine modulated five band systems, the analysis filters were chosen to be a cosine modulated version of a baseband filter. The baseband filter $h_0(n)$ had a nominal cutoff of $\frac{\pi}{10}$ and the analysis filters were defined as

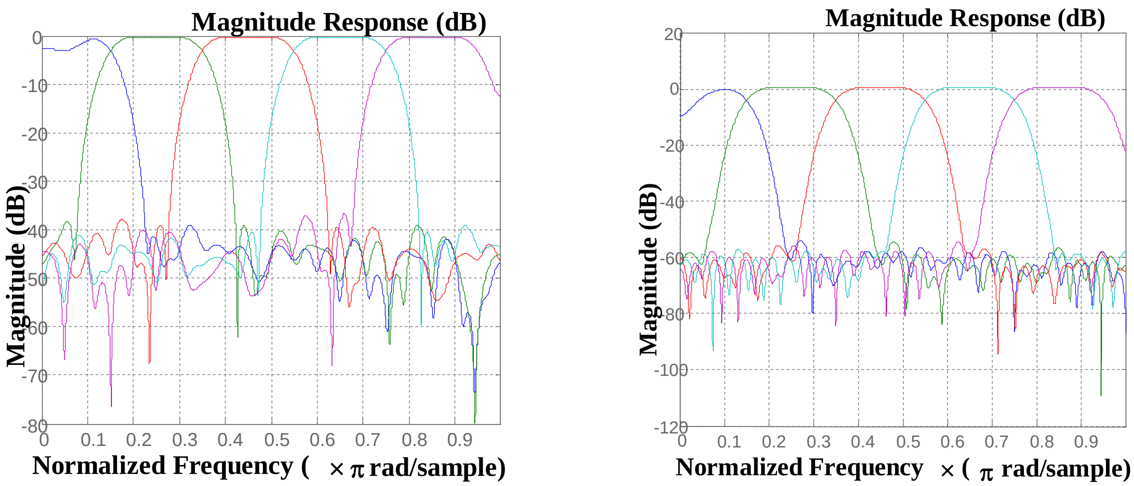
$$h_k(n) = h_0(n)\cos\left[\left(k - \frac{1}{2}\right)\frac{n\pi}{M}\right] \quad \text{k= 1, 2…M} \qquad (14)$$

and the synthesis filters were defined as

$$g_k(n) = g_0(n)\cos\left[\left(k - \frac{1}{2}\right)\frac{(n-n_0)\pi}{M}\right], \quad \text{k= 1, 2…M} \qquad (15)$$

where $g_0(n)$ is the baseband synthesis filter and n_0 is an integer whose value depends on the length of the filters and the total system delay. In this example n_0 is 5. Figure (6) shows the frequency response of the analysis and synthesis filters. The reconstruction error and signal to noise ratios of the five band systems are presented in Table 1. Table 4 contains the coefficients of the analysis baseband and synthesis baseband filters.

FIGURE.6 (a) Analysis and(b)Synthesis filters of the system with cosine modulated 55-tap analysis and synthesis filters and a system delay of 54 samples.(n_0=5) (Table 4)



4. COSINE MODULATED 16-BAND SYSTEM

To illustrate the possibly of designing large systems, a 16- band example with 96 –tap analysis and synthesis filters is designed. This system is also based on cosine modulation and the system delay is 95. For this case, the baseband filter has a cut off frequency of $\frac{\pi}{32}$ and n_0 is 1. Figure (7) shows the frequency response of the analysis and synthesis filters.. Table 5 contains the coefficients of the analysis baseband and synthesis baseband filters.

FIGURE7 (a) Analysis and (b) Synthesis filters of the cosine modulated 16 band systems with 96 tap filters (Table5)

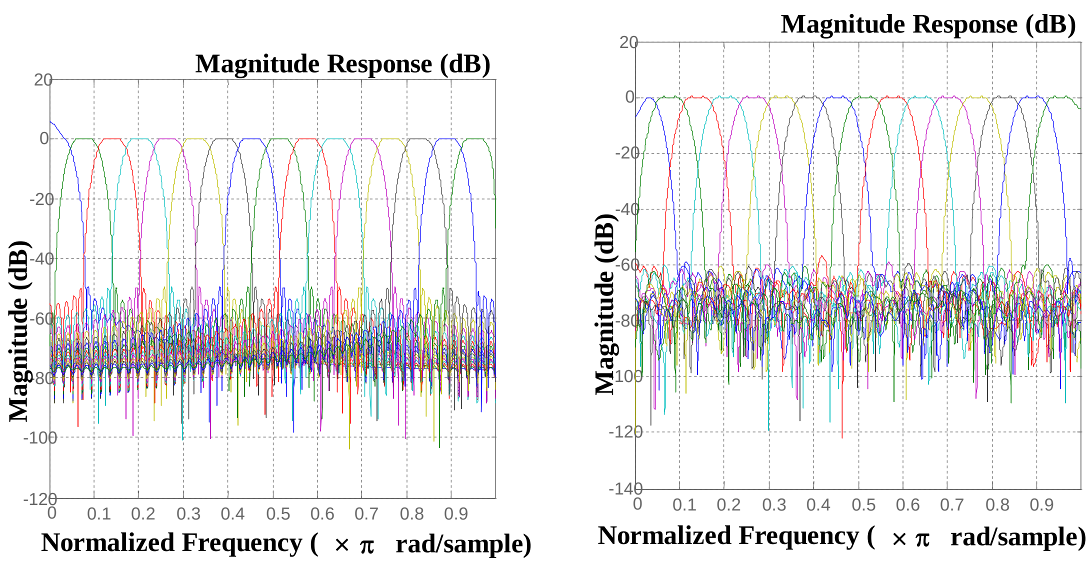


TABLE 1: Some System Specifications for the Five-Band Systems

	Basic System	Low Delay	Cosine Modulated
Filter Length	**55**	**55**	**55**
Reconstruction Error	**0.2501**	**0.1916**	**0.0022**
$SNR_r 1$	**100dB**	**97dB**	**87dB**
$SNR_r 2$	**100dB**	**97dB**	**87dB**
System Delay	**54**	**28**	**54**

Reconstruction error is calculated by considering the 8000 samples. The reconstruction error has also been tested with increasing number of samples which is an acceptable range that is minimum.

6. CONCLUSION

In this paper a new least P^{th} norm approach for FIR filter design is presented which provides optimal non linear phase designs that can minimize any norm from 2 (minimum error energy) to infinity (minimax / equiripple error). The resulting design procedure is very flexible and allows the direct imposition of many combinations of constraints and the realization of many different types of tradeoffs. The necessary assumptions and the design procedure for designing the filter have been explained. Four design examples have been included to illustrate the design procedure. The criteria used for the designed filter keeping in mind the properties of filter design are not violated, that is, in other words we have verified the mathematical results with *MATLAB* tool for filter design and analysis and verified for filter stability using the pole-zero plot, and the causality property using the impulse response. However this chapter helps to achieve the multirate analysis and synthesis approach which helps its application paradigm. In [2], a time domain design algorithm is described for the design of FIR filter bank systems. However, the design process involves calculating the pseudo inverse of large matrices, which is time consuming. In the design methods proposed here there is no need to calculate inverses of matrices. This leads to improvement in the computational efficiency during the implementation of the system. From Table 2.1, it can be seen that all the SNR_r ratios are over 80 dB, which are good enough for many applications. The tables show the filter coefficients and its responses and analysis and synthesis quality is verified with the help of SNR_r which is good as shown in the result tables. Hence we conclude that the P^{th} norm filter can be used for efficient multirate analysis and synthesis purpose.

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TABLE 2 The Filter Coefficients of the Basic Five-Band System with 55-Tap Filters and 54 Samples of Delay

$h_1(n)$	$h_2(n)$	$h_3(n)$	$g_1(n)$	$g_2(n)$	$g_3(n)$	$g_4(n)$	$g_5(n)$
-0.00028	-0.00086	0.00079	-0.00052	0.00000	-0.00010	0.00010	0.00005
0.00179	-0.00055	-0.00118	-0.00018	0.00028	-0.00004	-0.00025	-0.00024
0.00390	0.00183	-0.00092	0.00026	0.00122	0.00038	-0.00006	0.00055
0.00659	0.00274	0.00271	0.00114	0.00227	0.00045	0.00133	-0.00101
0.00914	-0.00082	-0.00105	0.00170	0.00100	-0.00048	-0.00253	0.00170
0.01103	-0.00358	-0.00548	0.00185	-0.00486	-0.00202	0.00106	-0.00247
0.01094	0.00156	0.00570	0.00089	-0.01216	-0.00078	0.00394	0.00326
0.00757	0.01014	0.00702	-0.00133	-0.01070	0.00588	-0.00835	-0.00369
0.00016	0.00615	-0.01036	-0.00471	0.00683	0.00528	0.00583	0.00356
-0.00947	-0.01356	-0.00318	-0.00769	0.02996	-0.01259	0.00394	-0.00272
-0.01808	-0.02811	0.00888	-0.00854	0.03177	-0.01407	-0.01223	0.00156
-0.02183	-0.01523	-0.00937	-0.00482	-0.00248	0.02061	0.01049	-0.00104
-0.01849	0.01479	0.00483	0.00425	-0.04680	0.02504	-0.00160	0.00257
-0.00774	0.02857	0.02963	0.01780	-0.04990	-0.02538	-0.00368	-0.00736
0.00777	0.01472	-0.02885	0.03160	0.00482	-0.03133	0.00247	0.01545
0.02318	-0.00049	-0.04936	0.04008	0.06295	0.02019	-0.00408	-0.02508
0.03188	0.00596	0.05160	0.03731	0.04863	0.02284	0.01345	0.03271
0.02855	0.01727	0.05431	0.02068	-0.03852	0.00047	-0.01819	-0.03438
0.01153	0.00247	-0.05419	-0.00759	-0.10023	0.00845	0.00454	0.02746
-0.01477	-0.02925	-0.03254	-0.03897	-0.04393	-0.03668	0.01689	-0.01300
-0.04073	-0.03751	0.02141	-0.06127	0.09353	-0.06186	-0.01957	-0.00362
-0.05390	-0.01337	-0.01495	-0.06168	0.16398	0.08060	0.00151	0.01358
-0.04417	-0.00134	0.04521	-0.03290	0.07047	0.12452	0.00285	-0.00763
-0.00752	-0.02725	0.07396	0.02355	-0.11024	-0.11847	0.02791	-0.01982
0.05103	-0.04110	-0.12571	0.09609	-0.20118	-0.17495	-0.05354	0.06735
0.11866	0.02507	-0.12172	0.16608	-0.11659	0.13660	0.00961	-0.12526
0.17794	0.13941	0.18909	0.21378	0.05184	0.19293	0.09474	0.17803
0.21289	0.16984	0.13991	0.22604	0.14821	-0.12833	-0.15167	-0.20941
0.21406	0.03603	-0.20962	0.20008	0.11738	-0.17042	0.06661	0.20817
0.18216	-0.16648	-0.12463	0.14551	0.03085	0.09765	0.11155	-0.17275
0.12732	-0.24591	0.18139	0.07977	-0.02625	0.11649	-0.21520	0.11195
0.06574	-0.12170	0.08776	0.02207	-0.04409	-0.05704	0.13380	-0.04200
0.01345	0.08371	-0.12135	-0.01430	-0.05647	-0.05275	0.05910	-0.01935
-0.01870	0.18187	-0.04759	-0.02511	-0.06624	0.02089	-0.18239	0.05876
-0.02844	0.11586	0.05737	-0.01638	-0.04277	0.00201	0.13937	-0.07151
-0.02115	-0.00788	0.01924	0.00051	0.01780	0.00132	-0.00361	0.06177
-0.00692	-0.06179	-0.01320	0.01355	0.06977	0.02270	-0.08522	-0.03975
0.00476	-0.03122	-0.00679	0.01577	0.06838	-0.00828	0.07329	0.01688
0.00843	0.00684	-0.00242	0.00677	0.02203	-0.02247	-0.01874	-0.00145
0.00447	-0.00116	0.00462	-0.00817	-0.02229	0.00511	-0.00653	-0.00359
-0.00287	-0.02966	-0.00296	-0.02187	-0.03408	0.00843	-0.00205	0.00036
-0.00815	-0.02791	-0.00474	-0.02843	-0.02252	0.00103	0.00436	0.00620
-0.00819	0.00770	0.01424	-0.02616	-0.01036	0.00646	0.01580	-0.01126
-0.00280	0.03515	0.00355	-0.01698	-0.00421	-0.00516	-0.03299	0.01207
0.00517	0.02512	-0.02065	-0.00564	0.00263	-0.01433	0.02229	-0.00874
0.01206	-0.00636	0.00003	0.00370	0.01141	0.00584	0.00656	0.00331
0.01448	-0.02364	0.01819	0.00822	0.01437	0.01426	-0.02450	0.00169
0.01136	-0.01380	-0.00271	0.00814	0.00752	-0.00424	0.01864	-0.00466
0.00410	0.00586	-0.01124	0.00490	-0.00269	-0.00974	-0.00160	0.00521
-0.00378	0.01439	0.00364	0.00114	-0.00743	0.00218	-0.00862	-0.00403
-0.00942	0.00787	0.00439	-0.00159	-0.00511	0.00482	0.00741	0.00224
-0.01155	-0.00149	-0.00235	-0.00246	-0.00070	-0.00076	-0.00183	-0.00084
-0.01018	-0.00536	-0.00058	-0.00221	0.00143	-0.00163	-0.00129	0.00001
-0.00688	-0.00351	0.00073	-0.00129	0.00115	0.00014	0.00125	0.00021
-0.00227	-0.00102	-0.00007	-0.0004	0.00033	0.00030	-0.00037	-0.00017

TABLE 3 The Filter Coefficients of The Low Delay Five-Band System With 55-Tap Filters and 28 Samples of delay.

$h_1(n)$	$h_2(n)$	$h_3(n)$	$g_1(n)$	$g_2(n)$	$g_3(n)$	$g_4(n)$	$g_5(n)$
0.00190	0.00175	0.00298	0.00108	-0.00295	0.00071	-0.00528	0.00179
0.00805	0.00636	-0.00027	-0.00425	-0.00587	-0.00031	0.01147	-0.00658
0.02201	0.00711	-0.01557	-0.00857	-0.00062	-0.00351	-0.00038	0.01183
0.04667	-0.00836	0.00110	-0.01285	0.01766	0.00128	-0.03232	-0.01701
0.08248	-0.03651	0.04663	-0.01696	0.03382	0.00910	0.05183	0.01976
0.12562	-0.04130	-0.00236	-0.01997	0.01981	-0.00289	-0.01530	-0.01840
0.16758	0.01352	-0.10054	-0.01769	-0.02759	-0.01487	-0.06317	0.01430
0.19663	0.09908	0.00325	-0.00791	-0.06528	0.00425	0.09787	-0.01019
0.20140	0.11600	0.16916	0.01025	-0.04579	0.01294	-0.03482	0.01057
0.17545	0.00115	-0.00270	0.03269	0.01418	-0.00358	-0.06085	-0.01710
0.12082	-0.16195	-0.22931	0.05290	0.03997	0.00799	0.06944	0.02867
0.04894	-0.19730	0.00033	0.06150	0.00076	-0.00058	0.01323	-0.03816
-0.02218	-0.03783	0.25098	0.05264	-0.02992	-0.05575	-0.05356	0.03837
-0.07379	0.16834	0.00270	0.02592	0.03148	0.00793	-0.03852	-0.02398
-0.09309	0.21061	-0.21455	-0.00902	0.13803	0.12590	0.15406	-0.00079
-0.07808	0.06019	-0.00423	-0.03698	0.13578	-0.01553	-0.11062	0.02516
-0.03834	-0.10421	0.12603	-0.04068	-0.03700	-0.19814	-0.09645	-0.03228
0.00875	-0.12269	0.00266	-0.01070	-0.23274	0.01909	0.25329	0.01022
0.04494	-0.03228	-0.01833	0.05124	-0.24357	0.24294	-0.18569	0.04443
0.05810	0.02299	0.00140	0.12888	-0.04667	-0.01575	-0.03318	-0.11931
0.04643	-0.00250	-0.06507	0.19858	0.16439	-0.23694	0.18176	0.19249
0.01818	-0.02436	-0.00533	0.23596	0.20420	0.00657	-0.15755	-0.23710
-0.01276	0.01781	0.09451	0.22803	0.08849	0.17779	0.04920	0.23595
-0.03333	0.06859	0.00627	0.17663	-0.02783	0.00362	0.02143	-0.18621
-0.03663	0.05075	-0.07012	0.09989	-0.05216	-0.08784	-0.04292	0.10473
-0.02406	-0.01659	-0.00344	0.02243	-0.02717	-0.00917	0.05808	-0.01769
-0.00359	-0.05152	0.01878	-0.03282	-0.02377	0.00281	-0.06377	-0.04708
0.01468	-0.02693	-0.00114	-0.05565	-0.03777	0.00740	0.02454	0.07500
0.02329	0.00619	0.02513	-0.04901	-0.02607	0.04763	0.03697	-0.06607
0.02027	0.00536	0.00436	-0.02759	0.00866	-0.00047	-0.05407	0.03616
0.00915	-0.00956	-0.04069	-0.00749	0.02090	-0.05475	0.00793	-0.00470
-0.00364	-0.00322	-0.00436	-0.00053	-0.00167	-0.00638	0.03844	-0.01138
-0.01216	0.01748	0.02905	-0.00765	-0.01830	0.03167	-0.02994	0.00793
-0.01354	0.02099	0.00177	-0.02217	0.00277	0.00862	-0.01417	0.00990
-0.00873	0.00249	-0.00648	-0.03265	0.03647	-0.00170	0.03533	-0.02900
-0.00125	-0.01291	0.00111	-0.03149	0.03866	-0.00545	-0.01968	0.03935
0.00491	-0.00987	-0.01013	-0.01677	0.00634	-0.01685	-0.00183	-0.03582
0.00736	0.00017	-0.00235	0.00518	-0.02202	-0.00027	0.00618	0.02298
0.00594	0.00222	0.01388	0.02570	-0.02161	0.01891	-0.00628	-0.00781
0.00233	-0.00171	0.00173	0.03633	-0.00721	0.00463	0.01243	-0.00157
-0.00125	-0.00197	-0.00832	0.03478	-0.00332	-0.01062	-0.01463	0.00358
-0.00319	0.00214	-0.00033	0.02326	-0.00814	-0.00546	0.00156	0.00044
-0.00312	0.00410	0.00113	0.00857	-0.00540	0.00130	0.01269	-0.00483
-0.00171	0.00154	-0.00059	-0.00362	0.00639	0.00337	-0.01172	0.00656
-0.00007	-0.00153	0.00270	-0.00921	0.01167	0.00343	-0.00211	-0.00344
0.00093	-0.00186	0.00065	-0.00877	0.00452	-0.00053	0.01015	-0.00198
0.00109	-0.00051	-0.00273	-0.00432	-0.00354	-0.00344	-0.00667	0.00751
0.00065	0.00030	-0.00027	0.00021	-0.00197	-0.00121	-0.00113	-0.00936
0.00007	0.00028	0.00125	0.00300	0.00377	0.00150	0.00443	0.00783
-0.00027	0.00009	-0.00007	0.00250	0.00402	0.00145	-0.00581	-0.00393
-0.00026	0.00001	-0.00012	-0.00011	-0.00163	-0.00001	0.00588	0.00148
-0.00009	0.00000	0.00007	-0.00327	-0.00511	-0.00087	-0.00356	-0.00033
0.00001	0.00000	-0.00011	-0.00356	-0.00304	-0.00037	-0.00209	0.00024
0.00002	0.00000	0.00000	-0.00221	0.00049	0.00026	0.00589	-0.00110
0.00000	0.00000	0.00003	-0.00110	0.00116	0.00019	-0.00424	0.00154

TABLE IV
The Filter Coefficients of the Cosine Modulated Five Band System with 55-tap filters and 54 samples of delay ($n_0 = 5$)

$h_0(n)$	$g_0(n)$
0.0002	-0.0011
0.0007	-0.0011
-0.0003	-0.0012
0.0006	-0.0016
0.0012	-0.0017
0.0012	-0.0011
0.0013	0.0004
0.0032	0.0029
0.0026	0.0066
0.0007	0.0115
-0.0010	0.0175
-0.0034	0.0242
-0.0084	0.0310
-0.0118	0.0365
-0.0149	0.0398
-0.0156	0.0396
-0.0132	0.0344
-0.0082	0.0239
0.0002	0.0070
0.0111	-0.0157
0.0285	-0.0439
0.0468	-0.0760
0.0652	-0.1101
0.0827	-0.1437
0.0970	-0.1743
0.1075	-0.1991
0.1123	-0.2162
0.1114	-0.2241
0.1056	-0.2219
0.0943	-0.2099
0.0789	-0.1896
0.0615	-0.1629
0.0438	-0.1320
0.0264	-0.1002
0.0114	-0.0697
-0.0001	-0.0426
-0.0077	-0.0209
-0.0122	-0.0051
-0.0138	0.0047
-0.0117	0.0092
-0.0099	0.0093
-0.0066	0.0064
-0.0028	0.0020
-0.0008	-0.0027
0.0012	-0.0069
0.0020	-0.0093
0.0023	-0.0104
0.0011	-0.0100
0.0009	-0.0086
0.0008	-0.0068
0.0005	-0.0049
-0.0001	-0.0031
0.0005	-0.0016
0.0002	-0.0005
0.0004	-0.0005

TABLE V
The Filter Coefficients of The Baseband Analysis and Synthesis of the 16-Band Cosine Modulated System with 96-Tap filters. $(n_0 = 1)$

$h_0(n)$		$g_0(n)$	
0.00020	-0.03234	0.00010	0.06316
0.00034	-0.03363	0.00025	0.05892
0.00051	-0.03473	0.00032	0.05445
0.00071	-0.03563	0.00050	0.04951
0.00094	-0.03632	0.00073	0.04462
0.00121	-0.03680	0.00107	0.03949
0.00151	-0.03706	0.00135	0.03438
0.00185	-0.03710	0.00178	0.02911
0.00221	-0.03693	0.00226	0.02404
0.00260	-0.03655	0.00282	0.01901
0.00301	-0.03597	0.00364	0.01435
0.00344	-0.03520	0.00456	0.00977
0.00388	-0.03425	0.00569	0.00556
0.00432	-0.03313	0.00681	0.00152
0.00475	-0.03187	0.00835	-0.00209
0.00516	-0.03048	0.00992	-0.00524
0.00554	-0.02899	0.01190	-0.00792
0.00587	-0.02740	0.01405	-0.01031
0.00615	-0.02574	0.01637	-0.01229
0.00636	-0.02403	0.01893	-0.01387
0.00649	-0.02230	0.02176	-0.01496
0.00652	-0.02055	0.02479	-0.01573
0.00645	-0.01881	0.02795	-0.01612
0.00626	-0.01710	0.03123	-0.01624
0.00594	-0.01543	0.03480	-0.01608
0.00549	-0.01381	0.03858	-0.01576
0.00490	-0.01226	0.04232	-0.01504
0.00415	-0.01079	0.04611	-0.01428
0.00326	-0.00941	0.05009	-0.01337
0.00221	-0.00812	0.05385	-0.01235
0.00102	-0.00694	0.05762	-0.01116
-0.00032	-0.00585	0.06136	-0.01005
-0.00181	-0.00488	0.06480	-0.00888
-0.00343	-0.00400	0.06809	-0.00777
-0.00517	-0.00323	0.07102	-0.00662
-0.00703	-0.00255	0.07358	-0.00566
-0.00897	-0.00197	0.07580	-0.00471
-0.01100	-0.00148	0.07753	-0.00380
-0.01309	-0.00107	0.07891	-0.00319
-0.01521	-0.00074	0.07976	-0.00255
-0.01735	-0.00047	0.08005	-0.00198
-0.01949	-0.00026	0.07973	-0.00153
-0.02159	-0.00011	0.07892	-0.00108
-0.02364	-0.00000	0.07758	-0.00070
-0.02562	0.00007	0.07574	-0.00045
-0.02750	0.00011	0.07327	-0.00024
-0.02925	0.00013	0.07032	-0.00000
-0.03087	0.00013	0.06690	0.00012