Refining Underwater Target Localization and Tracking Estimates

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Abstract

Improving the accuracy and reliability of the localization estimates and tracking of underwater targets is a constant quest in ocean surveillance operations. The localization estimates may vary owing to various noises and interferences such as sensor errors and environmental noises. Even though adaptive filters like the Kalman filter subdue these problems and yield dependable results, targets that undergo maneuvering can cause incomprehensible errors, unless suitable corrective measures are implemented. Simulation studies on improving the localization and tracking estimates for a stationary target as well as a moving target including the maneuvering situations are presented in this paper.

Keywords: Localization, Tracking, Maneuvering, Kalman filter.

1. INTRODUCTION

Underwater noise sources can be categorized mainly as natural and man-made, based on their source of origin. Processing of the received acoustic signals helps in locating, tracking or even identification of the noise source. Localization and tracking of underwater targets bear lots of significance and has attracted great attention in the past few decades due to its importance in oceanographic, fisheries and military applications. One of the main requisites of surveillance operations is the precise position estimates of targets, which is implemented using certain geometrical constructions and well known algorithms on real time data from such systems. Various techniques for underwater localization and tracking have been devised, one of which utilizes the acoustic emanations from the targets in question by making use of passive listening concepts [1-3].

Improving the localization and tracking estimates of targets using various techniques in an underwater scenario is a problem of unfathomable extent, owing to the characteristics of the ambient environment. As the localization estimates may vary due to sensor and environmental errors, Kalman filtering techniques are applied to obtain reliably accurate estimates of localization. The results of tracked targets can be mostly misleading, if enough measures for minimizing errors in every stage of the system are not employed. One of the major problems faced by underwater target tracking systems is the effects of noises of various forms, right from the ambient noises to system induced noises, which have to be dealt with for reliable results. Adaptive filters like the Kalman filter are very powerful tools to ward off the noises that affect the reliability of such a system [4, 5].

Target tracking systems basically produce a stream of data related to the position of the target. This problem can be further divided into one dimensional motion of the target with inherent noises of different forms such as process noise and measurement noise. A study of one dimensional system is carried out and then extended to two dimensions, which can further be generalized to a multi-dimensional system depending on the nature of the problem.

The observed errors in the case of a maneuvering target are far more complex in nature than the one in the case of a target which is moving with constant velocity and hence need to be mitigated by using suitable estimation techniques. The main cause of such errors in tracking targets is their stochastic maneuvering, which becomes difficult to be identified by the tracking device. This major issue is also associated with tracking of maneuvering targets with highly adaptive generic filters like the Kalman filter, since these filters end up producing an output, erroneously considering the measured values in response to the maneuvering target, as noise. Hence optimizing the performance of the Kalman filter in a maneuvering target scenario warrants certain modifications which are discussed in the final section of this paper.

2. KALMAN FILTER

Kalman filter is a recursive approximation algorithm that provides an efficient computational means to estimate the state of a dynamic system from a series of incomplete and erroneous measurements [6-9]. This filter supports estimation of past, present and even future states and it can do so even when the precise nature of the system is unknown. Unlike most of the data processing concepts, the Kalman filter does not require all the previous data to be stored and reprocessed for each new measurement which simplifies the practical implementation of the filter. This optimal linear estimator helps to refine the localization measurements and leads to more reliable position information by judiciously taking care of the variances in the measurements.

2.1 System Model

A Given a physical system, a mathematical model is developed that adequately represents some aspects of the behavior of the system, termed as system model. The structure and modes of responses of a system can be investigated using such mathematical models supplemented with appropriate mathematical tools. In order to observe actual system behavior, measurement devices are constructed to output data signals, proportional to certain variables of interest. These output signals and the known inputs to the system are the only information that is discernable about the system behavior [10, 11]

The general problem of estimating the state variable $\boldsymbol{x}$, of a discrete-time controlled random process, that is governed by the linear stochastic difference equation can be expressed as,

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{B}\boldsymbol{u}_k + \boldsymbol{w}_k, \tag{1}$$

with a measurement $\boldsymbol{z}_k$, that is

$$\boldsymbol{z}_k = \boldsymbol{H}\boldsymbol{x}_k + \boldsymbol{v}_k \tag{2}$$

The matrix $\boldsymbol{A}$ in the difference equation relates the state $\boldsymbol{x}$ at the current time step k to the state $\boldsymbol{x}$ at the next time step $k+1$, in the absence of either an optional control function $\boldsymbol{u}$ or process noise $\boldsymbol{w}$. The matrix $\boldsymbol{B}$ relates the optional control input to the state, while the matrix $\boldsymbol{H}$ in the measurement equation relates the state to the measurement. The process is presumed to be stationary and hence the matrices are considered as constants. The normal probability distribution random variables $\boldsymbol{w}_k$ and $\boldsymbol{v}_k$ represent the process and measurement noise respectively and are assumed to be independent and white with constant covariance.

2.2 Algorithm

The Kalman filter is essentially a set of mathematical equations that implements a predictor-corrector type estimator which minimizes the mean square error. The time update equations or the predictor equations are responsible for projecting forward the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations or corrector equations are responsible for mapping the predicted values into the a priori estimate to obtain an improved a posteriori estimate.

The following are the Predictor/corrected Kalman filter Equations.

Predictor Equations:

$$\hat{\boldsymbol{x}}_{k+1}^{-} = \boldsymbol{A}\hat{\boldsymbol{x}}_k + \boldsymbol{B}\boldsymbol{u}_k \tag{3}$$

$$\boldsymbol{P}_{k+1}^{-} = \boldsymbol{A}\boldsymbol{P}_k \boldsymbol{A}^T + \boldsymbol{Q} \tag{4}$$

Corrector Equations:

$$\boldsymbol{K}_{k+1} = \boldsymbol{P}_{k+1}^{-} \boldsymbol{H}^T (\boldsymbol{H}\boldsymbol{P}_{k+1}^{-} \boldsymbol{H}^T + \boldsymbol{R})^{-1} \tag{5}$$

$$\hat{\boldsymbol{x}}_{k+1} = \hat{\boldsymbol{x}}_{k+1}^{-} + \boldsymbol{K}_{k+1} \ (z_{k+1} - \boldsymbol{H}\, \hat{\boldsymbol{x}}_{k+1}^{-}) \tag{6}$$

$$\boldsymbol{P}_{k+1} = (I - \boldsymbol{K}_{k+1} \boldsymbol{H})\boldsymbol{P}_{k+1}^{-} \tag{7}$$

$\hat{\boldsymbol{x}}_{k+1}^{-}$, above is the a priori state estimate at step $k+1$, which is the estimate of the state based on measurements at previous time-steps and $\hat{\boldsymbol{x}}_{k+1}$ is the a posteriori state estimate at step $k+1$, given measurement z_{k+1}. The a priori estimate error covariance is given by $P_{k+1}^{-} = E\,[\,e_{k+1}^{-}\, e_{k+1}^{-\,T}],$ and the a posteriori estimate error covariance by, $\boldsymbol{P}_{k+1} = E\ [\boldsymbol{e}_{k+1}\ \boldsymbol{e}_{k+1}^{T}],$ where the a priori estimate error is $e_{k+1}^{-} = \boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1}^{-}$ and the a posteriori estimate error is $\boldsymbol{e}_{k+1} = \boldsymbol{x}_{k+1} - \hat{\boldsymbol{x}}_{k+1}$. The matrix $\boldsymbol{K}$ is the Kalman gain or blending factor that minimizes the a posteriori error covariance. The difference $(z_{k+1} - \boldsymbol{H}\,\hat{\boldsymbol{x}}_{k+1}^{-})$ is called the measurement innovation or the residual. The residual reflects the discrepancy between the predicted measurement and the actual measurement. A residual of zero means that the two are in perfect agreement. The $\boldsymbol{Q}$ and $\boldsymbol{R}$ values represent process and measurement noise covariance respectively.

3. SCENARIO OVERVIEW

Simulation of underwater target localization is carried out using an ocean surveillance system consists of sensor networks that has to be deployed in the ocean which compute location of the target by measuring angles to it, from known positions of the sensor nodes using passive listening concepts [12, 13]. The results of localization are applied to the Kalman filter so as to minimize the error leading to more accurate estimates of the localization information. This paper considers stationary target as well as moving target, represented in Cartesian co-ordinate system for analysis. Suitable transformations can be used if the measurement data are in a format other than the Cartesian system. However, the tracking system and design challenges are relatively insensitive to the choice of the co-ordinate system [14].

A target moving with nearly constant velocity is characterized by a state vector with position and velocities as elements. The observations made can be assumed as a linear combination of the state vector corrupted by additive measurement noise. The Kalman gain is used to derive the filtered estimates of the state vector which in turn is used to compute the estimates predicted for the next measurement state.

The residual value is the difference between the observed and predicted values. In addition to being used for updating the filtered estimates, the residual values can be checked for consistency. This consistency check can be used to adjust the filter parameters when large residual values are interpreted as due to increased target dynamics or the detection of maneuvering of the target. The estimation accuracy provided by the Kalman filter through the covariance matrix is useful for detection of maneuver. Upon detecting such maneuver, the Kalman filter also provides an efficient way to adapt to a scenario of varying target dynamics [15-17].

3.1 Improving localization Estimates of a Stationary Target

The latitude longitude pair obtained from the localizer [12, 13] may not be accurate due to the variations in the estimation of direction of arrival of the signals emanating from the target and the mathematical approximations involved in the range computation. These inaccuracies are resolved to a certain extent by applying the concepts of Kalman filter, making use of which the refinement of localization of the underwater target is carried out by reducing the mean square error. Here the target is assumed to be stationary and the filter is applied in both dimensions independently in order to get more accurate position estimates.

3.2 Tracking of a Moving Target

In this model, the state vector consists of the target position and velocity. The elementary laws of motion can be applied for computing the velocity υ for an arbitrary time step k+1 and can be written as $\upsilon_{k+1}=\upsilon_k+Tu_k$ where $\boldsymbol{u}$ is acceleration and T is the time interval. This velocity will be perturbed by noise due to the wave action and other physical parameters of the ocean. Hence a more realistic equation for velocity υ is

$$\upsilon_{k+1}=\upsilon_k+Tu_k+\tilde{\upsilon}_k \tag{8}$$

where $\tilde{\upsilon}$ is the velocity noise. A similar equation for position s can be expressed as,

$$s_{k+1}=s_k+T\upsilon_k+\frac{1}{2}T^2u_k+\tilde{s}_k \tag{9}$$

where $\tilde{s}$ is the position noise.

For an n dimensional system, the state vector at time step k, can be described as,

$$\boldsymbol{x}_k=\begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_n \\ \\ \upsilon_1 \\ \upsilon_2 \\ \vdots \\ \upsilon_n \end{bmatrix} \tag{10}$$

For a target moving in one dimension,

$$\boldsymbol{x}_k=\begin{bmatrix} s_1 \\ \upsilon_1 \end{bmatrix} \tag{11}$$

Since the measurement vector contains only the position element, the linear system equations can be represented as,

$$\boldsymbol{x}_{k+1}=\begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}\boldsymbol{x}_k+\begin{bmatrix} T^2/2 \\ T \end{bmatrix}\boldsymbol{u}_k+\boldsymbol{w}_k \tag{12}$$

$$\boldsymbol{z}_k=\begin{bmatrix} 1 & 0 \end{bmatrix}\boldsymbol{x}_k+\boldsymbol{v}_k \tag{13}$$

Process noise $\boldsymbol{w}_k$, represents the trajectory perturbations due to uncertainty in the target state whereas the measurement noise $\boldsymbol{v}_k$, represents the inability of the tracking device to precisely measure the position of the target due to unavoidable errors in the measurement system. Both these noises are assumed to be random Gaussian processes. The acceleration u can be assumed to be zero without disturbing the generality of the system for a target moving with a constant velocity.

When the target is moving in two dimensions with a constant velocity, the state, prediction and correction equations of the model are the same as that of the one dimensional scenario, except that all the vectors are of dimension 2.

The true position of the target at the time k+1, given the position at time k is:

$$\boldsymbol{x}_{k+1} = \boldsymbol{A}\boldsymbol{x}_k + \boldsymbol{w}_k \tag{14}$$

The state vector at time step k , when n= 2 is,

$$\boldsymbol{x}_k = \begin{bmatrix} s_1 \\ s_2 \\ v_1 \\ v_2 \end{bmatrix} \tag{15}$$

and the state transition model $\boldsymbol{A}$ is:

$$\boldsymbol{A} = \begin{bmatrix} I_2 & T.I_2 \\ 0 & I_2 \end{bmatrix} \tag{16}$$

where $\boldsymbol{I}_2$ represents the identity matrix of order 2.

The state vector keeps track of the positions of the target and velocities in different dimensions which usually are the X and Y dimensions. The purpose of the Kalman filter is to estimate the true state vector given a series of discrete measurements. The state transition model updates the state vector in each time step by updating each position by adding the time interval between each measurement multiplied by the velocity in the same dimension.

Again, the measurement vector is a function of the state vector and a random noise process, expressed as,

$$\boldsymbol{z}_k = \boldsymbol{H}\boldsymbol{x}_k + \boldsymbol{v}_k \tag{17}$$

where the measurement vector is:

$$\boldsymbol{z}_k = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \tag{18}$$

and the observation model $\boldsymbol{H}$ is:

$$\boldsymbol{H} = \begin{bmatrix} I_2 & 0 \end{bmatrix} \tag{19}$$

As the velocity is not measured directly, the observation model $\boldsymbol{H}$ is operated on the state vector to obtain the measurement vector.

3.3 Tracking of a Maneuvering Target

The standard Kalman filter cannot be applied while considering a maneuvering target that executes a turn or an evasive action to elude the detection, since the target movement appears as an extensive process noise on the target model which cannot be circumvented by the process noise variance.

In order to detect a maneuver, the difference between each measurement and its corresponding predicted value is computed, which is called residual or innovation. When the number of components in each measurement is more than one, a normalized distance function or total distance, d^2 is computed. This is done by squaring the differences in each of the component measurements, dividing by the respective error variances and then summed to form a total normalized distance. A generalized form of normalized distance function can be formed with the application of Kalman filter by using the residual vector $\tilde{z}_k$ and the residual covariance matrix $\boldsymbol{S}$,

$$d_k^2 = \tilde{\boldsymbol{z}}_k^{\;T} \boldsymbol{S}^{-1} \tilde{\boldsymbol{z}}_k \tag{20}$$

$$\text{where } \tilde{z}_k = z_k - \boldsymbol{H}\hat{x}_k^- \text{ and } \boldsymbol{S} = \boldsymbol{S}_k^- = \boldsymbol{H}\boldsymbol{P}_k^- \boldsymbol{H}^T + \boldsymbol{R} \tag{21}$$

A maximum allowable value for the residual is set using the accuracy statistics of the prediction and measurement values and is normally set to at least thrice the residual standard deviation assuming zero mean Gaussian statistics, for one dimensional movement of the target. The computed differences are compared with the above derived maximum allowable error value and if the difference exceeds the same, a target maneuver is considered as detected.

Since in the subject case, the target has two dimensions of physical freedom, the normalized distance function is the sum of squares of two zero mean, unit standard deviation Gaussians, and thus featuring a chi-square probability distribution with degrees of freedom equal to the number of the measurement dimensions which in this case is 2. Based on this chi square table for d^2, a threshold can be determined to detect the target maneuver [14, 16]. Once the maneuver is detected, the Kalman filter parameters are reset and the filter is reinitialized using the last two measurements

4. SIMULATION

The simulation of improving localization and tracking estimates using Kalman filter has been implemented using Matlab. The erroneous localizer output is considered in Cartesian coordinates and its latitude and longitude values are taken as X and Y dimensional values separately and filtered using Kalman filter for reducing the error in both the dimensions, for an assumed stationary target scenario. Distinct values for the latitude and longitude pair with an error distribution around zero and a deviation of one were simulated from the localizer output ($10^{\circ}04'00''$ E and $76^{\circ}21'00''$ N). The simulated erroneous values were fed to the Kalman filter for refinement of the estimates. By iterating these values and minimizing the covariance, the Kalman filter eventually converges. Measured values for the tracking scenario are also simulated from the localizer output by adding appropriate random functions. It is assumed that the target is moving in a straight line with constant velocity and when the corresponding measured values are loaded, the Kalman filter generates the corrected values.

The algorithm implemented for the tracking of a maneuvering target is illustrated below, which detects the maneuvering of the target and upon detection it reinitializes the Kalman filter. For all scenarios the output figures vivify the effectiveness of the algorithm.

4.1 Algorithm for Tracking of a Maneuvering Target

```
           Start
LABEL :    Obtain position measurements
           Form state vector containing position and velocity
           Implement KF tracking algorithm on the state vector
           Compute the distance function
           Select a threshold from the chi-distribution table
           If threshold > distance, go to LABEL
```

Reset Kalman filter parameters
Reinitialize the filter from previous iterations
If surveillance active, go to LABEL
Stop

Using the chi-square distribution table, the value of the threshold, TH is taken as 10 which is equal to the value of the chi-square corresponding to a probability of 0.99, beyond which the target is detected to be under maneuvering.

5. RESULTS AND DISCUSSIONS

5.1 Improving localization Estimates of a Stationary Target

In the case of a stationary target and when simulated with erroneous latitude data, the output of the Kalman filter is depicted in Figure. 1 where the erroneous data is generated by adding randomness to the latitude value $10^{o}04'00''$ from the localizer output. When the Kalman gain and the error covariance converge and remain stable, the output of the Kalman filter is considered to be reliable in the subsequent iterations. It can be seen that after 31st iteration the Kalman filter converges and generates the latitude value very close to the true value. The same algorithm is extended to the erroneous longitude values too, generating the corrected longitude value.

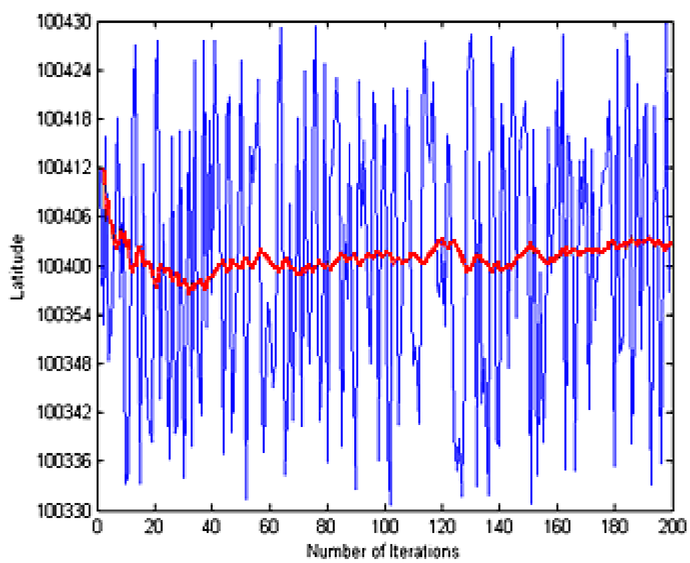

FIGURE 1: Kalman Filter output for the latitude data

A set of randomly fluctuating positional values indicated by 'o' markings, in Figure. 2, have been used for generating the corrected positional values by the Kalman filter. These values are simulated by adding random error to the assumed values of latitude and longitude data, viz., $10^{o}04'00''$ and $76^{o}21'00''$ respectively. The estimated Kalman output comprise of the points marked with '.' markings within the circled region in this figure. The filter converges and the estimated output of the Kalman filter is the position with latitude $10^{o}03'59.6''$ and longitude $76^{o}21'0.2''$ which is very close to the true value.

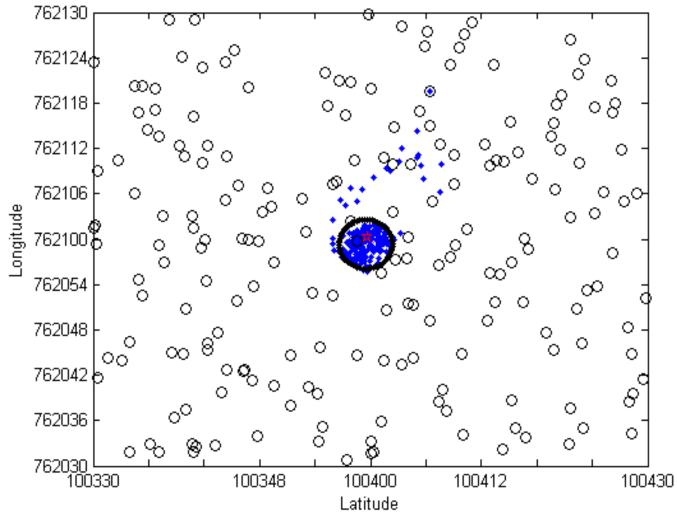

FIGURE 2: KF applied to the noisy measurements of a stationary target

5.2 Tracking of a Moving Target

In the second case analyzed herein, it is assumed that the target is moving in a straight line with a constant velocity. The true position of the target, the measured position and the estimated position in latitude and longitude dimensions are charted out in Figure. 3 and Figure. 4. The true positions and the estimated positions are almost close to be distinguished from one another after convergence of the Kalman filter while the '+' marks are the measured positions. The fact that Kalman filter reduces the minimum mean square errors with elapsed time is clearly proven in the given plots. The initial predictions are not accurate as shown, but the filter adapts and converges after few iterations.

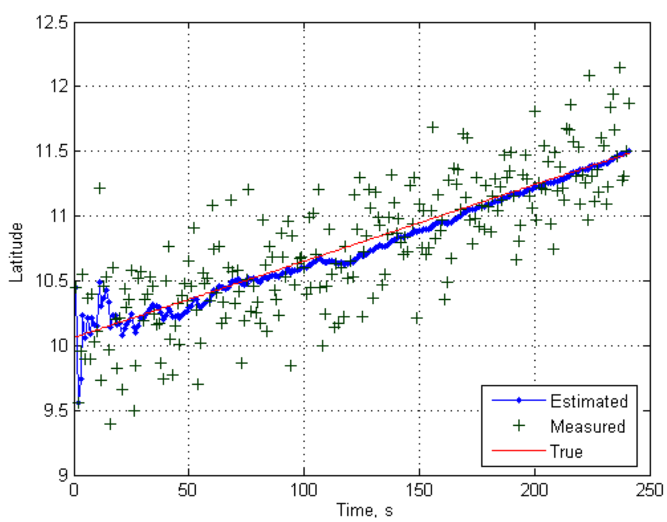

FIGURE 3: True, measured and estimated positions in X dimension

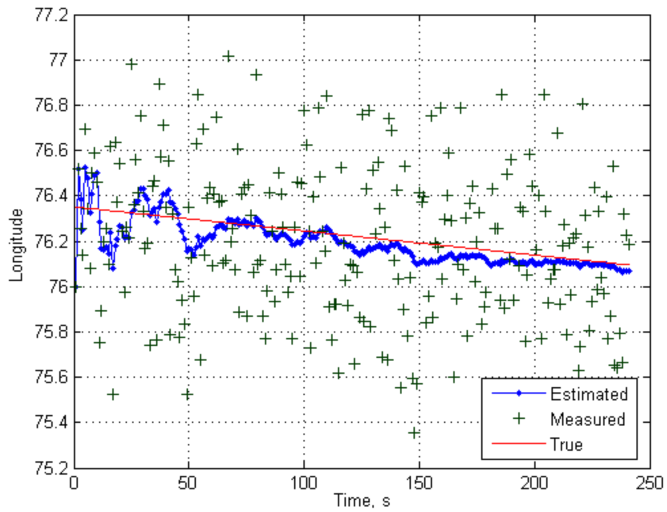

FIGURE 4: True, measured and estimated positions in Y dimension

In order to compare the accuracy of the estimates generated by the Kalman filter, Figure. 5 depicts the error values between the true position and the measured position as well as between the true position and the estimated position as provided by the filter. It is clear from this plot that the positions that are estimated or predicted by the Kalman filter are much closer to the true positions almost at every data points. This plot also shows how the Kalman estimates improve over time.

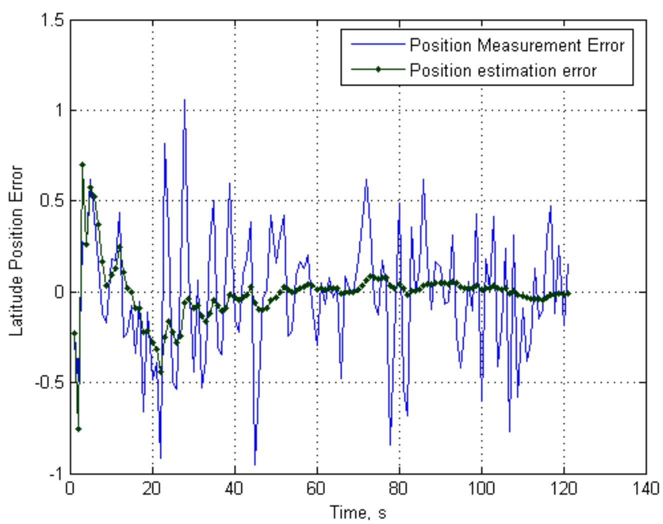

FIGURE 5: The residual plot of measurement and estimation with respect to the true values

The relative accuracy of the Kalman output is demonstrated in Figure. 6, which shows the target velocity estimate which is a part of the state variable x, along with the position estimate. Here the estimated velocity is plotted, as the system does not provide any velocity data measurements. As seen from the plot, the velocity estimates are remarkably more stable in both the dimensions. Though the initial predictions are not accurate, the filter adapts and gets tuned to the variations and limits the error range after a few iterations. The readings reinforce the fact that the Kalman filter is a powerful approach and reduces the error considerably in both dimensions and also that the accuracy of the filter improves over time, as vivified in the 2D plot shown in Figure. 7.

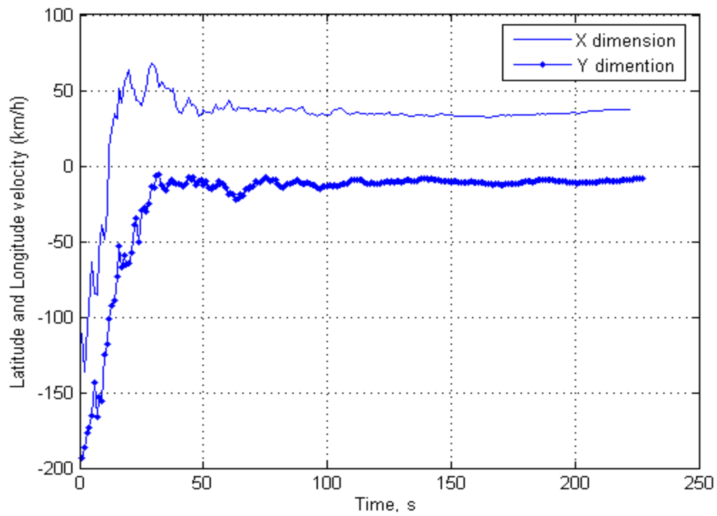

FIGURE 6: Velocity variation in X and Y dimensions

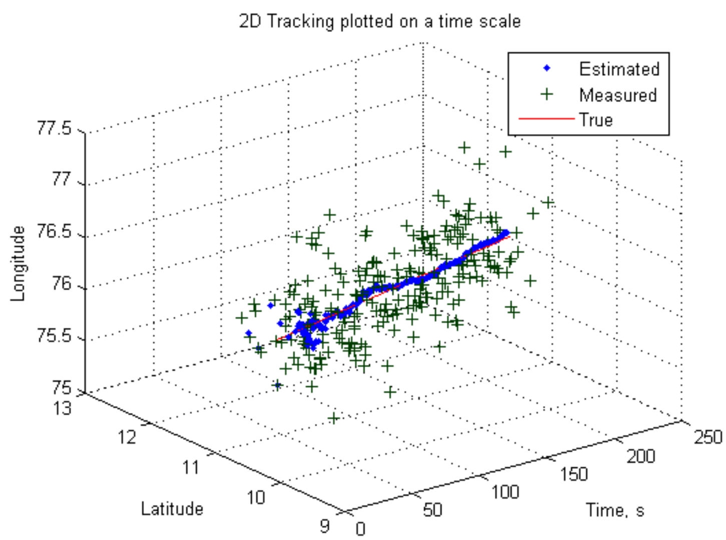

FIGURE 7: Two dimensional tracking over time

5.3 Tracking of a Maneuvering Target

This case assumes the system model to be the same as that of the two dimensional scenario, but with the target presumed to be maneuvering twice. Maneuver of the target is detected by comparing the distance function with the threshold determined by the chi square probability distribution. The threshold is set as 10 in this simulation, which corresponds to a probability of 0.99 in the chi-square distribution function. Once maneuver is recognized, the Kalman filter parameters are reset and the filter is reinitialized using previous measurements.

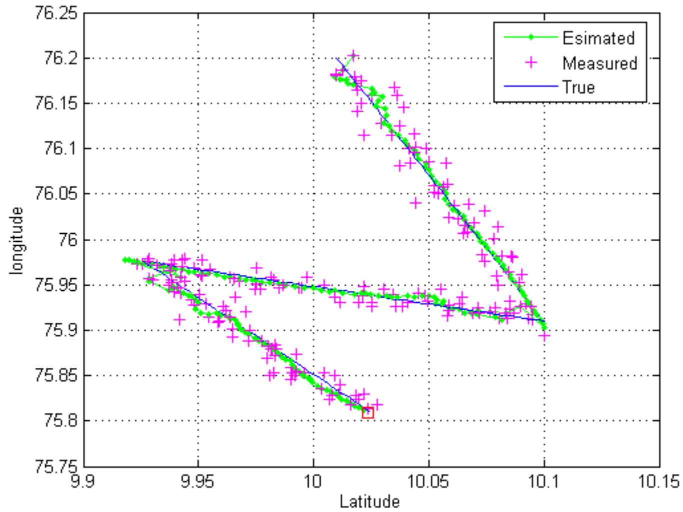

FIGURE 8: True, measured and estimated positions of a maneuvering target

As depicted in Figure. 8, the filter resets upon detecting a maneuver and thus it provides more accurate predictions over time. The position residual plot of the target maneuvering scenario depicted in Figure. 9 shows detection of the target maneuvers as graphical peaks at positions 63 and 128, which closely correlates with the simulated maneuvers at the positions 60 and 125 respectively. The validity of chi-square test relies on the assumption that the process is Gaussian and independent, which is not necessarily valid in practice. Nevertheless chi-square tests are used in these situations because of its simplicity even though it is not necessarily optimal. Also the measurements expressed in Cartesian coordinates are not independent, but the effect of ignoring this fact is negligible in practice [14, 17].

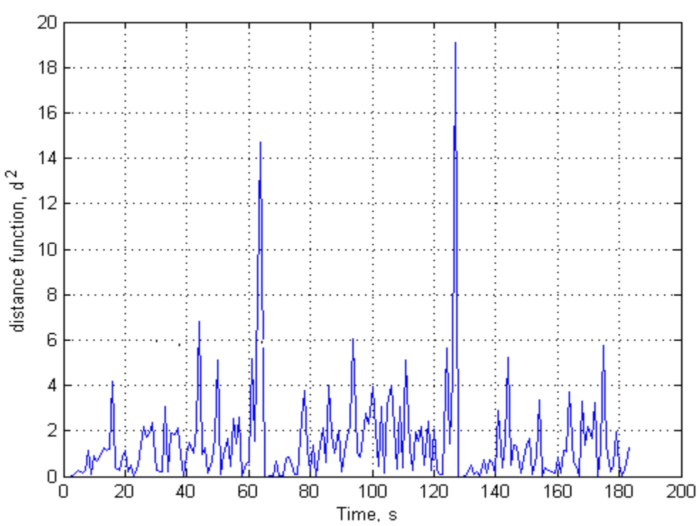

FIGURE 9: Position residual graph showing the maneuvering points

6. CONCLUSIONS

Implementing Kalman filters to target tracking systems yield reliable results, given that the nature of the system can be modeled suitably. Applying such an adaptive filtering to a simulated stationary and moving system has yielded encouraging results, even when stochastic maneuvers were introduced on the target. Various techniques that were implemented in the filter to circumvent the errors induced due to generic noises and the maneuvering of the target have been studied. The system can be extended to multiple dimensions of correlated parameters by appending desirable modifications in the system model. More reliable results can be obtained with the incorporation of other efficient techniques like neural networks and fuzzy logics.

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