Time Domain Signal Analysis Using Modified Haar and Modified Daubechies Wavelet Transform

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Abstract

In this paper, time signal analysis and synthesis based on modified Haar and modified Daubechies wavelet transform is proposed. The optimal results for both analysis and synthesis for time domain signals were obtained with the use of the modified Haar and modified Daubechies wavelet transforms. This paper evaluates the quality of filtering using the modified Haar and modified Daubechies wavelet transform. Analysis and synthesis of the time signals is performed for 10 samples and the signal to noise ratio (SNR) of around 25-40 dB is obtained for modified Haar and 24-32 dB for modified Daubechies wavelet. We have observed that as compared to standard Haar and standard Daubechies mother wavelet our proposed method gives better signal quality, which is good for time varying signals.

Keywords: Modified haar, Modified daubechies, Analysis, Synthesis.

1. INTRODUCTION

Wavelet analysis is a mathematical technique used to represent data or functions. The wavelets used in the analysis are functions that possess certain mathematical properties, and break the data down into different scales or resolutions [1]. Wavelets are better able to handle spikes and discontinuities than traditional Fourier analysis making them a perfect tool to de-noise noisy data. Therefore, the wavelet transform is anticipated to provide economical and informative mathematical representation of many objects of interest [2]. In this paper, signal data refer to data with some type of time or spatial relationship. The majority of signal data we encounter in practical situations are a combination of low and high frequency components. The low frequency component is somewhat stationary over the length of the signal data. Wavelet analysis employs two functions, often referred to as the father and mother wavelets, to generate a family of functions that break up and reconstruct a signal. The father wavelet is similar in concept to a moving average function, while the mother wavelet quantifies the differences between the original signal and the average generated by the father wavelet. The combination of the two functions allows wavelet analysis to analyze both the low and high frequency components in a signal simultaneously.

The wavelet transform is an emerging signal processing technique that can be used to represent real-life nonstationary signals with high efficiency [3]. Indeed, the wavelet transform is gaining momentum to become an alternative tool to traditional time-frequency representation techniques such as the discrete Fourier transform and the discrete cosine transform. By virtue of its multi-resolution representation capability, the wavelet transform has been used effectively in vital applications such as transient signal analysis [4], numerical analysis [5], computer vision [6], and image compression [7], among many other audiovisual applications. Wavelets (literally "small waves") are a relatively recent instrument in modern mathematics. Introduced about 20 years ago, wavelets have made a revolution in theory and practice of non-stationary signal analysis [8][9]. Wavelets have been first found in the literature in works of Grossmann and Morlet [10].Some ideas of wavelets partly existed long time ago. In 1910 Haar published a work about a system of locally-defined basis functions. Now these functions are called Haar wavelets. Nowadays wavelets are widely used in various signal analysis, ranging from image processing, analysis and synthesis of speech, medical data and music [11][12].

In this paper we use modified Haar and modified Daubechies wavelet by considering odd number of coefficients and implement it in time signal analysis and synthesis. The two sets of coefficients (low pass and high pass filter coefficients) obtained that define the refinement relation act as signal filters. A set of simultaneous equations are formulated and solved to obtain numerical values for the coefficients. The quality of filtering using the modified Haar and modified Daubechies wavelet transform is evaluated by calculating the SNR for 10 samples.

The organization of this paper is as follows: In section 2, the standard Haar and Daubechies mother wavelets is discussed. Section 3 explains the mathematical analysis, in section 4 results for modified Haar and modified Daubechies mother wavelets are presented. In Section 5 the results are discussed, Section 6 gives the observations and in Section 7 conclusions of this work are summarized.

2 STANDARD HAAR AND STANDARD DAUBECHIES WAVELETS

The key idea to find the wavelets is self-similarity. We start with a function $\phi(t)$ that is made up of a smaller version of itself. This is the refinement (or 2-scale, dilation) equation given by

$$\phi(t)=\sum_{k=0}^{N-1} h(k)\sqrt{2}\ \ \phi(2t-k)=\sum_{k=0}^{N-1} c_k \phi(2t-k) \tag{1}$$

where $c_k = h(k)\sqrt{2}$. We call c_k as un-normalized coefficients and $h(k)$ as the normalized coefficients.

$$\psi(t)=\sum_{k=0}^{N-1} \mathrm{g}(k)\sqrt{2}\ \ \phi(2t-k)=\sum_{k=0}^{N-1} c_k{}'\ \phi(2t-k) \tag{2}$$

where

$$c_k{}'= g(k)\sqrt{2}$$

First, the scaling function is chosen to preserve its area under each iteration,

so $$\int_{-\infty}^{\infty}\phi(t)dt = 1. \tag{3}$$

The scaling relation then imposes a condition on the filter coefficients.

$$\int_{-\infty}^{\infty}\phi(t)dt = \int_{-\infty}^{\infty}\sum_{k=0}^{N-1} c_k \phi(2t-k)dt$$
$$= \sum_{k=0}^{N-1} c_k \int \phi(2t-k)\, dt$$
$$= \frac{1}{2}\sum c_k \int \phi(y)\, dy$$

Since $\int \phi(t)\, dt = \int \phi(y)\, dy$, we obtain

$$\sum_{k=0}^{N-1} c_k = 2 \tag{4}$$

Utilizing the relation $c_k = h(k)\sqrt{2}$, we get the relation in terms of normalized coefficient as:

$$\sum_{k=0}^{N-1} h(k) = \frac{2}{\sqrt{2}} = \sqrt{2}\,. \tag{5}$$

Therefore for Haar scaling function

$$h(0) + h(1) = \sqrt{2}\,, \tag{6}$$
$$h(0) - h(1) = 0 \tag{7}$$

Solving we get

$$h(0) = h(1) = \frac{1}{\sqrt{2}}$$

Secondly, if N=4, the equations for the filter coefficients are

$$h(0) + h(1) + h(2) + h(3) = \sqrt{2} \tag{8}$$
$$h(0) - h(1) + h(2) - h(3) = 0 \tag{9}$$
$$h(0)h(2) + h(1)h(3) = 0 \tag{10}$$

The solutions are $h(0) = \frac{1+\sqrt{3}}{4}, h(1) = \frac{3+\sqrt{3}}{4}, h(2) = \frac{3-\sqrt{3}}{4}, h(3) = \frac{1-\sqrt{3}}{4}$

The corresponding wavelet is Daubechies-2(db_n) wavelet that is supported on intervals $[0,3]$. This construction is known as Daubechies wavelet construction. In general, db_n represents the family of Daubechies Wavelets and n is the order. The family includes Haar wavelet since Haar wavelet represents the same wavelet as db1.

3 IMPLEMENTATION

3.1 MATHEMATICAL ANALYSIS

Consider Haar scaling function $\phi(t)$ defined in equation (11) and shown in figure (1)

$$\phi(t) = 1 \qquad 0 \le t \le 1$$
$$\qquad 0 \qquad else\ where \tag{11}$$

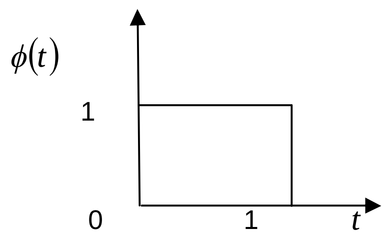


FIGURE 1: Haar Scaling Function

Consider functions of the type $\phi(t-1), \phi(t-2), \phi(t+1)$ or in general $\phi(t-k)$.These functions are called as translates of $\phi(t)$. In function $\phi(t)$**,** the function exists practically for values of t in the range $[0,1]$. Beyond this range, function value is zero.

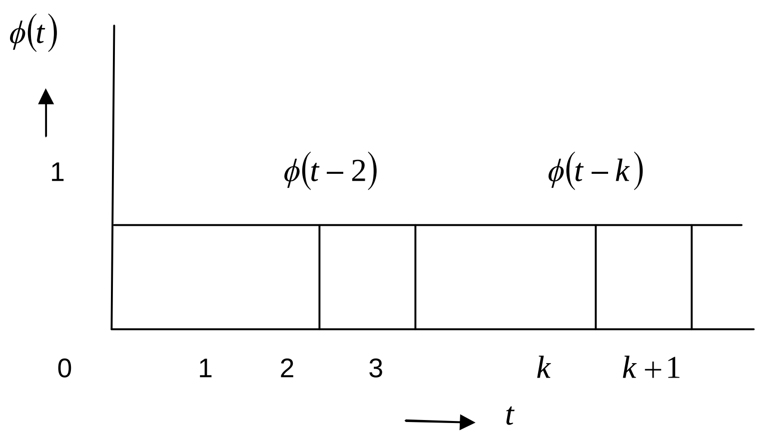


FIGURE 2: Translations Of Haar Scaling Function $\phi(t)$

The domain of the function is [0, 1].Note that the function is time limited and have finite energy. That is $\int_{-\infty}^{\infty} |f(t)|^2 dt$ exists and is finite.

Consider a set of orthonormal functions $\{.....\phi(t+1), \phi(t), \phi(t-1)....\}$, which are translates of a single function $\phi(t)$**.**

Let V_0 be the space spanned by the set of bases $\{.....\phi(t+1), \phi(t), \phi(t-1)....\}$.We denote this as

$$V_0 = Span\{\overline{\phi(t-k)}\} \quad (12)$$

Consider a function $f(t)=\sum_{k=-\infty}^{\infty} a_k \phi(t-k)$ where a_k's are real numbers (scalars) which we call as coefficients of $\phi(t-k)$'s. For one set of a_k's, we have one particular signal. But assume that we are continuously changing a_k's to generate continuously new functions or signals. The set of all such signals constitute the function space V_0.

3.2 FINER HAAR SCALING FUNCTIONS

Let us now scale the Haar basis function and form a new basis set. We scale $\phi(t)$ by 2 and form functions of the type $\phi(2t+1), \phi(2t), \phi(2t-1), \phi(2t-2)$ or in general $\phi(2t-k)$. These functions are again overlapping and are, therefore orthogonal among them.
We call the space spanned by this set of function $\{\phi(2t-k), k \in N\}$ as V_1. Figure (3) shows the new set of bases. Formally,

$$V_1 = \underset{k}{Span}\{\overline{\phi(2t-k)}\} \quad (13)$$

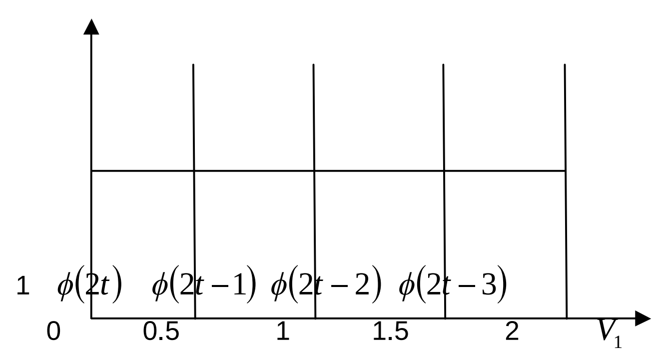


FIGURE 3: Haar Scaling Functions Which Form the Basis For V_1

Any signal in such space can be written as:

$$f_1(t)=\sum_{k=-\infty}^{\infty} a_k \phi(2t-k) \quad (14)$$

By varying a_k's in equation (14), we can generate new functions and set of all such possible functions constitute the space V_1. A signal in such a space is illustrated in figure (4)

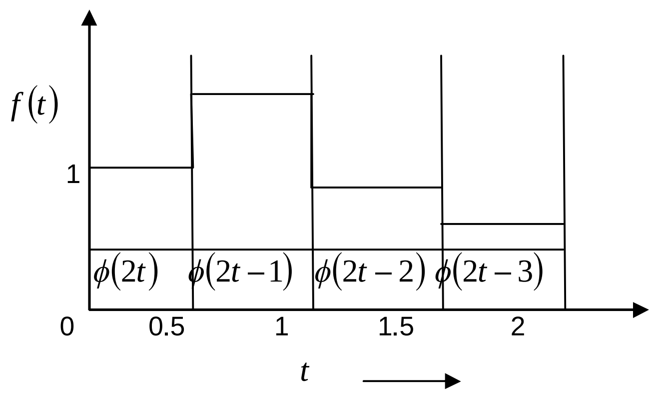


FIGURE 4: A Signal Which Is Element of Space V_1

Similarly V_2 is the space spanned by $\phi(2^2 t - k)$, that is, $V_2 = \underset{k}{Span}\{\phi(2^2 t - k)\}$

Generalizing V_j is the space spanned by $\phi(2^j t - k)$.

$$V_j = \underset{k}{Span}\{\phi(2^j t - k)\} \quad (15)$$

4 MODIFIED MOTHER WAVELETS

4.1 MODIFIED HAAR WAVELET TRANSFORM

We now illustrate how to generate modified Haar and Daubechies wavelets. First, consider the above constraints on the a_k for N=3.

The stability condition enforces $h(0) + h(1) + h(2) = 1.414$ (16)

the accuracy condition implies $h(1) - h(2) + h(3) = 0$. (17)

Solving these equations the different sets of infinitely many solutions (lowpass and high pass filter coefficients) obtained for Modified Haar is

Set 1 $h\{k\} = \{0.354, 0.707, 0.353\}$ *and* $g\{k\} = \{0.354, -0.707, 0.353\}$

Set2 $h\{k\} = \{0.200, 0.707, 0.507\}$ *and* $g\{k\} = \{0.200, -0.707, 0.507\}$.

Set 3 $h\{k\} = \{0.392, 0.707, 0.315\}$ *and* $g\{k\} = \{0.392, -0.707, 0.315\}$.

4.2 MODIFIED DAUBECHIES (db2) WAVELET TRANSFORM

Consider the above constraints on the a_k for N=5, the equations for the filter coefficients are

$$h(0)+h(1)+h(2)+h(3)+h(4) \quad = \quad 1.414 \tag{18}$$
$$h(0)-h(1)+h(2)-h(3)+h(4) \quad = \quad 0 \tag{19}$$

Solving these equations, the different sets of infinitely many solutions (lowpass and high pass filter coefficients) obtained for modified Daubechies are

Set1

$$h\{k\}=\{0.157,0.292,0.25,0.415,0.30\} \ and \ g\{k\}=\{0.157,-0.292,0.25,-0.415,0.30\}$$

Set2

$$h\{k\}=\{0.217,0.354,0.215,0.353,0.275\} and \ g\{k\}=\{0.217,-0.354,0.215,-0.353,0.275\}$$

Set3

$$h\{k\}=\{0.217,0.292,0.215,0.415,0.275\} and \ g\{k\}=\{0.217,-0.292,0.215,-0.415,0.275\}$$

Set4

$$h\{k\}=\{0.23,0.354,0.235,0.353,0.242\} \ and \ g\{k\}=\{0.23,-0.354,0.235,-0.353,0.242\}$$

5 RESULTS

TABLE 1: Modified Haar Wavelet Transform

Sample	SNR using Haar	SNR using modified Haar Set	SNR using modified Haar Set 2	SNR using modified Haar Set 3
T1	22.52	37.16	26.66	35.01
T2	22.04	37.07	26.38	34.84
T3	20.99	31.92	24.97	30.96
T4	23.24	37.56	27.39	35.55
T5	21.85	37.43	26.01	34.86
T6	22.63	37.56	26.80	35.29
T7	22.52	35.60	26.59	34.03
T8	21.30	33.21	25.67	32.10
T9	21.30	33.21	25.67	32.10
T10	21.58	35.24	26.20	33.66

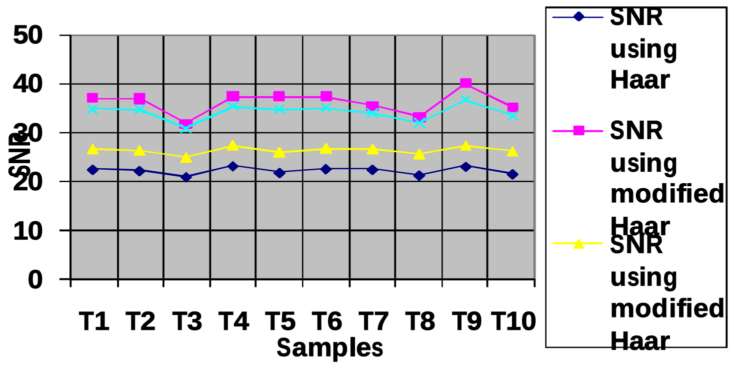


FIGURE 5: Graphical Presentation for Snr Calculation Using Modified Haar Wavelet

TABLE 2: Modified Daubechies Wavelet Transform

Sample	SNR using Standarddb2	SNR using modified db2 Set 1	SNR using modified db2 Set 2	SNR using modified db2 Set 3	SNR using modified db2 Set 4
T1	18.01	25.42	28.77	27.84	29.88
T2	17.71	25.19	28.68	27.70	29.85
T3	16.81	22.88	24.56	24.08	25.11
T4	18.74	26.05	29.17	28.30	30.19
T5	17.30	24.87	28.55	27.51	29.83
T6	18.12	25.63	29.15	28.17	30.33
T7	18.09	25.16	27.99	27.21	28.86
T8	17.38	24.08	26.32	25.71	27.01
T9	18.53	26.44	30.85	29.56	32.50
T10	17.60	24.77	27.62	26.84	28.50

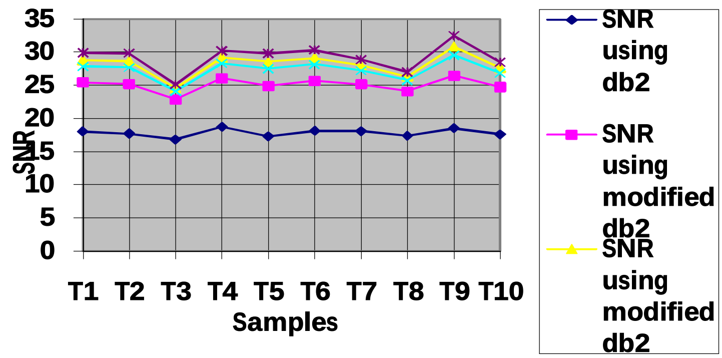


FIGURE 6: Graphical Presentation for SNR Calculation Using Modified Db2 Wavelet

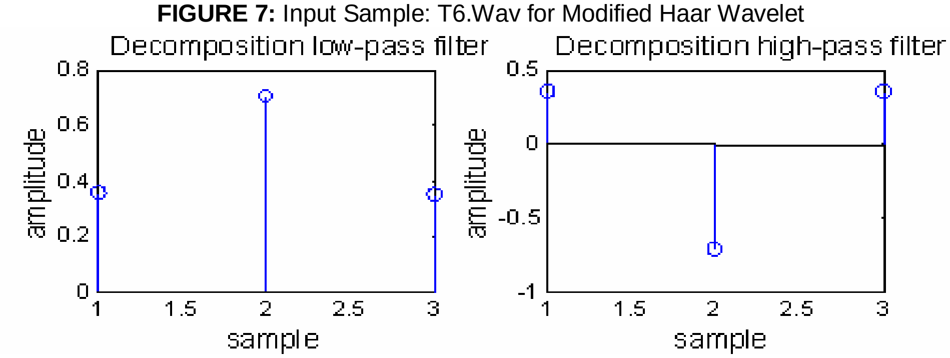
6. OBSERVATIONS

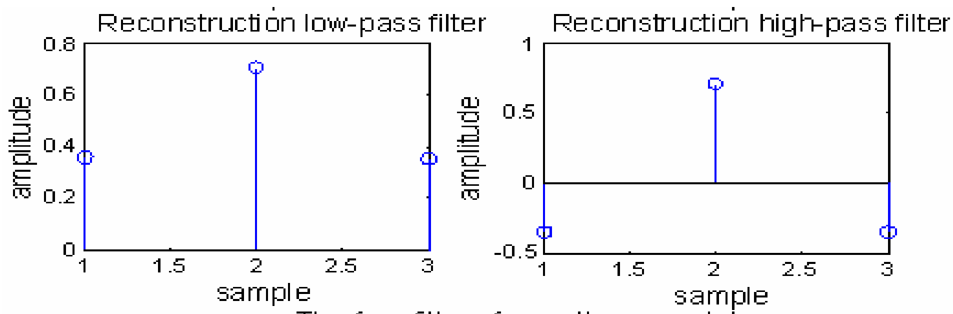
From table 1 we observe that the SNR is improved for Modified Haar as compared to standard Haar wavelet. The SNR was calculated by considering different sets of values for Modified Haar and we observe that Set 1 of Modified Haar gives better SNR values.

From table 2 we observe that the SNR is improved for Modified Daubechies (db2) as compared to standard Daubechies (db2) wavelet. The SNR was calculated by considering different sets of values for Modified Daubechies (db2) and we observe that Set 4 of Modified Daubechies (db2) gives better SNR values.

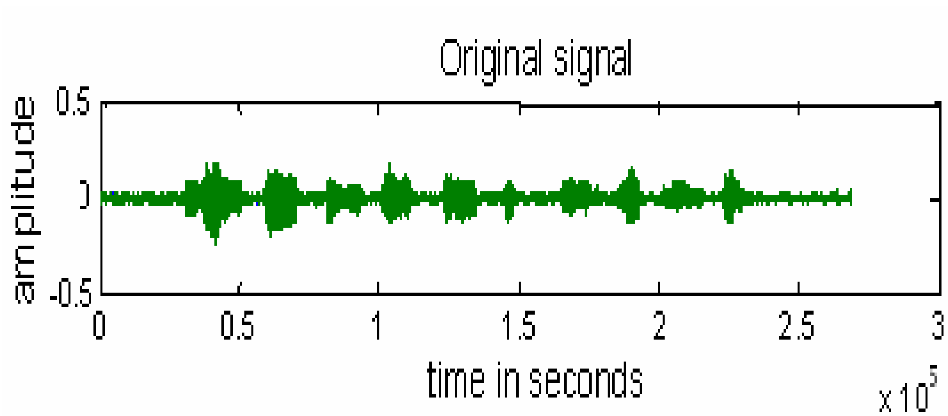
Following example shows how the analysis and synthesis is carried out using modified haar and modified Daubechies wavelet transform.

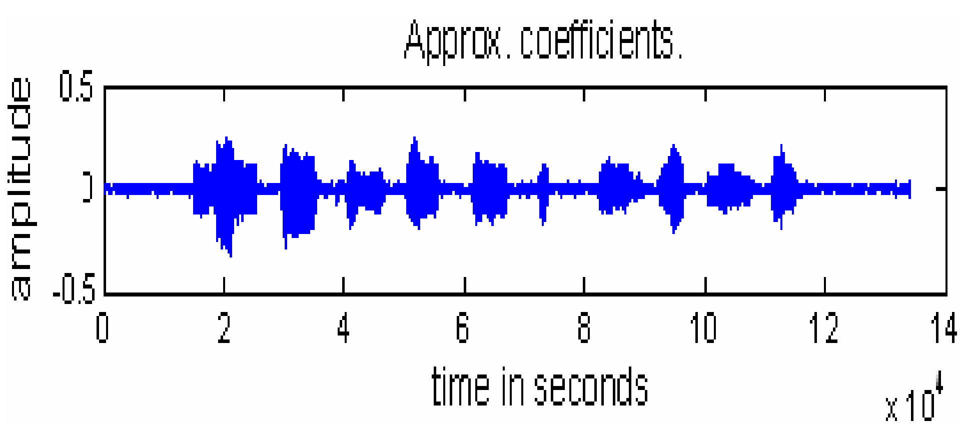
FIGURE 7: Input Sample: T6.Wav for Modified Haar Wavelet

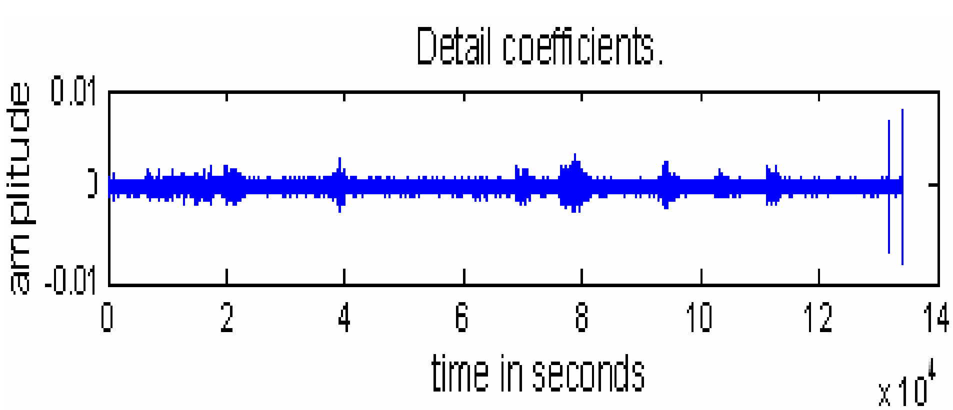




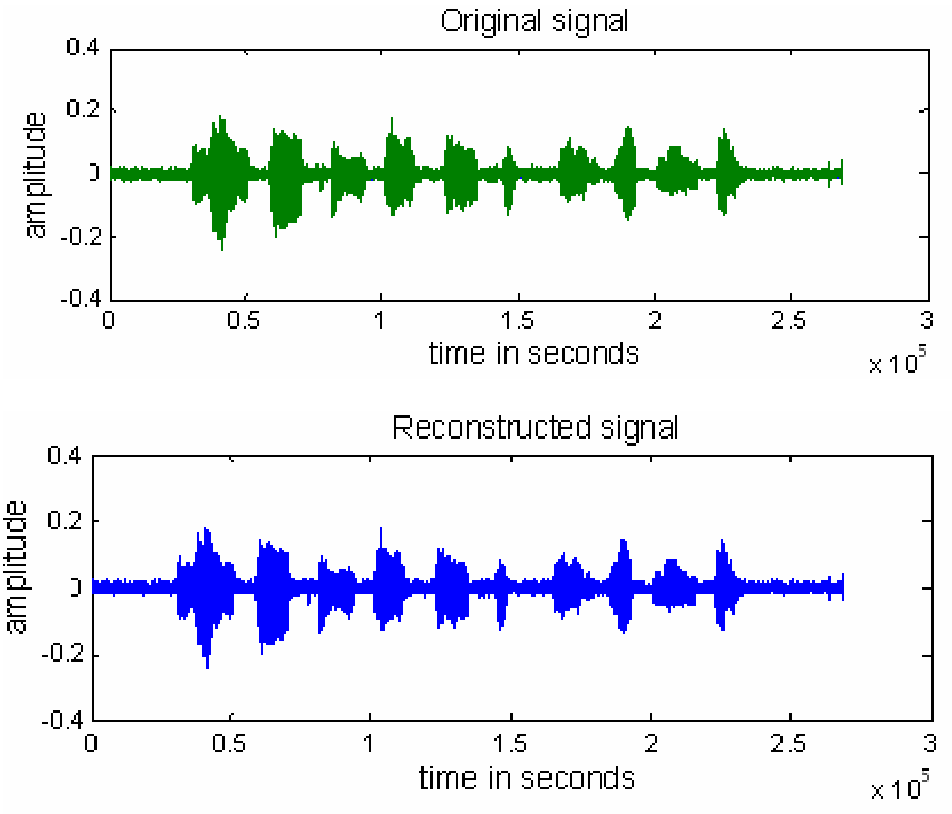
The four filters for mother wavelet





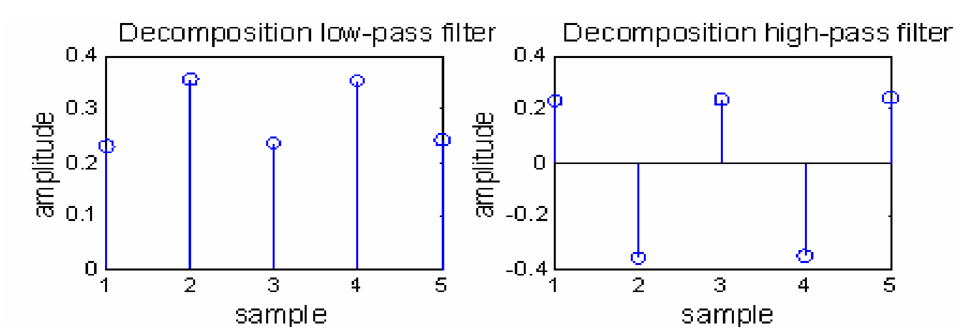


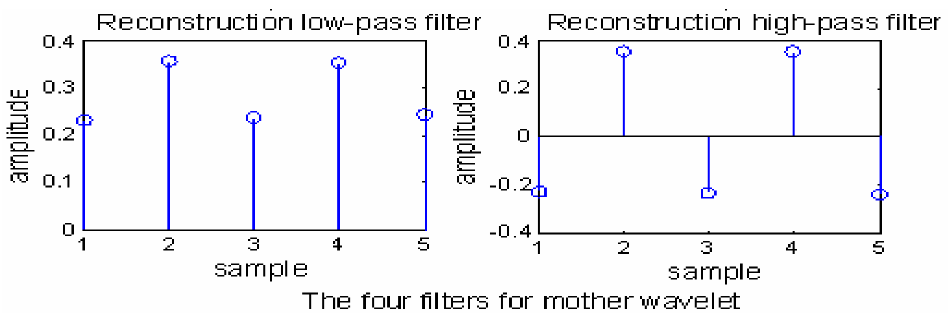
MODIFIED HAAR MOTHER WAVELET ANALYSIS AND SYNTHESIS

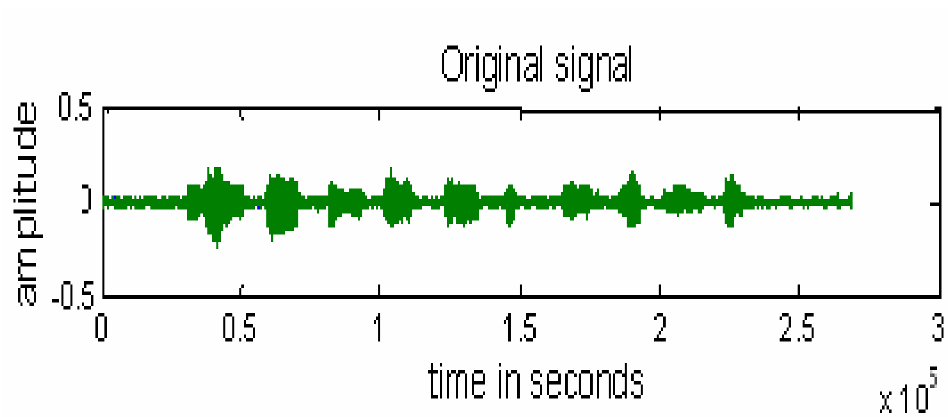


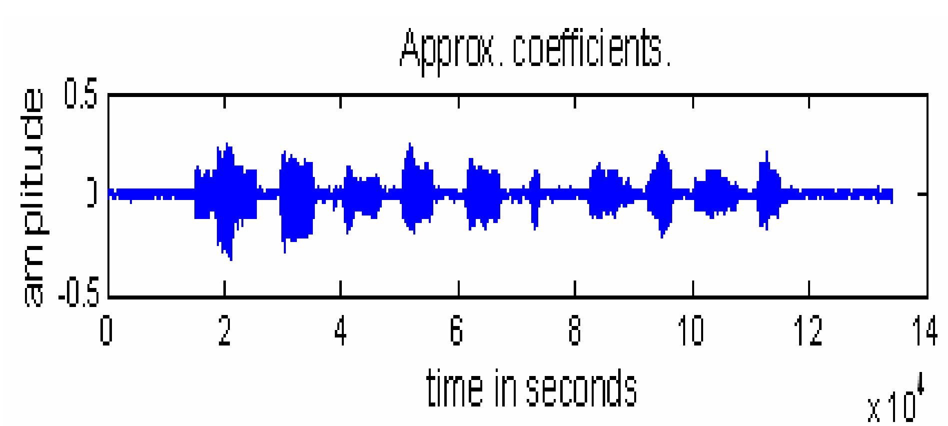
WAVELET FILTERING

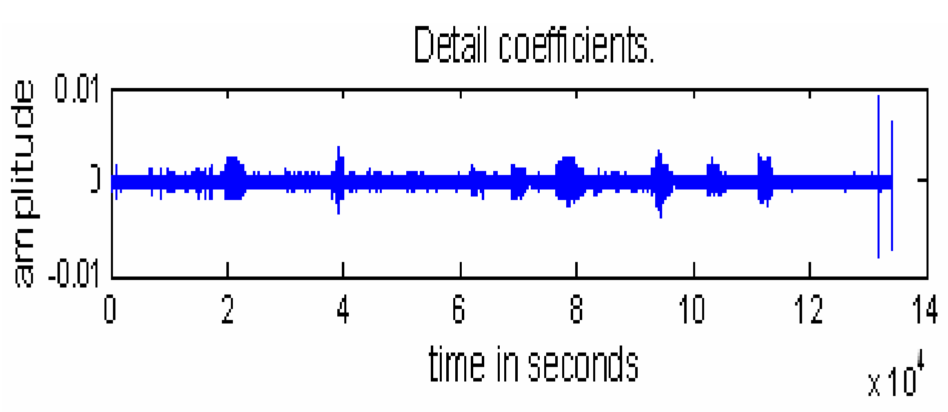
FIGURE 8: Input Sample: T6.Wav for Modified Daubechies (Db2) Mother Wavelet



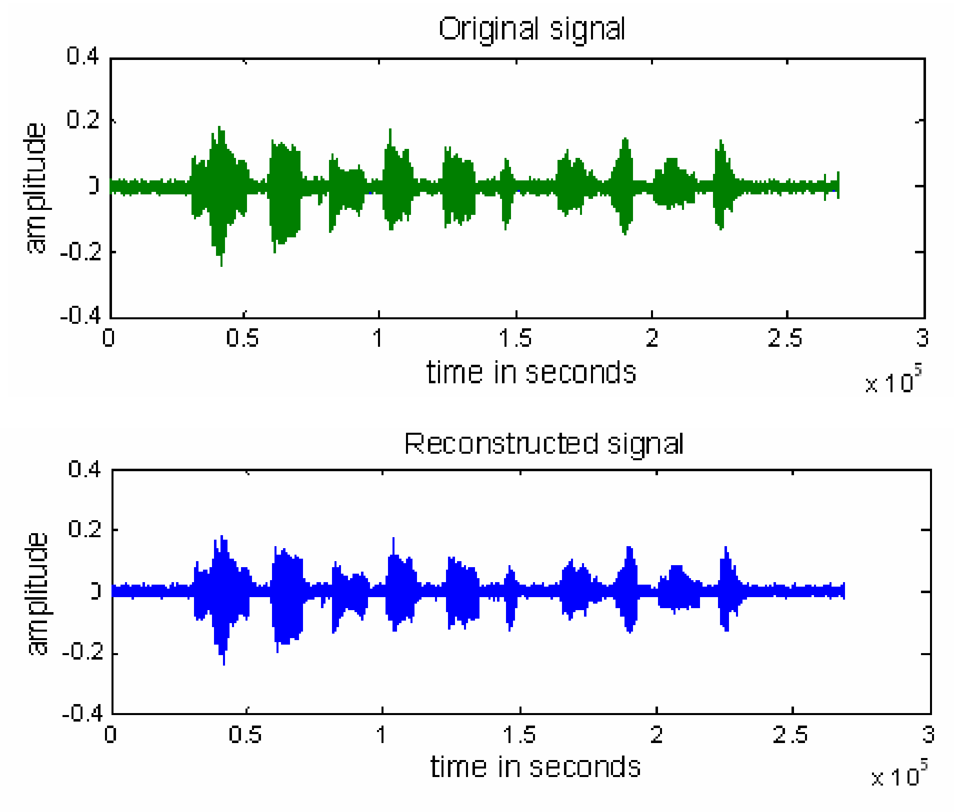








MODIFIED DAUBECHIES (db2) MOTHER WAVELET ANALYSIS AND SYNTHESIS



WAVELET FILTERING

7. CONCLUSION

We have presented a method for analysis and synthesis of time signals using modified Haar and modified Daubechies wavelet filtering techniques by considering odd number of coefficients and implement it in time signal analysis and synthesis. The two sets of coefficients (low pass and high pass filter coefficients) obtained that define the refinement relation act as signal filters. Analysis and synthesis of time signals is performed for 10 samples and the signal to noise ratio (SNR) of around 25-40 dB is obtained for modified Haar and 24-32 dB for modified Daubechies as compared to standard Haar and Daubechies mother wavelet, which is good for time varying signals. Hence we conclude that as compared to standard Haar and standard Daubechies mother wavelet our modified method gives better signal quality, and that the system will behave stable with wavelet filter and can be used for time signal analysis and synthesis purpose.

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