# Electrocardiogram Denoised Signal by Discrete Wavelet Transform and Continuous Wavelet Transform

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**Abstract**

One of commonest problems in electrocardiogram (ECG) signal processing is denoising. In this paper a denoising technique based on discrete wavelet transform (DWT) has been developed. To evaluate proposed technique, we compare it to continuous wavelet transform (CWT). Performance evaluation uses parameters like mean square error (MSE) and signal to noise ratio (SNR) computations show that the proposed technique out performs the CWT.

**Keywords:** Continuous Wavelet Transform, Denoising, Discrete Wavelet Transform, Electrocardiogram Signal.

## 1. INTRODUCTION

Electrocardiogram gives information from generation and propagation of electric signals in the heart. This activity is measured and recorded for more than a hundred years [1]. One cardiac cycle in an ECG signal consists of the P, Q, R, S, and T waves, and gives important information on diagnosing cardiac diseases. Most of the clinically useful information in the ECG is found in the amplitudes and intervals defined by its features. ECG signal are characteristically corrupted by noise from electric interference, baseline wandering, and electromyography [2]. Processing is necessary to cancel these noises while conserving information. Therefore the development of accurate and quick method for automatic ECG denoising is of major importance.

In this paper, we propose a denoising technique using discrete wavelet transform (DWT), to evaluate proposed technique we compare it to continuous wavelet transform. The method described is robust and simple to implement. Finally, the ECG signals used in the experiments are obtained from MIT-BIH arrhythmia database.

## 2. MATERIAL

### 2.1 Wavelet Transform

The Fourier transform is a useful tool to analyze the frequency components of the signal, but it is well suited only to the study of stationary signals where all frequencies have an infinite coherence time. But the wavelet transform replaces Fourier transform's waves by a family generated by translations and dilations of a window called a wavelet, it take two arguments: time and scale. The original wavelet transform is called mother wavelet and is employed for generating all basis functions [3]. A set of functions is constructed by scaling and shifting the mother wavelet $\psi(t)$.

Those functions are expressed as follow:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}}\ \psi\left(\frac{t-b}{a}\right) \qquad (1)$$

Where $a \in R^{+} - \{0\}, b \in R.$

### 2.2 Continuous Wavelet Transform

The original signal can be reconstructed by an appropriate integration and this is performed after projecting the given signal on a continuous family of frequency bands. A continuous wavelet transform (CWT) is used to divide a continuous-time function into wavelets. Mathematically, the continuous wavelet transform of a function x(t) is defined as the integral transform of f(t) with a family of wavelet functions, $\psi_{a,b}(t)$:

$$\text{CWT(a,b)}=\frac{1}{\sqrt{a}}\int_{-\infty}^{+\infty} x(t)\ .\ \psi^{*}\left(\frac{t-b}{a}\right) dt \qquad (2)$$

Were the superscript * is the complex conjugate and $\psi_{a,b}{}^{*}$ represents a translated and scaled complex conjugated mother wavelet.

The mother wavelet $\psi$ is invertible when it verifies the condition of admissibility which is stated as [4]:

$$\int_{-\infty}^{+\infty} \frac{\left|\widehat{\psi}(\omega)\right|}{\omega}\, d\omega < \infty \qquad (3)$$

### 2.3 Discrete Wavelet Transform

The Discrete Wavelet Transform (DWT), which is based on sub-band coding is found to yield a fast computation of wavelet transform. It is easy to implement and reduces the computation time and resources required [5].

The DWT is defined by the following equation:

$$\omega(j,k) = \sum_j \sum_k x(k).\, 2^{-j/2}\ \psi(2^{-j}.\, n - k) \qquad (4)$$

Where $\psi(t)$ is a time function with finite energy and fast decay called the wavelet. The DWT analysis can be performed using a fast, pyramidal algorithm related to multirate filter banks [6].

The goal of using DWT in an algorithm of filtering biomedical signals is the possibility of choosing the signal's coefficients with a significant energy and discards the others that have a very low percentage of all energy.

Many mother wavelets are used for computing the wavelet transform and Morlet is one of them. It is expressed as follow [7].

$$\varphi(t) = \frac{1}{\sqrt{a}} \cdot \exp\left[-\left(\frac{t}{T_0}\right)^2\right] \exp(j\omega_0 t) \qquad (5)$$

This analytic wavelet contains a central frequency and having an exponentially decaying time support, while the orthonormal wavelet frequency support of discrete wavelet transform and WPT covers a broader bandwidth. Therefore the proposed Morley's wavelet is more "frequency focused" along each scale [8].

### 2.4 Database

The preliminary tests of the two denoising techniques application were mad on the ECG signals of MIT-BIH Arrhythmia Database. This Database contains many data sets of electrocardiogram signals, mostly abnormal or unhealthy electrocardiograms, but it also contains normal electrocardiograms that can be used as a reference base [9].

## 3. THE PROPOSED DENOISING TECHNIQUE

The objective is to apply the technique of discrete wavelet transform and the technique of continuous wavelet transform on electrocardiograms signals and compared the obtained results to decide that these two techniques gives a better result. In order to evaluate the performance of these proposed techniques, we used the:

- signal to noise ratio (SNR) improvement measure:

$$\mathrm{imp[dB]} = \mathrm{SNR_{output}} - \mathrm{SNR_{input}} \quad (6)$$

$$= 10 \times \log\left[\frac{\sum_i |x_d(i) - x(i)|^2}{\sum_i |x(i) - x_n(i)|^2}\right]$$

Where xd and x represent respectively the denoised and the clean ECG signals and xn denotes the noisy ECG one.

- Mean Square Error (MSE) expressed by the following equation is also used:

$$\mathrm{MSE} = \frac{1}{N}\sum_{n=0}^{N-1}\big(s(n) - \tilde{s}(n)\big)^2 \quad (7)$$

Where $s(n)$ and $\tilde{s}(n)$ are respectively the clean and the denoised signals.

Figure 1 shows the block diagram of the proposed technique and they will be detailed in the next paragraph.

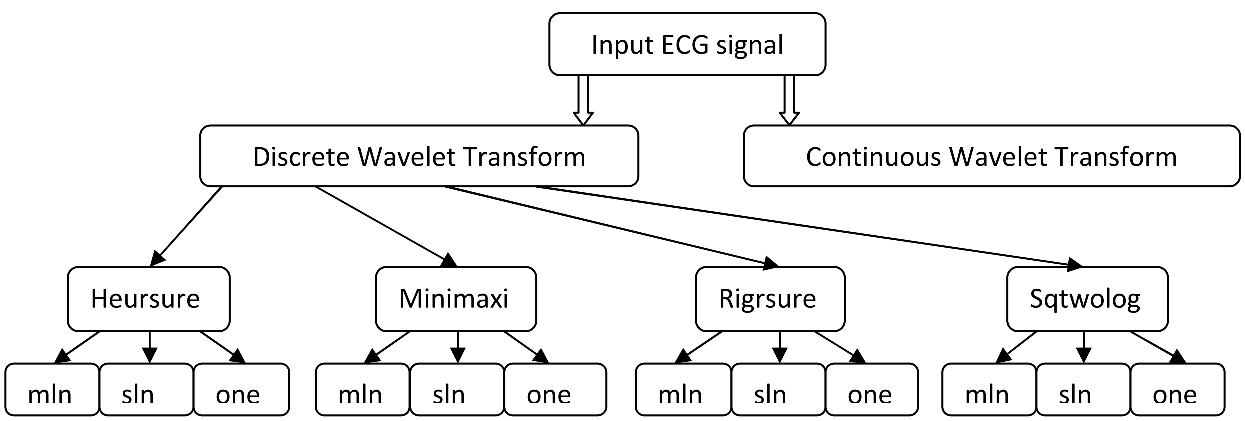


**FIGURE 1:** The Two Methods Chosen For Dinoising**.**

The ECG signals taken from MIT-BIH arrhythmia database are converted in to Matlab format (.mat files) and are sampled at 360 Hz with a resolution of 11 bits.

- 'Rigrsure' uses for the soft threshold estimator, a threshold selection rule based on Stein's Unbiased Estimate of Risk (quadratic loss function). One gets an estimate of the risk for a particular threshold value t. Minimizing the risks in t gives a selection of the threshold value.

- 'Sqtwolog' uses a fixed-form threshold yielding minimax performance multiplied by a small factor proportional to log(length(X)).

- 'Heursure' is a mixture of the two previous options. As a result, if the signal to noise ratio is very small, the SURE estimate is very noisy. If such a situation is detected, the fixed form threshold is used.

- 'Minimaxi' uses a fixed threshold chosen to yield minimax performance for mean square error against an ideal procedure. The minimax principle is used in statistics in order to design estimators. Since the de-noised signal can be assimilated to the estimator of the unknown regression function, the minimax estimator is the one that realizes the minimum of the maximum mean square error obtained for the worst function in a given set.

- 'One' for no rescaling.

- 'Sln' for rescaling using a single estimation of level noise based on first-level coefficients.

- 'Mln' for rescaling done using level-dependent estimation of level noise.

## 4. RESULTS AND DISCUSSION

Table 1 gives the average results of calculation of SNR applied to 30 ECG signals using the technique DWT.

| **Mean Results of 30 ECG Signals Tested** | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Input SNR (dB)** | **Output SNR (dB)** | | | | | | | | | | | |
| | **Heursure** | | | **Minimaxi** | | | **Rigrsure** | | | **Sqtwolog** | | |
| | **Mln** | **sln** | **One** | **Mln** | **Sln** | **One** | **mln** | **sln** | **one** | **mln** | **sln** | **One** |
| -5 | 2.28 | 2.5 | 1.55 | 2.7 | 3.04 | 1.55 | **3.38** | -0.11 | 2.01 | 2.3 | 3.11 | 2.01 |
| 0 | 5.9 | 7.16 | 3.16 | 6.64 | 7.25 | 3.14 | **7.63** | 3.44 | 3.63 | 4.17 | 6.8 | 3.61 |
| 5 | 11.46 | 10.34 | 3.82 | 10.41 | 11.41 | 3.82 | **11.98** | 8.91 | 4.32 | 7.91 | 8.35 | 4.32 |
| 10 | 15.01 | 14.47 | 4.07 | 13.92 | 15.38 | 4.07 | **15.93** | 13.78 | 4.57 | 11.02 | 14.2 | 4.57 |

**TABLE 1:** Comparison Improvement SNR vs SNR (DWT Technique).

Table 1 shows that this method achieves a very good performance when we use the 'rigrsure' and 'mln' argument. Then we use these results when we will make a comparison with the results of the second method (CWT).

Table 2 gives the average results of calculation of SNR applied to 30 ECG signals using the technique CWT.

| Mean Results of 30 ECG Signals Tested | |
|---|---|
| **Input SNR (dB)** | **Output SNR (dB)** |
| -5 | -0.29 |
| 0 | 4.53 |
| 5 | 10.81 |
| 10 | 16.9 |

**TABLE 2:** Comparison Improvement SNR vs SNR (CWT Technique).

Figure 2 illustrate the comparison between the improvement values of SNR obtained by discrete wavelet transform denoising technique and those obtained by the second technique based on continuous wavelet technique.

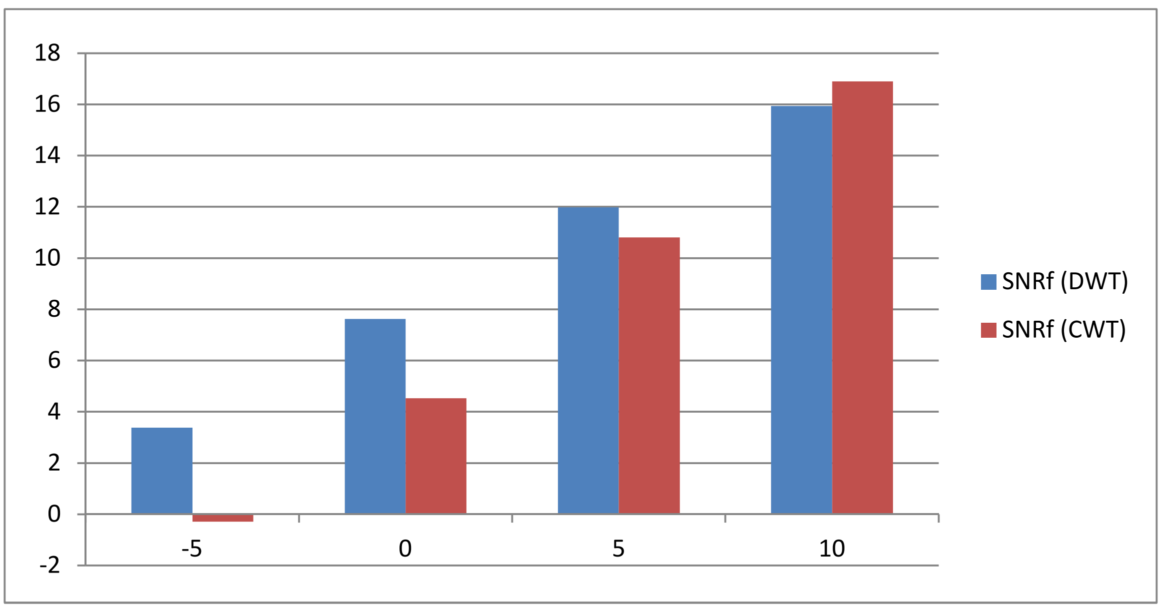


**FIGURE 2:** Output SNR Improvement vs SNR.

Table 3 gives the average results of calculation of MSE applied to 30 ECG signals using the technique DWT.

| Mean Results of 30 ECG Signals Tested | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Input SNR (dB) | MSE*($10^{-4}$) | | | | | | | | | | | |
| | heursure | | | Minimaxi | | | rigrsure | | | Sqtwolog | | |
| | mln | sln | one | mln | Sln | one | mln | Sln | one | mln | sln | One |
| -5 | 11.21 | 10.52 | 12.91 | 10.45 | 9.48 | 12.91 | **8.78** | 22.4 | 11.5 | 10.88 | 9 | 11.5 |
| 0 | 5.07 | 3.64 | 9.04 | 4.21 | 3.55 | 9.04 | **3.32** | 7.38 | 8.12 | 5.86 | 3.94 | 8.12 |
| 5 | 1.35 | 1.55 | 7.84 | 1.78 | 1.37 | 7.84 | **1.21** | 2.47 | 7.02 | 3.1 | 1.64 | 7.02 |
| 10 | 0.61 | 0.68 | 7.32 | 0.81 | 0.55 | 7.32 | **0.48** | 0.71 | 6.72 | 1.52 | 0.72 | 6.74 |

**TABLE 3:** Comparison Improvement SNR vs SNR (DWT Technique).

Table 3 shows that this method achieves a very good performance when we use the 'rigrsure' and 'mln' argument. Then we use these results when we will make a comparison with the results of the second method (CWT).

Table 4 gives the average results of calculation of MSE applied to 30 ECG signals using the technique CWT.

| Mean Results of 30 ECG Signals Tested | |
|---|---|
| Input SNR (dB) | MSE |
| -5 | 0.0331 |
| 0 | 0.0192 |
| 5 | 0.0160 |
| 10 | 0.0154 |

**TABLE 4:** MSE variation vs SNR (CWT technique)

Figure 3 illustrate the comparison between the improvement values of MSE obtained by discrete wavelet transform denoising technique and those obtained by the second technique based on continuous wavelet technique.

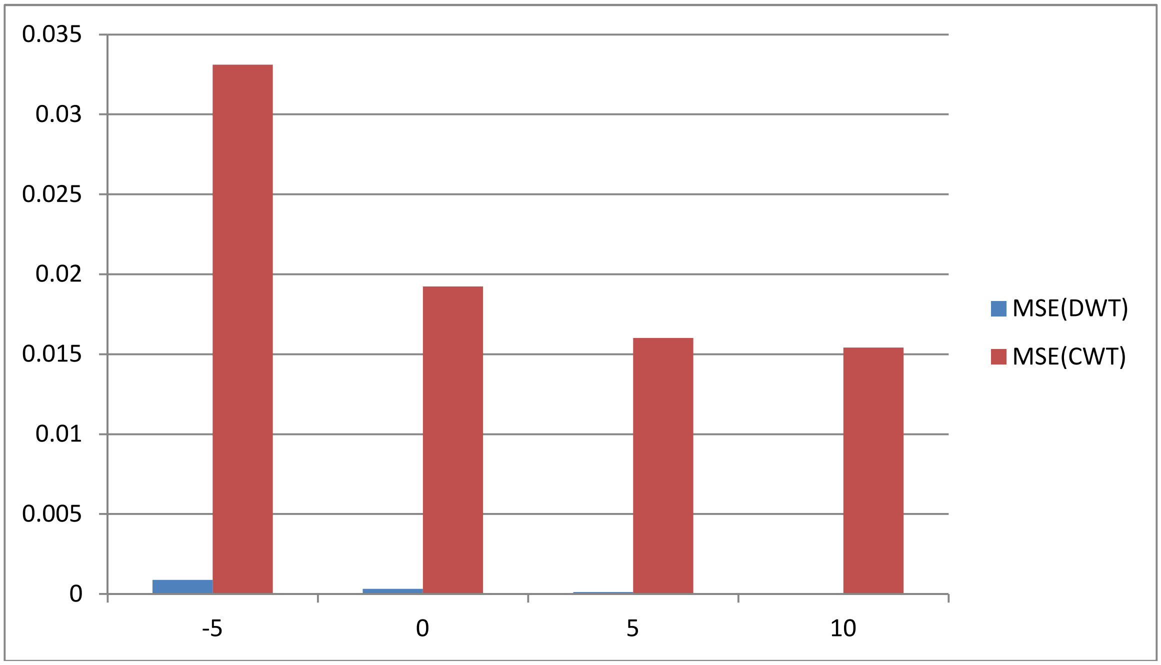


**FIGURE 3:** MSE versus Input SNR of the Two Methods.

The obtained values show that the discrete wavelet transform denoising technique gives better results than continuous wavelet technique.

Figure 4 and Figure 5 show some examples of denoised of ECG signals corrupted by white Gaussian Noise at 5db.

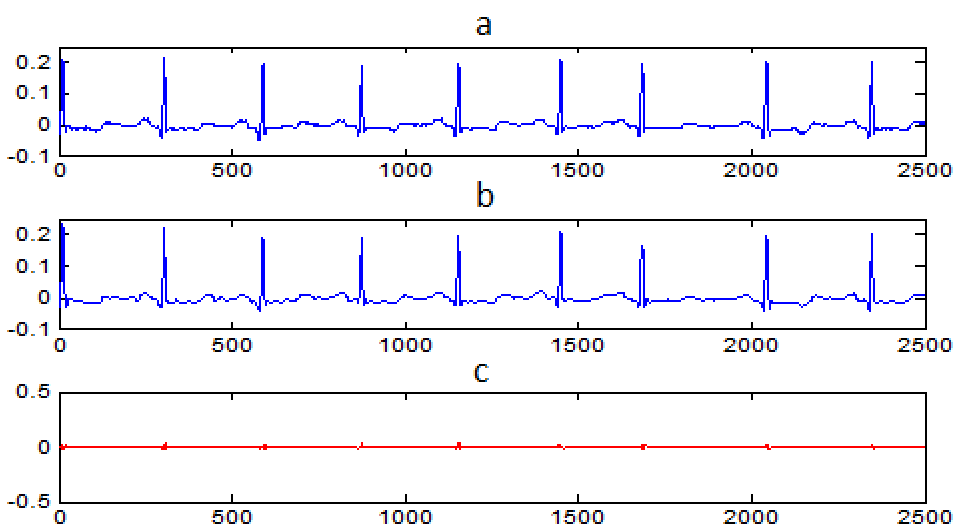


**FIGURE 4:** (**a**) Original ECG signal, (**b**) Filtered ECG signal by DWT, (**c**) Mean Square Error.

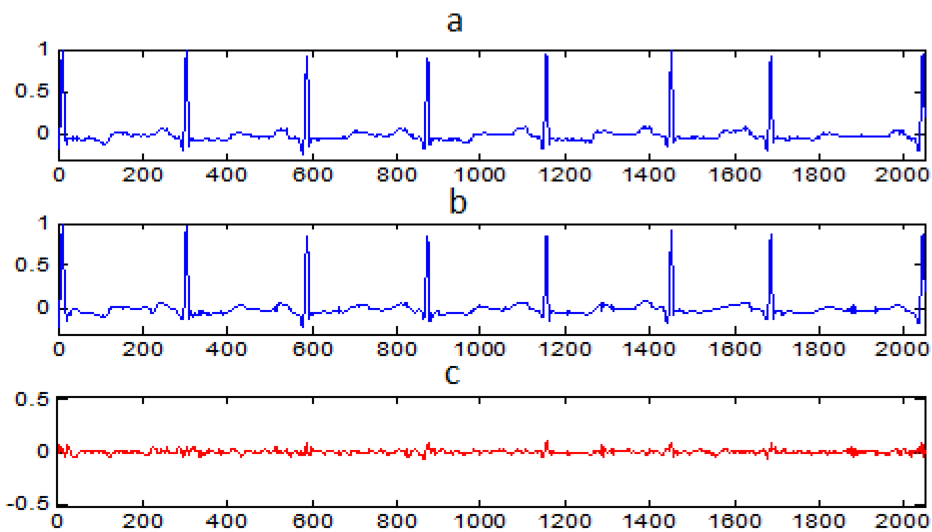


**FIGURE 5:** (**a**) Original ECG signal, (**b**) Filtered ECG signal by CWT, (**c**) Mean Square Error.

## 5. COMPARATIVE STUDY

In this part benchmarking research with other authors was provided to strengthen the results found in this work.

Burhan Ergen [10] proves in their work that the rigresure method has showed a better performance than the other methods in terms of SNR level especially when the decomposition level is higher than 6.

Vijay S. Chourasia et al. [11] shows that the developed wavelet “fetal” along with rigresure and soft threshold provides the best performance for denoising the fetal phonocardiography signals.

Zhang xizheng et al.[12] shows, four threshold denoising and default threshold denoising methods (rigrsure, minimaxi, sqtwolog and heursure) are improved SNR, and have removed some noise. From the denoising effect, the principle of rigrsure threshold and minimaxi threshold denoising are better than the other methods, because the two denoised SNR higher than heursure and sqtwolog denoising methods and its RMSE are smaller.

## 6. CONCLUSION

ECG signals are corrupted by noise from electric interference, electromyography, and baseline wandering. In this paper we propose a denoising technique based on smoothing of discrete wavelet transform coefficients. To evaluate the proposed technique, we compare it to smoothing of continuous wavelet transform. The results obtained from the SNR computation show that the SNR improvement values obtained by the proposed technique outperform the denoising technique based on continuous wavelet transform. Results obtained from MSE computation, also show that the MSE improvement values obtained by using the proposed technique are better than those obtained by second technique and this is by reducing the difference between the original and denoised signals.

## 7. REFERENCES

[1] M. Fira and L.Goras, "Biomedical Signal Compression based on Basis Pursuit", International Journal of Advanced Science and Technology, Vol. 14, January 2010.

[2] K.D. Chinchkhede, G.S. Yadav, S.R Hirekhan, D.R Solanke, "On the Implementation of FIR Filter with Various Windows for Enhancement of ECG signal", International Journal of Engineering Science and Technology (IJEST), Vol. 3 No. 3 March 2011.

[3] Z. Chik, T. Islam, S.A. Rosyidi, H. Sanusi, M.R. Taha, M.M. Mustafa, "Comparing the Performance of Fourier Decomposition and Wavelet Decomposition for Seismic Signal Analysis", European Journal of Scientific Research ISSN 1450-216X Vol.32 No.3 (2009), pp.314-328.

[4] T.Mourad, A.Aouinet, B.Riadh, and C.Adnane, "ECG Analysis Based on Wavelet Transform", International journal of computer science issues, Vol.9, Issue 1, No 3, January 2012.

[5] J.Zhoo, H. Guo, and X.Sun, "Macrozooplankton Image Edge Detection Using Packet Decomposition", Advanced Materials Research, Vols. 108-111 (2010) pp 44-49.

[6] S.G Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation" IEEE. Transactions on Pattern Analysis and Machine Intelligence, Vol.11 (1989) pp 674-693.

[7] A.Teolis, "Computational Signal Processing With Wavelets", Springer, Birkhauser Engineering, 1998.

[8] H.Salah, T.Mourad, A.Akram and C.Adnane, "A Hybrid Electrocardiogram Denoising Technique Based on Continuous Wavelet Transform Translation Invariant", International Review on Computers and Software, Vol.6 March 2011.

[9] P.S Addision "Wavelet Transform and the ECG": a review. Physiological Measurement, Vol.26 (2005) pp 155-199.

[10] E. Burhan, "Signal and Image Denoising Using Wavelet Transform", Signal and Image Denoising Using Wavelet Transform, Advances in Wavelet Theory and Their Applications in Engineering, Physics and Technology, Dr. Dumitru Baleanu (Ed.), ISBN: 978-953-51-0494-0, InTech

|11] V. S. Chourasia and A. K. Tiwari, "Design Methodology of a New Wavelet Basis Function for Fetal Phonocardiographic Signals", The Scientific World Journal, Vol.2013 (2013).

[12] X. Zhang, Y. Rui and W. Wang, "An New Filtering Methods in the Wavelet Domain for Bowel Sounds", (IJACSA) International Journal of Advanced Computer Science and Applications, Vol. 1, No. 5, November 2010.