Construction of The Sampled Signal Up To Any Frequency While Keeping The Sampling Rate Fixed

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Abstract

In this paper we will try to develop a method that will let us construct up to any frequency by some additional work we propose. With this method we can construct up to any frequency by adding more hardware to the system with the same sampling rate. By increasing the hardware complexity and keeping the same sampling rate we can reduce the information loss in a proportional manner.

Keywords: Sampling, Filtering, Aliasing, Reconstruction, Heterodynes.

1. INTRODUCTION

Technically, we are proposing to decompose a signal into several separate signals by band passing the original signal. We then propose to process each signal and then to recombine the several processed signals. All this processing is being done in order to use a smaller sampling rate. There are several problems associated with this procedure that must be discussed in a paper on the procedure. First, practical band pass filters are only approximations of ideal band pass filters. We must discuss the effect of this approximation. Second, there is a lot of processing in the proposed method. We then must compare this method with others.

2. METHODOLOGY

In this method the input signal x(t) is applied to many channels . The first channel with a pre-pass (low pass) filter with a frequency band from 0 to Fs/2 Hz. the second channel with a second pre-pass (band pass) filter with a frequency band from Fs/2 to Fs . The third channel with a pre-pass (band pass) and a frequency band from; Fs to 3*Fs/2, and so on up to n-channels; where the number of channels n is choose to satisfy the maximum frequency component of the output for the reconstructed signal. In this way we can divide the spectrum of the signal into intervals each with Fs/2 band starting from 0. Now if we sample the output of each filter at Fs then taking the DFT for each channel samples we will get a spectrum from 0 to Fs/2 for each channel output. At this point if the spectrum of odd channels i.e. the frequency band that lies in range [m*Fs <f< (m+1/2)*Fs] is placed into the interval band of the pre pass filter for that channel. And the mirror image of the spectrum around the (Fs/2) axis for even channels i.e. (m-1/2)*Fs<f<m*Fs is placed into the interval band of the pre pass for that channel. We will get the spectrum for x(t) constructed up to any frequency if the channels cover that frequency . As we can see by some additional work developed, we can increase frequency limit for reconstruction of signals without increasing the sampling frequency Fs.

In this approach to the solution I tried to follow the guidelines. I divided the problem into two parts. One part is to try to solve the problem in the frequency domain the second is to try to solve it in the time domain.

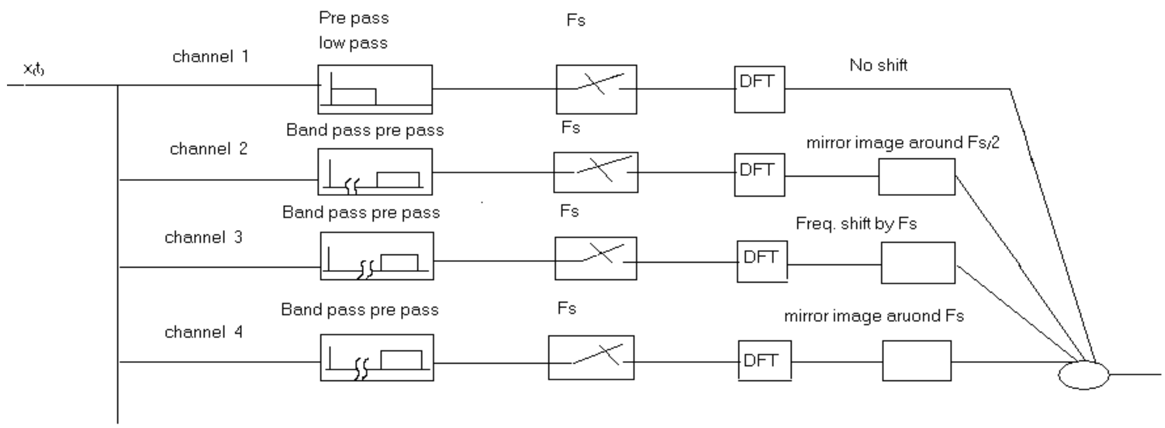

FIGURE 1: The System Block Diagram.

Because it is very hard, in practice, to build those ideal band pass filters. We suggest solving this problem by super heterodyning. By that I mean we build a high quality low pass filter with sharp transition with pass band from 0 to Fs/2 Hz. For the second channel we project the spectrum in the band Fs/2 to Fs into the band of the low pass filter. For the third channel we project the spectrum in the band Fs to 3*Fs/2 into the band of the low pass filter. And so on. I hope this will make us avoid the problem of having too many band pass filters which are difficult to have them ideal in practice.

let us suggest another method. Say we sample the original signal at a given rate, say Fs. Then shift the signal in time by a given amount and sample again at Fs. Again shift the signal in time and sample again at Fs. The samples from the various samplings of the original signal can then be combined by interlacing them to form a sample sequence equivalent to several times Fs. Note that the several samplings of the original signal can be done in parallel. My proposed technique would thus take no more time than sampling the original signal at the several times Fs. If this were done, my proposed technique is equivalent to replacing a sampler with a high sampling rate with several at a lower sampling rate. This procedure would thus avoid the problems involved with the other proposed procedure that I mentioned above.

I feel the time delay has to be logarithmic. The first channel is sampled every Ts. The second channel we delay by Ts/2 and sample again at Fs. For the third channel we have to delay now by +Ts/4 and –Ts/4 and sample each delayed wave at Fs. The two delays for the third channel is important to keep the sampling rate fixed and to be able to reconstruct the original signal by having equidistance samples, and so on.

Let me propose an alternative to this technique. Which does not require ideal band pass filters and reconstruction of the samplings of each band passed and frequency shifted waveform? I suggest just sampling the original waveform at a rate above Fn, the Nyquist rate, in the following manner: Call some time instant of the waveform t = 0. Then ;

1. Sample at the rate Fn with the first sample at t = 0.

2. Sample at the rate Fn with the first sample at t = 1/(k)(Fn).

3. Sample at the rate Fn with the first sample at t = 2/(k)(Fn).

4. Sample at the rate Fn with the first sample at t = 3/(k)(Fn)

and so forth until

k-1. Sample at the rate Fn with the first sample at t = (k-1)/(k)(Fn).

Then concatenate the various sample sequences. Note that the resulting sequence obtained is the sequence that would be obtained by sampling the waveform at the rate (k)(Fn).Note that this proposed technique does not require any filters, band shifting or time shifting of the waveform. Also, all the k samplings can be done at the same time if k samplers were used.

3. THEORY PROOF

To proof this proposed method we will take a cosine wave on the input of each interval band channel, as follows;

Channel 1

For an input; X1(t) = cos (2*pi*f1*t)

Where, 0< f1 <Fs/2 and Fs is the sampling frequency

If the channel output is sampled at Fs rate then the output sampled signal will be

X1(n) = cos (2*pi*f1*(n/Fs)) and the reconstructed signal is the same as the input signal i.e.

X1(t) = cos (2*pi*f1*t)

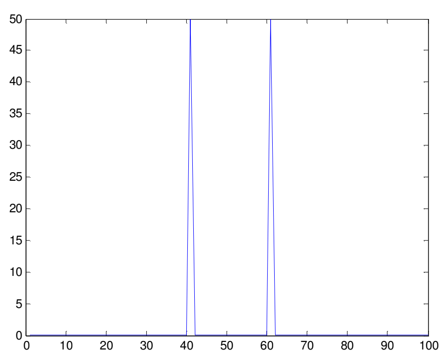

FIGURE 2: Digital Frequency (f1/Fs)=.4.

Channel 2

For the input ; X2(t) =cos(2*pi*f2*t)

Where Fs/2<f2<Fs and Fs is the sampling frequency

The sampled signal will be

X2(n) =cos(2*pi*(f2/Fs)*n)

X2(n) =cos(2*pi*((f2/Fs)-1)*n)

Reconstruction

X2 (t) = cos (2*pi*(f2-Fs)*t)

Which is the mirror image of the input around Fs/2

We can get the input using the developed algorithm

X2(t)=cos((2*pi*(Fs/2+(Fs/2-(f2-Fs)*t))

X2(t)=cos(2*pi*f2*t)

Example : for f2 =70 Hz

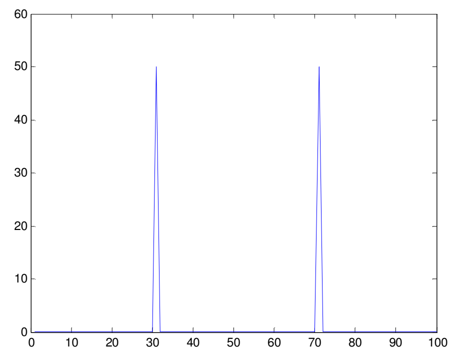

FIGURE 3: Digital frequency (f2/Fs) = .7

Channel 3

For the input ;X3(t)=cos(2*pi*f3*t)

Where Fs<f3<3*Fs/2 and Fs is the sampling rate

The sampled signal is

X3(n)=cos(2*pi*(f3/Fs)*n)

X3(n)=cos(2*pi*((f3/Fs)-1)*n)

The reconstructed signal is

X3(t)=cos(2*pi*(f3-Fs)*t)

Using our methodology and Shifting by Fs to place it in ch3 interval

X3(t)=cos(2*pi*f3*t) which is the input

Example : For F3 = 130 Hz

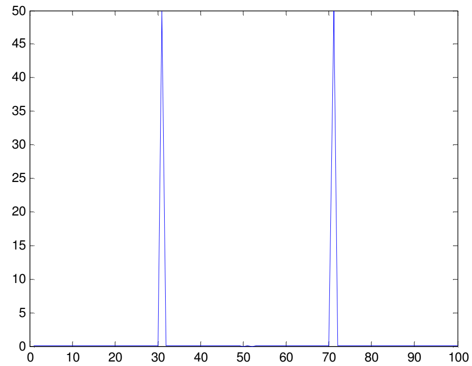

FIGURE 4: Digital Frequency (f3/Fs) = 1.3.

Channel 4

For the input X4(t)=cos(2*pi*f4*t)

where 3*Fs/2<f4<2*Fs and the sampling rate is Fs

The digital signal after sampling is

X4(n)=cos(2*pi*(f4/Fs)*n)

X4(n)=cos(2*pi*(2-(f4/Fs)*n)

Reconstruction

X4(t)=cos(2*pi*(2Fs-f4)*t)

Using our methodology For ch4 we have to take the mirror image around Fs

X4(t)=cos(2*pi*f4*t) which is the same as the input

Example : for f4=190Hz

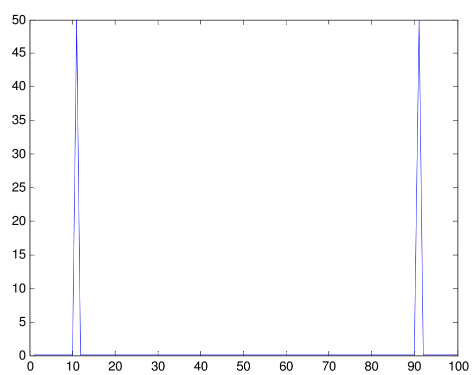

FIGURE 5: Digital Frequency (f4/Fs) = 1.9.

X(t)=X1(t)+X2(t)+X3(t)+X4(t), constructed up to 2*Fs with Fs fixed

Using the same method we can construct up to any frequency as long as we cover the band with enough channels.

4. DISCUSSION

To have a feeling for this method think of a register were increasing the number of bits is like increasing the number of channels. Imagine that we are counting in the decimal system . This method is like inventing the zero. In this system each digit can take ten values. This is smeller to the sampling rate Fs . Increasing the hardware complexity by increasing the number of channels (i.e. increasing the number of bits (or digits) in a register) will enable us to construct up to higher frequency (more numbers or more accuracy)

5. IMPACT OF PROPOSED RESEARCH WORK

This work support real time DSP. I will give a simple example. Let as say we have a sampler at 5kHz and we want to process 20 kHz signal in real time. We can make four channels each one 5kHz band and proceed as explained above. This like replacing the sampler by 20 kHz sampler. The result of the simulation is given in figure 6 were we have two frequencies 4kHz and 16kHz sampled at 5kHz and reconstructed using our method at real time with no loss of information.

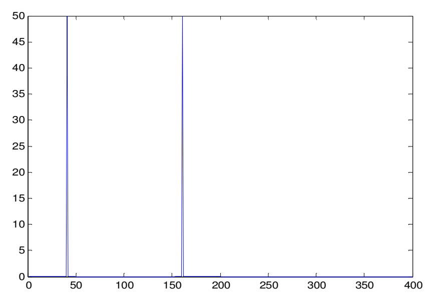

FIGURE 6: Test Using Two Frequencies.

6. COMPARATIVE EVALUATION

This idea was developed from Proakis Digital Communication book and the chapter on Multichannel and Multicarrier systems. There is a large amount of literature on multicarrier digital communication systems. Such systems have been implemented and used for over 35 years. One of the earliest systems, described by Doeltz et al. (1957) and called Kineplex, was used for digital transmission in the HF band. Other early work on multicarrier system design have been reported in the paper by Chang (1966). The use of DFT of multicarrier systems was proposed by Weinstein and Ebert (1971).

7. CONCLUSION

In this paper we were able to develop a procedure that will make us able to construct up to any frequency keeping the sampling frequency fixed. This is done by some additional steps we propose. We provide the proof for this method. A block diagram was given to the final system.

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