

## Adaptive Variable Step Size in LMS Algorithm Using Evolutionary Programming: VSSLMSEV

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### Abstract

The Least Mean square (LMS) algorithm has been extensively used in many applications due to its simplicity and robustness. In practical application of the LMS algorithm, a key parameter is the step size. As the step size becomes large /small, the convergence rate of the LMS algorithm will be rapid and the steady-state mean square error (MSE) will increase/decrease. Thus, the step size provides a trade off between the convergence rate and the steady-state MSE of the LMS algorithm. An intuitive way to improve the performance of the LMS algorithm is to make the step size variable rather than fixed, that is, choose large step size values during the initial convergence of the LMS algorithm, and use small step size values when the system is close to its steady state, which results invariable step size Least Mean square (VSSLMS) algorithms. By utilizing such an approach, both a fast convergence rate and a small steady-state MSE can be obtained. Although many VSSLMS algorithmic methods perform well under certain conditions, noise can degrade their performance and having performance sensitivity over parameter setting. In this paper, a new concept is introduced to vary the step size based upon evolutionary programming (VSSLMSEV) algorithm is described. It has shown that the performance generated by this method is robust and does not require any presetting of involved parameters in solution based upon statistical characteristics of signal.

**Keywords:** Adaptive Equalization, LMS Algorithm, Step Size, Evolutionary Programming, MSE.

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### 1. INTRODUCTION

The recent digital transmission systems impose the application of channel equalizers with short training time and high tracking rate. These requirements turn our attention to adaptive algorithms, which converge rapidly. One of the most important advantages of the digital transmission systems for voice, data and video communications is their higher reliability in noise environment in comparison with that of their analog counterparts. Unfortunately most often the digital transmission of information is accompanied with a phenomenon known as intersymbol interference (ISI). Briefly this means that the transmitted pulses are smeared out so that pulses that correspond to different symbols are not separable. Depending on the transmission media the main causes for ISI are: cable lines – the fact that they are band limited; cellular communications – multipath propagation. Obviously for a reliable digital transmission system it is crucial to reduce

the effects of ISI and it is where the adaptive equalizers come on the scene. Two of the most intensively developing areas of digital transmission, namely digital subscriber lines and cellular communications are strongly dependent on the realization of reliable channel equalizers. One of the possible solutions is the implementation of equalizer based on filter with finite impulse response (FIR) employing the well known LMS algorithm for adjusting its coefficients. The LMS algorithm is one of the most popular algorithms in adaptive signal processing. Due to its simplicity and robustness, it has been the focus of much study and its implementation in many applications. The popularity stems from its relatively low computational complexity, good numerical stability, simple structure, and ease of implementation in terms of hardware. The essence of LMS algorithm is to update the adaptive filter coefficients recursively along the negative gradient of estimate error surface. Conventional algorithm uses a fixed step-size to perform the iteration, and to get a compromise between the conflict of fast convergence and small steady-state MSE. A small step-size could ensure small MSE with a slow convergence, where as a large step-size will provide a faster convergence and better tracking capabilities at the cost of higher steady-state MSE. Therefore, fixed step-size LMS algorithm definitely cannot settle this contradiction. Consequently, many variable step-size algorithms were proposed to solve the problem. Though these algorithms could accelerate convergence and deduce steady-state MSE to some extent, they failed to analyze the optimality of variable step-size LMS further. LMS algorithm is described by the following equations:

$$e(n) = d(n) - X^T(n) * W(n) \quad \text{-----} \quad (1)$$

$$W(n+1) = W(n) + \mu e(n) X(n) \quad \text{-----} \quad (2)$$

where  $\mu$  is learning step,  $X(n)$  is the input vector at sampling time  $n$ ,  $W(n)$  is the coefficient vector of the adaptive filter,  $d(n)$  is the expected output value  $e(n)$  is the deviation error, dimension of  $W(n)$  is the length of the adaptive filter.

## 2 RELATED WORK

In this work we introduced a novel method to obtain an optimal step-size and an algorithm for LMS. The algorithm runs iteratively and convergence to the equalizer coefficients by finding the optimal step-size which minimizes the steady-state error rate at each iteration. No initialization for the step-size value is required. Efficiency of the proposed algorithm is shown by making a performance comparison between some of the other LMS based algorithms and optimal step-size LMS algorithm [1]. A variation of gradient adaptive step-size LMS algorithms are presented. They propose a simplification to a class of the studied algorithms [2]. Adaption in the variable step size LMS proposed by [3] based on weighting coefficients bias/variance trade off. Authors in [4] examine the stability of VSLMS with uncorrelated stationary Gaussian data. Most VSLMS described in the literature use a data-dependent step-size, where the step-size either depends on the data before the current time (prior step-size rule) or through the current time (posterior step-size rule). It has often been assumed that VSLMS algorithms are stable (in the sense of mean-square bounded weights), provided that the step-size is constrained to lie within the corresponding stability region for the LMS algorithm. The analysis of these VSLMS algorithms in the literature typically proceeds in two steps [5], [6]. First, a rigorous stability analysis is attempted, apparently leading to conditions for MSE bounded weights and bounded MSE, and second, an approximate performance analysis is carried out, including convergence to and characterization of the asymptotic weight mean, covariance, and MSE. Thus one can at least guarantee stability (MS bounded weights) rigorously would seem to support the performance analysis. Two methods of variable step-size normalized least mean square (NLMS) and affine projection algorithms (APA) with variable smoothing factor have presented in [8]. With the Simulation results they have illustrated that the proposed algorithms have improvement in convergence rate and lower small adjustment error.

## 3. BASIC CRITERIA FOR PERFORMANCE

The performance of the LMS adaptive filter can be characterized in three important ways: i) the adequacy of the FIR filter Model (ii) the speed of convergence of the system and iii) the small

adjustment steady-state.

### 3.1 Speed of Convergence

The rate at which the coefficients approach their optimum values is called the speed of convergence. As per the previous analytical result obtained, there exists no one quantity that characterizes the speed of convergence, as it depends on the initial coefficient values, the amplitude and correlation statistics of the signals, the filter length  $L$ , and the step size  $\mu(n)$ .

However, we can make several qualitative statements relating the speed of convergence to both the step size and the filter length. All of these results assume that the desired response signal model is reasonable and the errors in the filter coefficients are uniformly distributed across the coefficients on average.

- The speed of convergence increases as the value of the step size is increased, up to step sizes near one-half the maximum value required for stable operation of the system. This result can be obtained from a careful analysis for different input signal types and correlation statistics. For typical signal scenarios, it is observed that the speed of convergence of the excess MSE actually decreases for large enough step size values.
- The maximum possible speed of convergence is limited by the largest step size that can be chosen for stability for moderately correlated input signals. In practice, the actual step size needed for stability of the LMS adaptive filter is smaller than one-half the maximum values when the input signal is moderately correlated. This effect is due to the actual statistical relationships between the current coefficient vector and the signals. Since the convergence speed increases as  $\mu$  is increased over this allowable step size range, the maximum stable step size provides a practical limit on the speed of convergence of the system.

### 3.2 Choice of Step Size

Based on the previous result obtained, the speed of convergence as the step is increased. We have seen that the speed of convergence increases as the step size is increased, up to values that are roughly within a factor of 1/2 of the step size stability limits. Thus, if fast convergence is desired, one should choose a large step size according to the limits. However, we also observe that the small adjustment increases as the step size is increased. Therefore, if highly accurate estimates of the filter coefficients are desired, a small step size should be chosen. This classical trade off in convergence speed versus the level of error in steady state dominates the issue of step size selection in many estimation schemes. If the user knows that the relationship between input signal  $x(n)$  and desired signal  $d(n)$  is linear and time-invariant, then one possible solution to the above trade off is to choose a large step size initially to obtain fast convergence, and then switch to a smaller step size. The point to switch to a smaller step size is roughly when the excess MSE becomes a small fraction (approximately 1/10th) of the minimum MSE of the filter. This method of gear shifting, as it is commonly known, is part of a larger class of time-varying step size methods.

## 4. EVOLUTIONARY COMPUTATION

Evolutionary algorithms are stochastic search methods that mimic the metaphor of natural biological evolution. Evolutionary algorithms operate on a population of potential solutions applying the principle of survival of the fittest to produce better and better approximations to a solution. At each generation, a new set of approximations is created by the process of selecting individuals according to their level of fitness in the problem domain and breeding them together using operators borrowed from natural genetics. This process leads to the evolution of populations of individuals that are better suited to their environment than the individuals that they were created from, just as in natural adaptation. Evolutionary computation uses computational models of evolutionary processes as key elements in the design and implementation of computer-based problem solving systems. There are a variety of evolutionary computational models that have been proposed and studied (evolutionary algorithms). They share a common conceptual

base of simulating the evolution of individual structures via processes of selection and reproduction. These processes depend on the perceived performance (fitness) of the individual structures as defined by an environment. More precisely, evolutionary algorithms maintain a population of structures that evolve according to rules of selection and other operators such as recombination and mutation. Each individual in the population receives a measure of its fitness in the environment. Selection focuses attention on high fitness individuals, thus exploiting the available fitness information. Recombination and mutation perturb those individuals, providing general heuristics for exploration. Although simplistic from a biologist's viewpoint, these algorithms are sufficiently complex to provide robust and powerful adaptive search mechanisms.

```

Procedure EP; {
    t = 0;
    Initialize population P (t);
    Evaluate P (t);
    Until (done) {
        t = t + 1;
        Parent selection P (t);
        Mutate P (t);
        Evaluate P (t);
        Survive P (t);
    } }
    
```

Evolutionary algorithms (EA) differ substantially from more traditional search and optimization methods. The most significant differences of EA are:

- search a population of points in parallel, not just a single point.
- not require derivative information or other auxiliary knowledge; only the objective function and corresponding fitness levels influence the directions of search.
- use probabilistic transition rules, not deterministic ones.
- generally more straightforward to apply, because no restrictions for the definition of the objective function exist.
- Provide a number of potential solutions to a given problem. The final choice is left to the user. (Thus, in cases where the particular problem does not have one individual solution, for example a family of pareto-optimal solutions), as in the case of multi-objective optimization and scheduling problems, then the evolutionary algorithm is potentially useful for identifying these alternative solutions simultaneously.

#### 4.1. Algorithmic View of EP in VSSLMSEV

Evolutionary programming (EP), developed by Fogel et al. traditionally has used representations that are tailored to the problem domain. For example, in real valued optimization problems, the individuals within the population are real-valued vectors. Similarly, ordered lists are used for traveling salesman problems, and graphs for applications with finite state machines. EP is often used as an optimizer, although it arose from the desire to generate machine intelligence. The outline of the evolutionary programming algorithm is shown below

(i). Generate initial population of  $\mu$  individuals and set  $k=1$ , each individual is taken as a pair of real valued vectors  $(p_i, \eta_i), \forall i=\{1, \dots, \mu\}$

(ii). Evaluate the fitness score of each individual  $(p_i, \eta_i), \forall i=\{1, \dots, \mu\}$  of the population based on the objective function.

(iii). Each parent  $(p_i, \eta_i), \forall i=\{1, \dots, \mu\}$  creates a single offspring  $(p'_i, \eta'_i)$  by:

$$p'_i(j) = p_i(j) + \eta_i(j) \cdot N(0,1) \quad \text{-----(3)}$$

$$\eta'_i(j) = \eta_i(j) \exp(\tau' N(0,1) + \tau N_j(0,1)) \quad \text{-----(4)}$$

for  $j=1 \dots n$ . where  $N(0,1)$  is a Gaussian random variable with zero mean and unity standard deviation and  $N_j(0,1)$  is generated a new for each value of  $j$ .  $p_i(j)$ ,  $p'_i(j)$ ,  $\eta_i(j)$ ,  $\eta'_i(j)$  denote the  $j$ th component of vector  $p_i$ ,  $p'_i$ ,  $\eta_i$ ,  $\eta'_i$  respectively. The factor  $\tau$  and  $\tau'$  are commonly set to  $(\sqrt{2\sqrt{n}})^{-1}$  and  $(\sqrt{2n})^{-1}$ .

(iv). Calculate the fitness of each offspring  $(p_i, \eta_i), \forall i=\{1, \dots, \mu\}$

(v). From the union of parents  $(p_i, \eta_i)$  and offspring  $(p_i, \eta_i), \forall i=\{1, \dots, \mu\}$  select the  $\mu$  individuals have the maximum fitness to be parents of the next generation

**5. SIMULATION**

To see the performance of EVSSLMS, for channel, number of taps selected for equalizer is 11 taken. Input signal contains total 500 samples generated randomly through uniform distribution shown in fig1. Gaussian noise having zero mean and 0.01 standard deviation added with input signal as shown in fig2. channel characteristics is given by the vector:

[0.05 -0.063 0.088 -0.126 -0.25 0.9047 0.25 0 0.126 0.038 0.088]

**5.1 Case1: LMS with fixed step size**

A rule of thumb available in number of literatures applied for selecting the step size in order to ensure convergence and good tracking capability in slow varying channel.

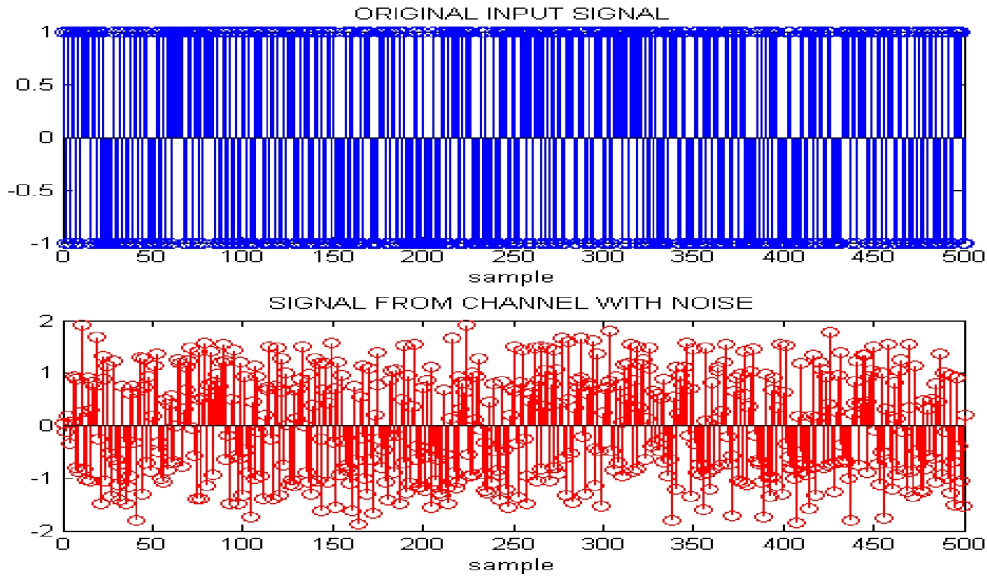
$$\Delta = 1 / (5 * (2K + 1) * P_R) \quad \text{-----(5)}$$

Where  $P_R$  denotes the received signal plus noise power, which can be estimated from the received signal. In the taken input signal value of  $\Delta$  from eq(5) is equal to 0.0088. Three different value of fixed step size applied with LMS to see the difference in performance: (a) 0.11 (b) 0.045 (c) 0.0088 and resulting performance has shown in the fig(3). From the result it is clear that it is very difficult to find the optimal step size.

**5.2 Case 2: LMS With Adaptive Variable Step Size**

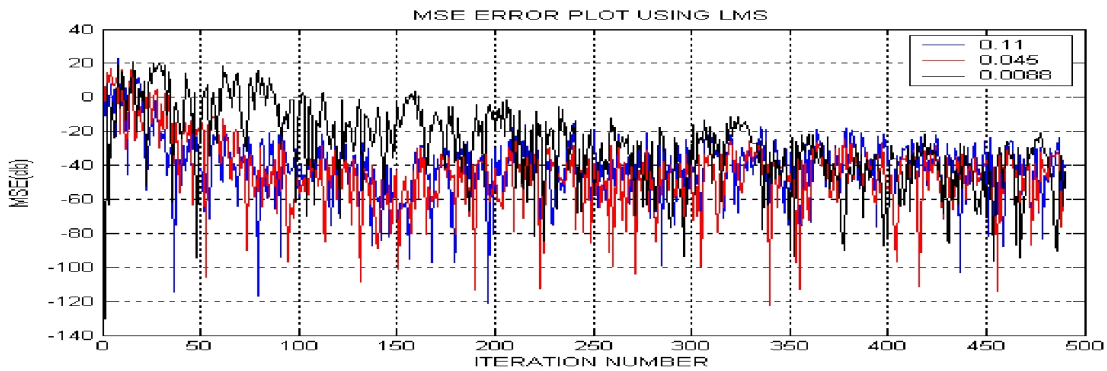
To over the problem of optimal step size evolutionary computation as given above applied. For each iteration one generation created and fittest step size in that generation taken for that particular iteration. From previous generation a new generation created by mutation process defined in eq (3) and in eq (4), for next iteration and process will keep continue until all iteration are not completed.

Parameter setting: Initial population is defined by uniform distribution random variable in the range of [0 1] and  $\eta_i$  is taken as  $0.000005, \forall i=\{1, 2, \dots, \mu\}$  and fitness of solution is defined by  $f = 1 / \text{MSE}$ .

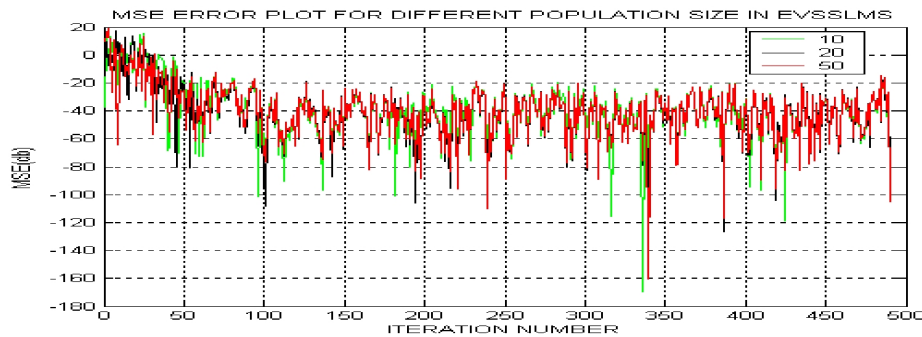


**FIGURE 1:** generated input signal and signal with noise from channel

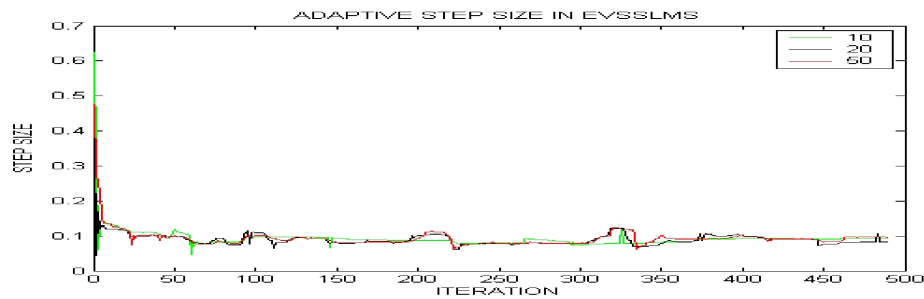
To see the performance of EVSSLMS three different population size (i) 10 (ii) 20 (iii)50, taken for same input signal as in case(1) and outcome has shown in fig3.it is clear from result performance is vary very little with higher population size. Plot of adapted vary step size also shown in fig4.the requirement of initial higher value and later lower value of step size easily captured by evolutionary programming.



**FIGURE 2:** fixed step size performance of LMS with step size equal to 0.11, 0.045 and 0.0088



**FIGURE 3:** performance of EVSSLMS with different population size 10,20 and 50.



**FIGURE 4:** Defined step size by EVSSLMS for different population size

## 6. CONCLUSION

The problem of optimal variable step size integrated with LMS algorithm has solved with the involvement of evolutionary programming. Presented method is robust and does not require the statistical characteristics of input signal as in the case of other existing solutions. Very good convergence and tracking capability can be achieved automatically by presented method. Performance of proposed VSSLMSEV also checked with different population size and it has shown that with less population performance is also equally well and in result higher speed of solution.

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