On The Fundamental Aspects of Demodulation

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Abstract

When the instantaneous amplitude, phase and frequency of a carrier wave are modulated with the information signal for transmission, it is known that the receiver works on the basis of the received signal and a knowledge of the carrier frequency. The question is: If the receiver does not have the a priori information about the carrier frequency, is it possible to carry out the demodulation process? This tutorial lecture answers this question by looking into the very fundamental process by which the modulated wave is generated. It critically looks into the energy separation algorithm for signal analysis and suggests modification for distortionless demodulation of an FM signal, and recovery of sub-carrier signals.

Keywords: Teager Energy Operator, Modulation, Mixer, Energy Seperation Algorithm, AM, FM.

1. INTRODUCTION

The purpose of communication is to overcome the barrier of distance in such a way as to achieve simultaneous transmission of many messages. What is done in this respect is to first select high frequency signals, called carriers and then to modulate amplitude, phase and frequency as the case be with the information bearing signals and transmitted over a channel. Or all the modulated carriers, called sub-carriers, are clubbed together and used to modulate a carrier having still higher frequency to be finally transmitted over a channel, which is known as sub-carrier multiplexing.

The function of the receiver is to operate on the received signal so as to reconstruct a recognizable form of the original message. It is worthwhile to note that at the receiving end the demodulation process starts by tuning the carrier which requires a precise knowledge of the carrier frequency and then operate to recover the message. For example,

a. For an *AM* wave the signal is first captured by tuning the carrier frequency and then to use the envelope detector to extract the baseband signal and
b. For an *FM* wave the signal is first captured by tuning the carrier frequency and then to use the slope circuit plus the envelope detector to recover the baseband signal.

Thus the entire process of reconstructing the baseband signal is a two-step process based on a priori information of the carrier and the modulation formats. That is, given the right signal and carrier frequency, the amplitude and the instantaneous frequency are estimated. This paper, on the other hand, looks into the problem of reception from a different angle. That is, it first estimates the energy required for generating the modulated signal and separate into amplitude and frequency. On elaboration, it is stated as follows:

a. It does not presume a precise knowledge of the carrier frequency.
b. It minimizes the multi-step process into a single step for recovering the baseband signal.
c. It suggests a novel method for recovering the baseband signal from an *FM* wave without distortion.

It begins with an approach based on the use of energy tracking operator known as '*Teager Energy Operator*'. This is followed by some applications of TEO and a proposed modification of the conventional *energy separation algorithm* for the demodulation of FM signals [1-5].

2. TEAGER ENERGY OPERATOR

We look into the process of analyzing the received signal from the point of view of energy required to generate the signal. To begin let us consider an un-modulated signal like,

$$x(t) = ACos\left(\omega_c t + \theta\right) \qquad (1)$$

The differential equation of the system that is capable of generating such a waveform is,

$$\frac{d^2x}{dt^2} + \omega_c^2 x = 0 \qquad (2)$$

This is the system equation of an un-driven linear un-damped linear oscillator. It could be mechanical system consisting of a mass '*m*' attached to a spring of constant '*k*' or a lossless charged capacitor '*C*' is discharging through a lossless inductance '*L*'. In the case of mass-spring system '*x*' is the displacement and capacitor-inductance system '*x*' is the voltage across the plates of the capacitor. Thus,

$$\omega_c^2 = \sqrt{\frac{k}{m}} \; or \; \frac{1}{LC} \qquad (3)$$

The differential equation (2) can also be expressed as

$$\frac{m}{2}\left(\frac{dx}{dt}\right)^2 + \frac{k}{2}x^2 = \text{Constant energy} = E = \frac{m}{2}\left(A\omega_C\right)^2 \qquad (4)$$

Or,

$$\frac{L}{2}\left(\frac{dV}{dt}\right)^2 + \frac{V^2}{2C} = \text{Constant energy} = E = \frac{L}{2}\left(A\omega_C\right)^2 \qquad (4a)$$

Thus the energy of the linear oscillator is proportional to the square of both the amplitude and the frequency. Further, using (2) and (4) or (4a) it easy to show that,

$$\left(\frac{dx(t)}{dt}\right)^2 - x(t)\cdot\left(\frac{d^2x(t)}{dt^2}\right) = \frac{2E}{m} = \frac{2E}{L} = \left(A\omega_C\right)^2 \qquad (5)$$

The left hand side is defined as a nonlinear signal operator and is denoted as $\psi[x(t)]$ which is also called a *Teager Energy Operator (TEO)* [2-4]

$$\psi\left[x(t)\right] = \left(\frac{dx(t)}{dt}\right)^2 - x(t)\cdot\left(\frac{d^2x(t)}{dt^2}\right) \qquad (6)$$

It is used to for tracking the energy of a source generating an oscillation, also used for signal and speech *AM-FM* demodulation [1, 4, 7,9]. It is interesting to note that another operator known as Lie bracket,

$$[x, y] = \dot{x}y - x\dot{y} = -[y, x] \qquad (6a)$$

This operator can be used to derive (6) by putting, $y = \dot{x}$. Lie bracket is used to measure the instantaneous differences in the relative rate of change between two signals (*x,y*). Thus one can verify,

$$[x, \dot{x}] = \Psi[x(t)] \qquad (6b)$$

It is to be noted that the output of the *TEO* is equal to the energy (per half unit mass or half unit inductance). Therefore, it must have a positive value. The energy function is a very local property of the signal depending only on the signal and its first two time derivatives. A close look at the *TEO* reveals that it involves nonlinear operation on the signal. This is also true for the discrete version of the *TEO* [cf. (7), (7a)].

The discrete version of the *Teager Energy Operator* can be shown to be given by,

$$\psi[x(n)] = x^2(n) - x(n-1)x(n+1) \qquad (7)$$

Incidentally, when Teager proposed this algorithm he did not provide details regarding its derivations. This has a reference to a letter written to J. F. Kaiser by T. M. Teager in 1985. The modified Teager-Kaiser energy operator can be defined as,

$$\psi_k\left(x(n)\right) = x^2(n-k) - x(n)x(n-2k) \qquad (7a)$$

For any signals *x* and and *y* it can be shown that,

$$\psi[x(t)y(t)] = x^2(t)\psi[y(t)] + y^2(t)\psi[x(t)] \qquad (8)$$

$$\psi[x(t)+y(t)] = \psi[x(t)] + \psi[y(t)] + 2\frac{dx}{dt}\cdot\frac{dy}{dt} - x\frac{d^2y}{dt^2} + y\frac{d^2x}{dt^2} \qquad (9)$$

The discrete operator has similar property, namely

$$\psi[x(n)y(n)] = x^2(n)\psi[y(n)] + y^2(n)\psi[x(n)] - \psi[x(n)]\psi[y(n)] \qquad (10)$$

Again energy operator is invariant with respect to ‘n’,

$$\psi[x(n+1)] = \psi[x(n)] \qquad (11)$$

Looking into (6), (7) and (7a) it is seen that *TEO* involves nonlinear operation. This can be easily implemented in the hardware.

3. DETECTION TECHNIQUES

In this section we shall discuss the various methodologies of demodulating the signals that are widely used in the field of communication.

3.1 Tone Signal Detection

A continuous wave tone signal can be expressed as,

$$x(t) = A\cos(\omega_C t + \varphi) \qquad (12)$$

Now applying the TEO on the continuous signal we can write that,

$$\psi\left[x(t)\right] = \left(A\omega_c\right)^2 \qquad (13)$$

At this point it is worthwhile to note that,

i. *ψ[x(t)]* is independent of the initial phase of oscillations.
ii. It is robust even the signal passes through zero, as no division operation is required.
iii. It is capable of responding very quickly to changes in amplitude and frequency

It can be easily shown that if we operate *dx/dt* by the energy operator then,

$$\psi\left[\frac{dx}{dt}\right] = \omega_C^2\left(A\omega_C\right)^2 \qquad (14)$$

Thus using the TEO one can express the amplitude and frequency of the tone signal as,

$$\left.\begin{aligned} A &= \sqrt{\frac{\psi\left[x(t)\right]^2}{\psi\left[dx/dt\right]}} \quad .....(15a) \\ \omega_C &= \sqrt{\frac{\psi\left[dx/dt\right]}{\psi\left[x(t)\right]}} ....(15b) \end{aligned}\right\}$$

3.2 Discrete Tone Signal Detection

Similar to the CW tone signal [1-5] the discrete tone signal can be expressed as,

$$x(n) = A\cos\left(\Omega n + \varphi\right) \qquad (16)$$

where $\Omega = (\omega_c) / 2\pi f_s$ and 'f_s' is the sample frequency. In this case it is easily shown that,

$$\psi\left[x(n)\right] = A^2 sin^2\left(\Omega\right) \qquad (17)$$

Using the relation, $y(n) = x(n+1) - x(n)$, it can be shown that,

$$\psi\left[y(n)\right] = 4A^2 sin^2\left(\frac{\Omega}{2}\right) sin^2(\Omega) \qquad (18)$$

Hence the frequency and amplitude of the discrete tone signal can be estimated as,

$$\left.\begin{aligned} \Omega &= \sin^{-1}\left[1-\frac{\psi\left(x(n+1)-x(n)\right)}{2\psi\left(x(n)\right)}\right] ......(a) \\ A &= \frac{\sqrt{\psi\left[x(n)\right]}}{\sin(\Omega)} ....(b) \end{aligned}\right\} \quad (19)$$

Using the identity,

$$\psi\left[x(n+1)-x(n)\right] = 2\psi\left[x(n)\right] - \left[x(n)x(n+1)\right] - \left[x(n-1)x(n+2)\right] \quad (20)$$

the relation (19a) can be expressed as,

$$\Omega = \cos^{-1}\left(\frac{x(n)x(n+1)-x(n-1)x(n+2)}{2\psi\left[x(n)\right]}\right) \quad (21)$$

And thus we can also write,

$$\omega_c = \left(2\pi f_s\right)\cdot\cos^{-1}\left(\frac{x(n)x(n+1)-x(n-1)x(n+2)}{2\psi\left[x(n)\right]}\right) \quad (22)$$

Now using the relation of the modified Teager-Kaiser energy operator as defined in (7) for the discrete time tone signal we can write that,

$$\psi_k\left(x(n)\right) = A^2\sin^2\left(k\Omega\right) \quad (23)$$

Now if the signal is passed through as FIR Filter with the transfer function like,

$$F(z) = \frac{1}{2}\left(z^{-l}+z^{-m}\right) \quad (24)$$

Then we have.

$$F\left(\Omega\right) = \frac{\psi_k\left[\frac{\left(x(n-l)+x(n-m)\right)}{2}\right]}{\psi_k\left[x(n)\right]} = \cos^2\left[\left(\frac{l-m}{2}\right)\Omega\right] \quad (25)$$

Using the above relation it can be easily shown that,

$$A^2 = \frac{\psi_k\left[x(n)\right]}{1-F(\Omega)} \quad (26)$$

3.3 Continuous FM Signal Detection

In this section we consider the application of the energy separation algorithm (*ESA*) for the demodulation of an *FM* signal. *ESA* is employed here for its simplicity, efficiency, low complexity and its excellent adaptive nature [5-20]. Let us consider the analog *FM* signal can be expressed like,

$$x(t) = cos\left[\omega_c t + \Delta\int_0^t e(t)dt + \theta\right] = cos\left[\varphi(t)\right] \qquad (27)$$

Therefore, the instantaneous frequency of the *FM* signal will be,

$$\omega(t) = \omega_c + \Delta e(t) \qquad (28)$$

Note that $\varphi(t)$ is the instantaneous phase of the *FM* signal. Applying *TEO* to the *FM* signal it is found that

$$\psi\left[cos\left(\varphi(t)\right)\right] = \left[\frac{d\varphi}{dt}\right]^2 + \frac{1}{2}\frac{d^2\varphi}{dt^2} sin\left(2\varphi(t)\right) \qquad (29)$$

Auditing the expression on the right hand side of (29) it is seen that the second term indicates the error signal. The first when square-rooted gives the demodulated output for the FM signal [10, 18, 19]. At this stage it is worthwhile to note that if the energy is applied to the Quadrature component of the signal it is seen

$$\psi\left[sin\left(\varphi(t)\right)\right] = \left[\frac{d\varphi}{dt}\right]^2 - \frac{1}{2}\frac{d^2\varphi}{dt^2} sin\left(2\varphi(t)\right) \qquad (30)$$

From (29) and (30) it is easy to show that,

$$\frac{d\varphi}{dt} = \sqrt{\frac{\psi\left[cos(\varphi)\right] + \psi\left[sin(\varphi)\right]}{2}} \qquad (31)$$

And,
$$\frac{1}{2}\frac{d^2\varphi}{dt^2} sin\left(2\varphi(t)\right) = \frac{1}{2}\left[\psi\left(cos\varphi\right) - \psi\left(sin\varphi\right)\right] \qquad (32)$$

It is worthwhile to note that in order to demodulate an *FM* signal, one has to take recourse to square-rooting (cf. 28). This can be overcome if the *TEO* is preceded by *mixers*. One can assume the outputs of two mixers as

$$\left.\begin{aligned} x_1(t) &= \cos\left[\omega_1 t + \Delta\int_0^t e(t)dt + \theta\right] = cos\left[\varphi_1(t)\right] \text{.......(a)} \\ x_2(t) &= \cos\left[\omega_2 t + \Delta\int_0^t e(t)dt + \theta\right] = cos\left[\varphi_2(t)\right] \text{.....(b)} \end{aligned}\right\} \qquad (33)$$

If these two mixer outputs are fed to two *TEO*s, then outputs are expressed in (34 a) and (34 b).

$$\left.\begin{aligned} \psi\left[x_1(t)\right] &= \left[\frac{d\varphi_1}{dt}\right]^2 + \frac{1}{2}\frac{d^2\varphi_1}{dt^2} sin\left(2\varphi_1(t)\right) \text{............}(a) \\ \psi\left[x_2(t)\right] &= \left[\frac{d\varphi_2}{dt}\right]^2 + \frac{1}{2}\frac{d^2\varphi_2}{dt^2} sin\left(2\varphi_2(t)\right) \text{...}(b) \end{aligned}\right\} \qquad (34)$$

After taking the difference of the outputs of the both *TEO* and passing through a low pass filter it is not difficult to show that,

$$\left\{\left[\psi\left(x_1(t)\right)\right]-\left[\psi\left(x_2(t)\right)\right]\right\}_{LPF} \cong \left(\omega_1-\omega_2\right)\left[\omega_1+\omega_2+2\frac{d\varphi}{dt}\right] \quad (35)$$

where, $\phi(t)=\Delta\int e(t)dt$.

From the above analysis it is verified that the use of square rooter is not necessary. The proposed algorithm along with the simulation results are shown in Fig. 1.

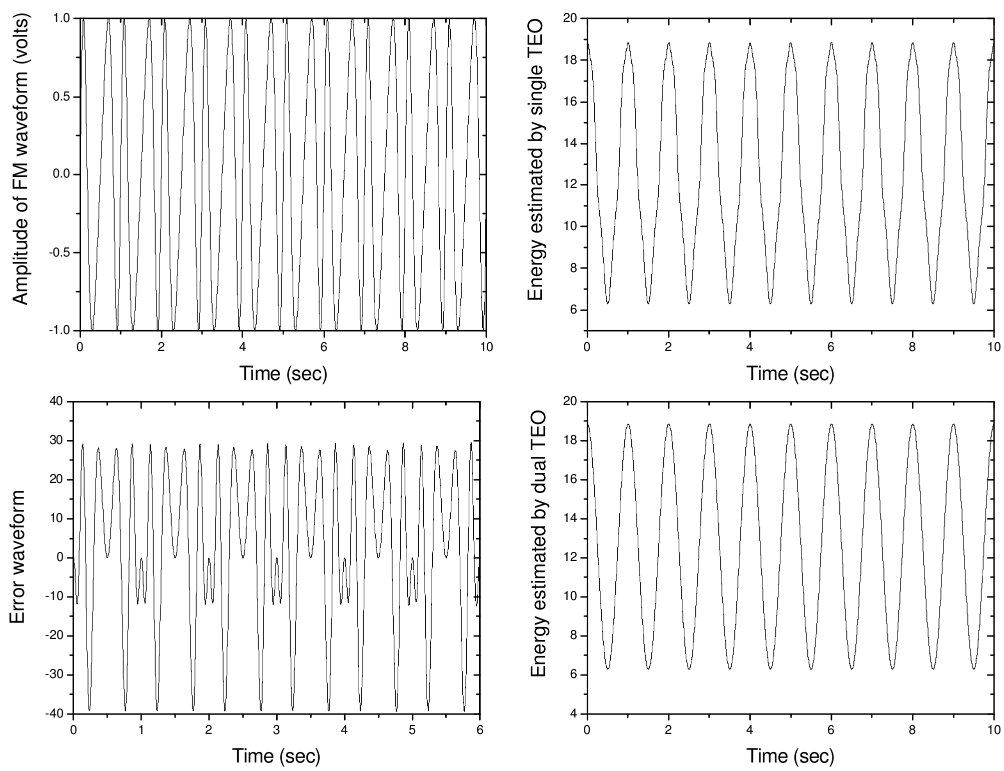

Figure 1: The figure shows the response of the TEO in presence of FM signal (top left). The single TEO based demodulator has distortion (bottom left) in the output waveform (top right) while the error waveform is completely absent in dual TEO based demodulator (bottom right).

3.4 Discrete Time FM Chirp Signal Detection

Let the frequency of the signal at the beginning of the time window be Ω_c ($\omega_c T$) and frequency decrease by $\Delta\Omega$ in N samples of the signal. That is,

$$\Omega_i\left(n\right)=\Omega_c-\frac{\Delta\Omega}{N}n \quad (36)$$

In that case the received signal can be written as,

$$x(n) = A\cos\left(\Omega_c t - \frac{\Delta\Omega}{N}n^2 + \theta\right) = A\cos\left[\phi(n) + \theta\right] \quad (37)$$

Therefore on operation of *TEO* we can write that,

$$\psi\left[x(n)\right] = A^2 \sin^2\left[\Omega_i\left(n\right)\right] + \frac{A^2}{2}\sin\left(\frac{\Delta\Omega}{N}\right)\sin\left[2\phi\left(n\right) + 2\theta + \frac{\Delta\Omega}{N}\right] \quad (38)$$

Likewise, if we apply *TEO* on an In-phase and Quadrature of the signal [10, 18, 19], one gets,

$$\left.\begin{aligned} \psi\left[x(n)\right]_{\theta=0} &= A^2 \sin^2\left[\Omega_i\left(n\right)\right] + \frac{A^2}{2} sin\left(\frac{\Delta\Omega}{N}\right) sin\left[2\phi\left(n\right) + \frac{\Delta\Omega}{N}\right] \text{....(a)} \\ \psi\left[x(n)\right]_{\theta=\frac{\pi}{2}} &= A^2 \sin^2\left[\Omega_i\left(n\right)\right] - \frac{A^2}{2} sin\left(\frac{\Delta\Omega}{N}\right) sin\left[2\phi\left(n\right) + \frac{\Delta\Omega}{N}\right] \text{...(b)} \end{aligned}\right\} \quad (39)$$

Similar to the continuous *FM* wave signal demodulation we get by adding (39a) and (39b),

$$\psi\left[x(t)\right]_{\theta=0} + \psi\left[x(t)\right]_{\theta=\frac{\pi}{2}} = 2A^2 \sin^2\left[\Omega_i\left(n\right)\right] \quad (40)$$

The right hand side gives the information of the chirp frequency. These two algorithms give the demodulation output and the error in demodulation if the concept of In-phase and Quadrature components is taken into consideration. The algorithms shown given in (40) is simulated in SIMULINK, and the results are shown in Fig. 2.

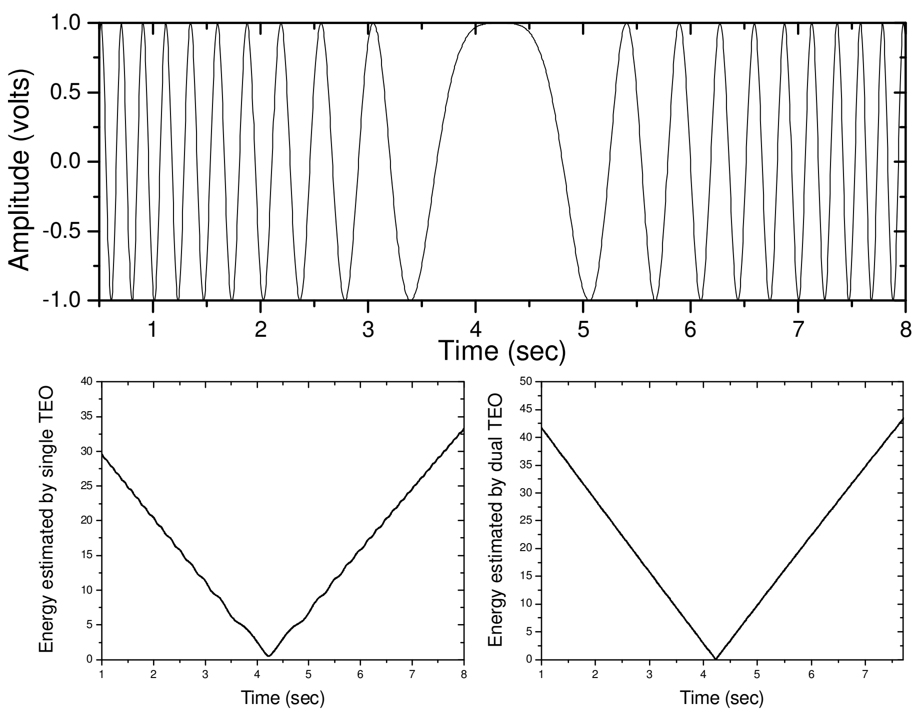

Figure 2: Energy operator based estimation of FM chirp signal. The output of single TEO based demodulator contains distortion in the waveform which is completely absent in the dual TEO based demodulator.

3.5 Carrier Frequency Estimation

The carrier frequency of the FM signal can also be easily determined which leads us to another added advantage of using TEO demodulator over conventional demodulator. Referring to the incoming FM signal is in the form of,

$$x(t) = A\cos\left[\varphi(t)\right] \qquad (41)$$

We can approximately write that,

$$y(t) = \frac{dx}{dt} \cong -A\omega_c \sin\left[\varphi(t)\right] \qquad (42)$$

It can be shown that,

$$\left.\begin{aligned} \psi\left[x(t)_I\right] + \psi\left[x(t)_Q\right] &= 2A^2\left[\frac{d\varphi}{dt}\right]^2 \\ \psi\left[y(t)_I\right] + \psi\left[y(t)_Q\right] &= 2A^2\omega_c^2\left[\frac{d\varphi}{dt}\right]^2 \end{aligned}\right\} \qquad (43)$$

Therefore from (42) and (43) we have,

$$\omega_c = \sqrt{\frac{\psi\left[y(t)_I\right] + \psi\left[y(t)_Q\right]}{\psi\left[x(t)_I\right] + \psi\left[x(t)_Q\right]}} \qquad (44)$$

Using this algorithm one can approximately determine the carrier frequency and then adjust the center frequency of a phase locked demodulator and extract the modulating signal.

3.6 Mixer With a Difference

Interference is an annoying factor that degrades the performance of a demodulator. In order to overcome this difficulty, the demodulator is usually preceded by a band pass limiter. Instead the net incoming signal is led to pass through a *TEO* whence it can be shown that the output is,

$$\begin{aligned} \psi\left[R(t)\right] \cong {} & \left(\omega_1 A\right)^2 + \left(\omega_1 \alpha A\right)^2 + \frac{1}{2}\left(\alpha A^2\right)\left(\omega_1 + \omega_i\right)^2 cos\left[\left(\omega_1 - \omega_i\right)t + \phi(t)\right] \\ & + \frac{1}{2}\left(\alpha A^2\right)\left(\omega_1 - \omega_i\right)^2 cos\left[\left(\omega_1 + \omega_i\right)t + \phi(t)\right] + \frac{1}{2}\alpha A^2 \frac{d^2\phi}{dt^2}\left[sin(\omega_1 + \omega_I)t + \cdots\right] \end{aligned} \qquad (45)$$

Where,

$$R(t) = Acos\left(\omega_1 t + \phi(t)\right) + \alpha Acos\left(\omega_i t\right) \qquad (46)$$

In general if the signals are cos $(\omega_1 t)$ and cos $(\omega_2 t)$, then

$$\psi\left[R(t)\right] = \left(\dot{\phi}_1 + \dot{\phi}_2\right)\cos\left(\phi_1 - \phi_2\right) + \left(\dot{\phi}_1 - \dot{\phi}_2\right)\cos\left(\phi_1 + \phi_2\right) \qquad (47)$$

It is interesting to note that the interference enhance the signal component and reduces the unwanted component. Not the multiplying factors $(\omega_1+\omega_i)^2$ and $(\omega_1-\omega_i)^2$. These are all illustrated in the Fig. 3. It is to be noted that the first difference frequency term dominates over the other terms.

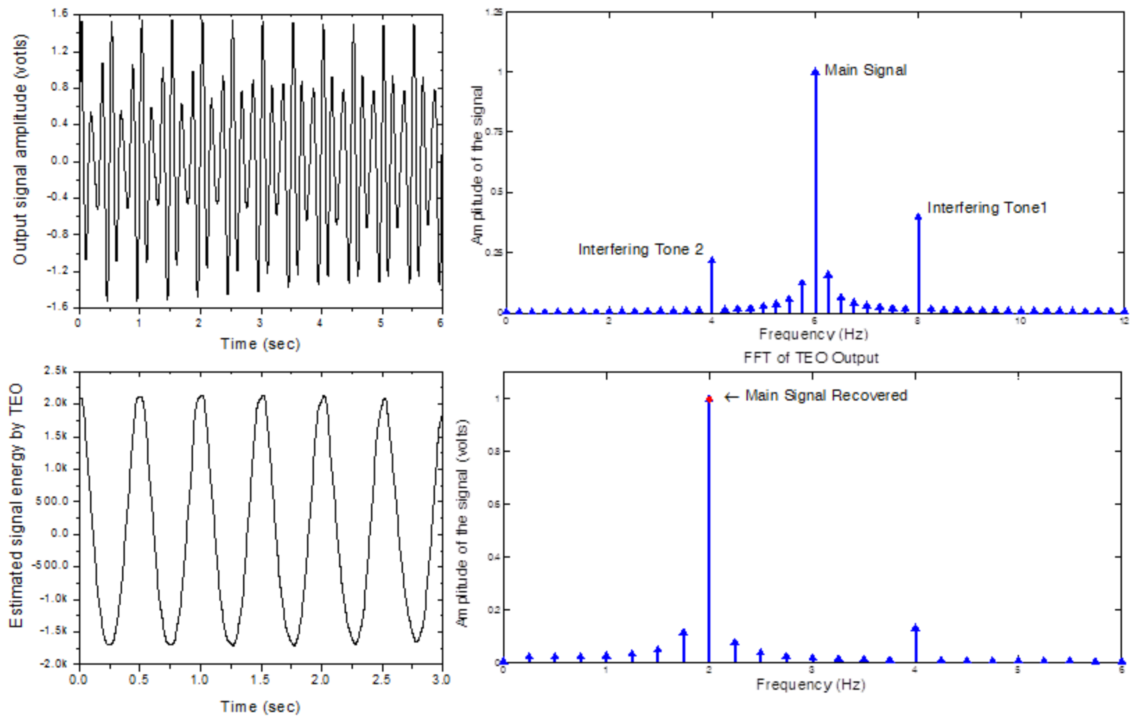

FIGURE 3: Response of TEO based mixer in time and frequency domain.

4. CONCLUSION

In this paper we have presented the non-conventional detection schemes of the commonly used signals. We have proposed a new variety of mixer in the paper along with a modified energy separation algorithm for the distortion-less demodulation of the FM signal. Demodulation of discrete FM chirp signals and subcarrier multiplexed signals are also discussed here.

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