Spectral Analysis of Sample Rate Converter

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Abstract

The aim of digital sample rate conversion is to bring a digital audio signal from one sample frequency to another. The distortion of the audio signal introduced by the sample rate converter should be as low as possible. The generation of the output samples from the input samples may be performed by the application of various methods. In this paper, a new technique of digital sample-rate converter is proposed. We perform the spectral analysis of proposed digital sample rate converter.

Keywords: Sample Rate Converter, Spectral Analysis, Upsample-Downsample Filter, Sigma-Delta Modulator, Frequency Detector.

1. INTRODUCTION

Sample Rate Conversion (SRC) is a process by which the audio sample rate gets changed without affecting the pitch of the audio [1]. This process is necessary in different situations: Digital Audio Workstation (DAW) users often record and edit at a high sample rate, and then down-sample the audio to get it onto various media. This sample rate conversion can either be done by the DAW during or after the bounce, or in a separate application after bouncing. In another scenario, sample rate conversion is necessary when audio material recorded for a specific media (e.g. CD) gets transferred to a different media (e.g. DVD, DAT or Digital Video). For example, a DVD audio project requires sample rate conversion from 96 kHz to 44.1kHz in order to be transferred to CD, and a CD audio project requires conversion from 44.1 kHz to 48 kHz to be transferred to Digital Video format.

It is very important for the sample rate conversion to be as transparent as possible. Ideally, when converting from an original into a new sample rate, we would like the converted signal fidelity to be as high as if we had directly sampled it from the original analog signal. This degree of perfect transparency is possible only in theory, since we would need a computer with infinite memory and infinite processing power to achieve it. In practice, however, a very high degree of transparency can be achieved with a high-quality sample rate converter.

In this paper, analysis results are shown for a new method of digital sample rate converter [2]. The operation principle of the new method of sample rate conversion is very simple. An input sample is directly transferred to the output, while per unit of time, a certain amount of these samples is omitted or repeated, depending on the difference in input and output sample frequencies. The omission, acceptance or repetition of a sample is called 'validation'. In order to get the simplest hardware implementation, the choice has been made to use only the take-over operation and the repetition operation in the current system solution. This means that the output sampling frequency of the sample rate converter is always larger than the input sample frequency.

The process of repeating samples inevitably introduces errors. The resulting output samples will have correct values, but as a result of the validation operation, they are placed on the output time grid with a variable time delay with respect to the input time grid. As a consequence, the output sequence should be viewed as the input sequence, having the correct signal amplitude, which is sampled at wrong time moments. The effect is the same as sampling the input signal by a jittered clock [3]. As a result, it can be stated that the time error mechanism introduced by the validation algorithm is time jitter.

If all input samples would be transferred to the output grid without the repetition or omission of a certain amount of them, then the output signal would be just a delayed version of the input signal, exhibiting the same shape. It is the repetition and omission (in the current system setup only the repetition) of input samples that give rise to a variation in time delay for each individual output sample. This variation in individual time delays introduces phase errors. As a result of this, the shape of the output signal will be distorted [4].

The time errors introduced by the conversion process can be reduced considerably by applying upsampling and downsampling techniques. The input sample rate of the converter will be higher so that the conversion errors are smaller, resulting in smaller time jitter. These techniques do not suffice when we want to achieve the very high analog audio performance required for professional applications [5]. By using a sigma-delta modulator (noise shaper) as control source for the conversion process, the time errors will be shaped to the higher frequency region. As a result, the audio quality (in the baseband) of the signal will be preserved, provided that enough bandwidth is created by upsampling of the input signal. The high frequency (out of base band) phase modulation terms can be filtered by a decimation filter or an analog low-pass filter which is directly placed after the sample-rate converter [6]. Figure 1 shows the block diagram of the complete sample-rate converter.

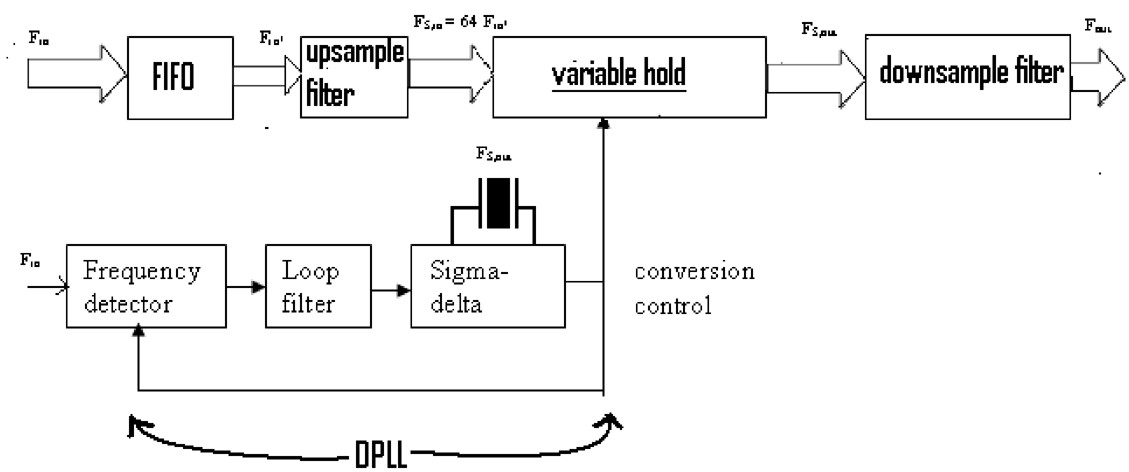

FIGURE 1: Block diagram of the sample-rate converter.

As has already been mentioned, only the input sample take over operation will be employed here in order to get the simplest hardware. This means that the input sample frequency of the converter must be always be smaller than the output sample frequency. With this restriction imposed, it is assured that all input samples are used in the output sequence, none of them being omitted. The extra output samples per unit of time are inserted in the output sequence by repetition of their previous output samples.

For practical problems in which only a finite number of samples of the analog signal are available, say $x(nT)$, $n = 0, 1, \ldots, N-1$, the band-limited analog signal and transform often are still modeled by the sampling representations but with the unobserved samples set equal to zero, i.e., $x(nT) = 0$ for $n < 0$ or $n > N-1$. With this model the Fourier transform in the band $Mod(\omega) < \pi / T$ is represented by the discrete Fourier transform (DFT). In either case, the problem of resampling with a different sample interval is in principle solved, because one can reconstruct the original analog signal, or an acceptable model of it, and then resample at will (Crochiere and Rabiner [1], [7], Pridham and Mucci [8], [9], Shaefer and Rabiner [10]) .

In spite of the availability and utility of the Fourier and sampling theorem representations it is sometimes preferable to employ a simpler interpolation scheme than that involving the sin **x/x** kernel in order to reduce the computational load. In such cases it is important to consider the approximation error and its influence on the ability of the new sample set adequately to represent the original signal.

2. THEORETICAL GUIDELINES FOR SPECTRAL ANALYSIS

In this part, the properties of the proposed sample-rate converter in the frequency domain will be investigated. It is observed that the first order approximation of the amplitude error is accurate enough, even for the worst case situation. The continuous-time description of the first-order model is:

$$y(t) = x(t) - \dot{x}(t).\Delta t(t)$$

(1)

Figure 2 gives a block diagram of this first-order model.

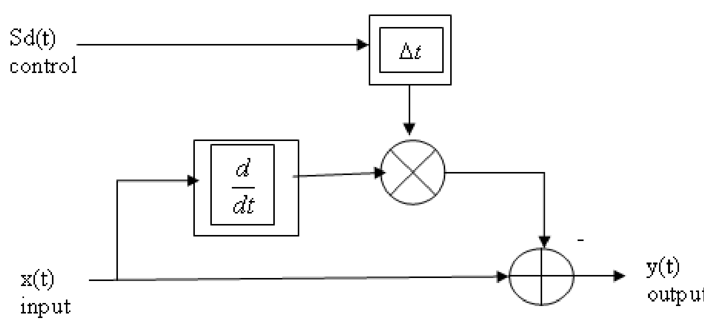

FIGURE 2: Block diagram of the first-order model.

When we want to convert (1) to discrete-time, we have to keep in mind that the output samples have a different sample time than the input samples. We will therefore enter two discrete-time variables; $k.T_{S,out}$ for the output samples and time delays, and $l.T_{S,in}$ for the input samples (shortly denoted by k and l). The discrete-time model now becomes:

$$y(k) = x(l) - \dot{x}(l).\Delta t(k)$$

(2)

Normally the derivative of a discrete-time signal is determined by the amplitude difference of the current sample and its previous sample, divided by the sample time. In our case it is more correct to use the difference between the next input sample and the present input sample, because the position in time of the present output sample is between the time moments of those two input samples. For the time derivative of the input signal x(l) we obtain:

$$\dot{x}(l) = \frac{x(l+1) - x(l)}{T_{S,in}} = F_{S,in}.[x(l+1) - x(l)]$$

(3)

Substituting this into (2), we get

$$y(k) = x(l) - F_{S,in}.[x(l+1) - x(l)].\Delta t(k)$$

(4)

In order to become known with the spectral density of the output signal y(k), we must firstly determine the correlation function:

$$R_{yy}(n) = E\{y(k).y(k+n)\}$$

(5)

This correlation function describes the correlation between the present output sample (k_0) and the (k_0+n)-th output sample and is therefore dependent on the output sample time $T_{S,out}$. Note that n must be an integer.

The problem arises that for a time step of n samples (=n.$T_{S,out}$) in the output signal we must know the corresponding time step in the input signal. Assume that this time step is equal to m.$T_{S,in}$, that is, y(k+n) corresponds to x(l+m). The relation between m and n then becomes:

$$n.T_{S,out} = m.T_{S,in} \Rightarrow m = \frac{T_{S,out}}{T_{S,in}}.n \Rightarrow m = \frac{F_{S,in}}{F_{S,out}}.n$$

(6)

The conversion factor for the sample-rate conversion process is not necessarily a rational number, which implies that m is not necessarily an integer. For the calculation of the discrete-time correlation function we need both n and m, as we have two discrete-time variables. The problem is that the discrete-time input signal x(l+m) is not defined when m is not an integer. We must therefore conclude that the correlation function $R_{yy}(n)$ of the discrete-time output signal can not be solved analytically.

Consider the discrete-time description of the first-order model (2). For the calculation of an output sample on time moment k (somewhere between l and l+1) the discrete-time derivative of the input signal x(l) on time moment k is needed. This derivative is determined using the two adjacent input samples (4). Suppose x(t) is the continuous-time signal constructed out of the input samples x(l) using *linear interpolation.* The continuous-time derivative of this input signal is in fact similar to the discrete-time derivative given by (3). In fact we deduce our discrete-time analysis from the continuous-time analysis. In order to find out the spectral properties of the sample-rate converter, it is therefore allowed that we use the *continous-time* description given by (1).

3. SIMULATION AND PERFORMANCE ANALYSIS

In this part, the properties of the sample-rate converter will be demonstrated and compared to the theoretical results. Firstly, the time domain simulations will be shown for a certain conversion factor. Next, the Fourier transformations of these time domain signals will give the frequency domain presentation of the sample-rate conversion process.

3.1 Time Domain Simulations

In this subsection the results of a time domain simulation are presented. The results have been obtained by simulating the sample-rate converter using a third-order sigma-delta modulator. The input signal consists of a single sinewave with a frequency Fin of 20kHz and an amplitude A of 1[Volt]. The output sampling frequency equals 128Fs=5.6448MHz while the input sampling frequency is chosen to be 30.13579Fs=1.328988MHz. The latter is chosen much smaller than 128Fs so that the distortion in the output signal due to much repetitions will be clearly visible. The DC input of the sigma-delta modulator can be calculated as 0.52912828125. The average number of repetitions will be 3.25. Figure 3 shows a plot of the three most important signals involved in the conversion process. The upper trace is the sigma-delta control signal, the signal in the middle is the output signal of the sample-rate converter and the lower trace shows the corresponding time error for each output sample.

It can be seen that the output signal of the converter is fairly distorted due to the large number of repetition samples. The plot style of the corresponding time error signal is staircase, because with this plot style the stepwise behaviour of this signal is illustrated best.

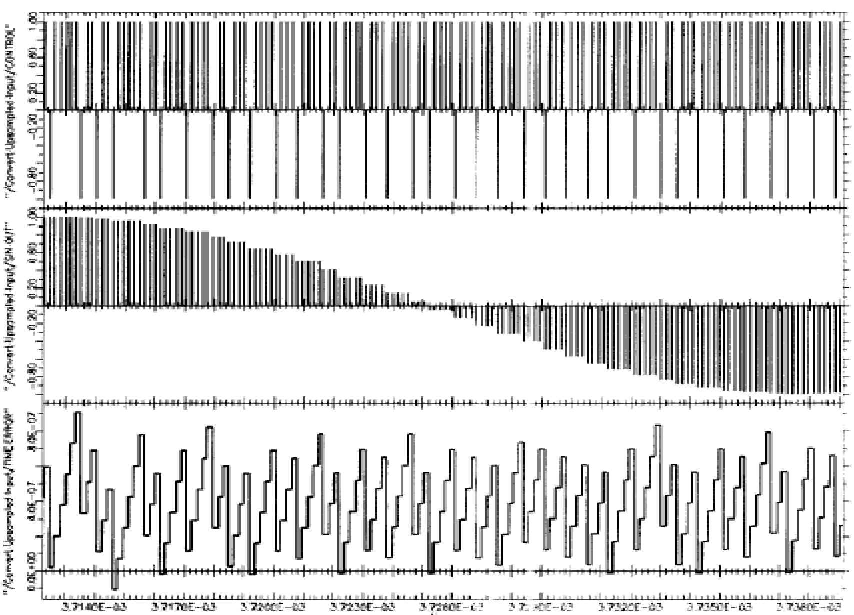

FIGURE 3: Time domain signals of the sample-rate conversion process: sigma-delta control, output sinewave, time error signal. $F_{S,in}$=30.13579F_s, FS,out=128F_s.

3.2 Frequency Domain Results

The frequency spectra of the signals in figure 3 are obtained by taking the Fourier transform of these signals. Figure 4 shows the spectra of the sigma-delta output signal and the time error signal, while figure 5 shows the frequency spectrum of the output signal. The plots have a logarithmic frequency-axis.

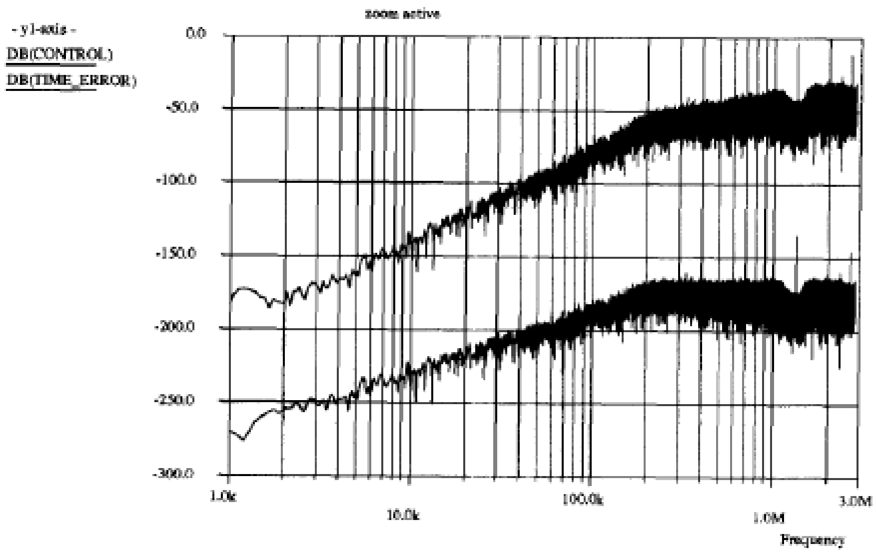

FIGURE 4: Sigma-delta spectrum (higher trace) and time error spectrum (lower trace) for $F_{S,in}$=30.13579F_s, $F_{S,out}$=128F_s and F_{in}=20kHz.

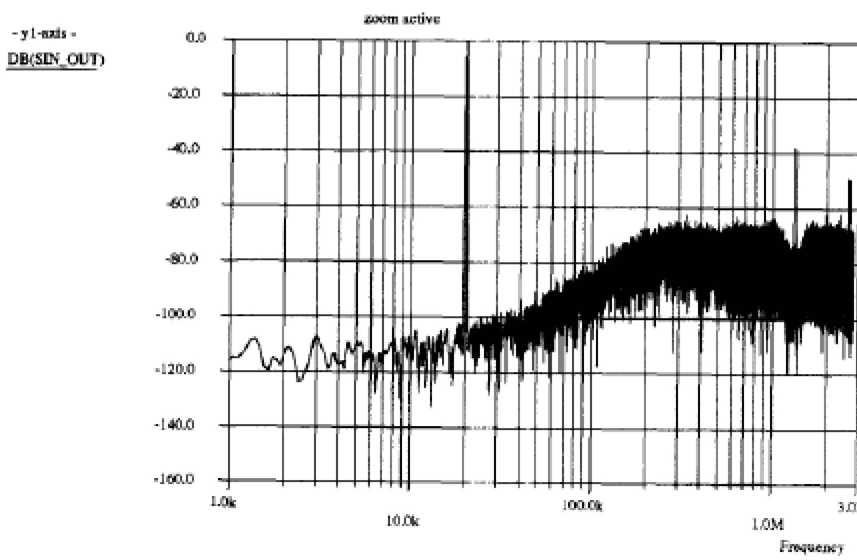

FIGURE 5: Frequency spectrum of the output signal for $F_{S,in}$=30.13579F_s, $F_{S,out}$=128F_s and F_{in}=20kHz.

In figures 6 and 7 a zoom-in is given of the figures 4 and 5 respectively, around the “first” spectral peak in the output spectrum of the sigma-delta modulator. The plots have a linear frequency-axis.

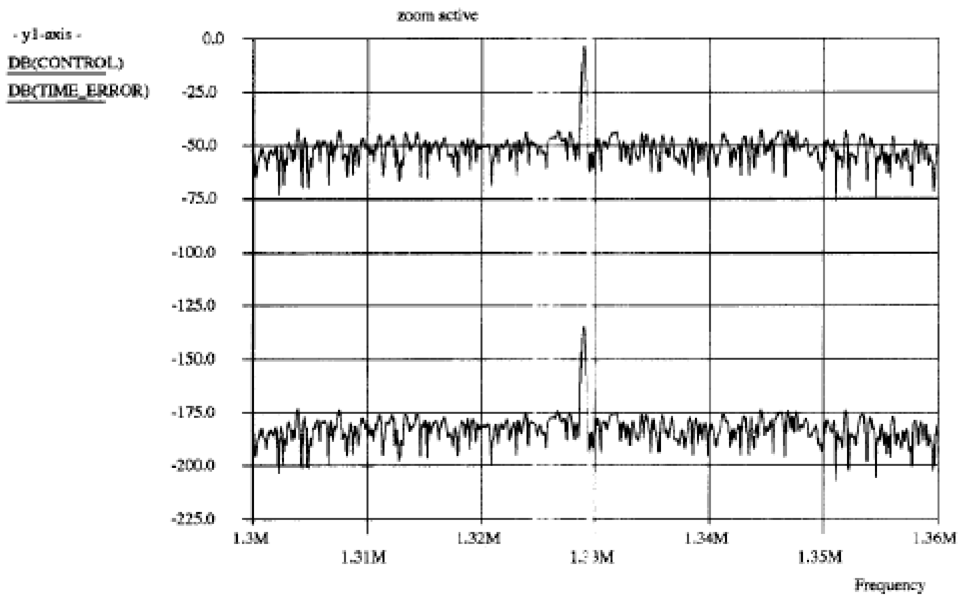

FIGURE 6: A zoom-in of figure 4

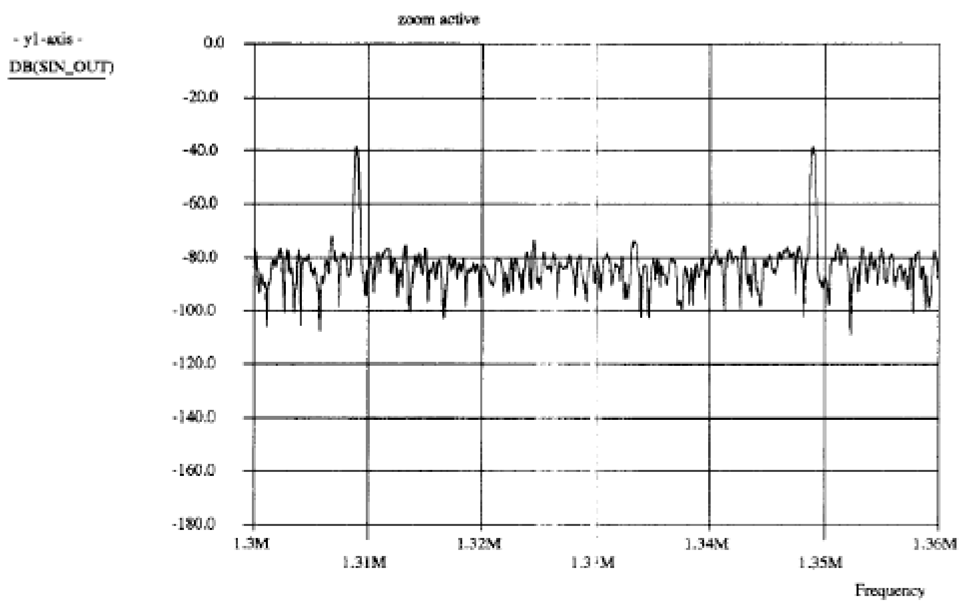

FIGURE 7: A zoom-in of figure 5.

From the figures 4 to 7, we can make the following observations:

- The slope in the sigma-delta spectrum is +60 dB per decade, which corresponds to a third order sigma-delta modulator, while the time error spectrum shows a slope of +40 dB per decade. The additional roll-off in the time error spectrum amounts 20 dB in comparison with the sigma delta spectrum, as was expected. The shaping of the time error is indeed one order lower than the shaping of quantization noise to the high frequency region, second order in this case.

- The time error spectrum indeed contains the same spectral components as the sigma-delta spectrum (figures 4 and 6), which was expected. The sigma-delta spectrum always contains a spectral peak at the input sampling frequency $F_{S,in}$, which is for this case 1.328988 MHz and at multiples of this frequency. These peaks can also be observed in the time error spectrum (having a different amplitude).

- The spectrum of the output signal of the sample-rate converter (figure 5) contains spectral peaks at $F_{S,in}$-F_{in} and $F_{S,in}$+F_{in} with F_{in}=20 kHz (figure 7). This corresponds to the theoretical considerations. The spectral peaks of the sigma-delta modulator are frequency shifted over a frequency fin to the right and to the left.

- For frequencies between 20 kHz and 200 kHz, the output spectrum shows a slope of about +40 dB per decade (figure 5). For frequencies above 200 kHz, the spectrum is flat. In the audio base band (0-20kHz), the spectrum is also fairly flat. It should be noticed that the negative frequncies must be taken into account when we look at frequency-shifted spectra [7].

4. CONCLUSION

It is concluded that the frequency domain results obtained in this paper correspond to the theoretical dependencies. It appears that due to the frequency-shift of the time error spectrum, the spectrum of the output signal of the sample-rate converter is fairly flat in the audio base band. The addition of the two frequency-shifted spectra causes more quantization noise to fall into this base band. As a result, the signal-to-noise ratio of the output signal will be smaller: the performance of the output sinewave is degraded.

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